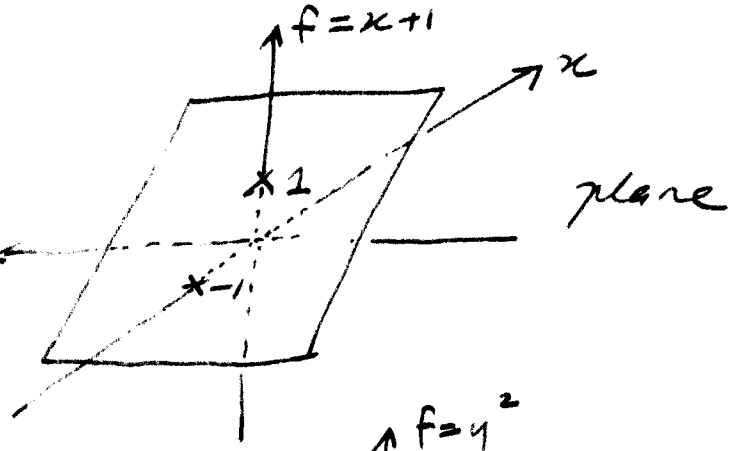
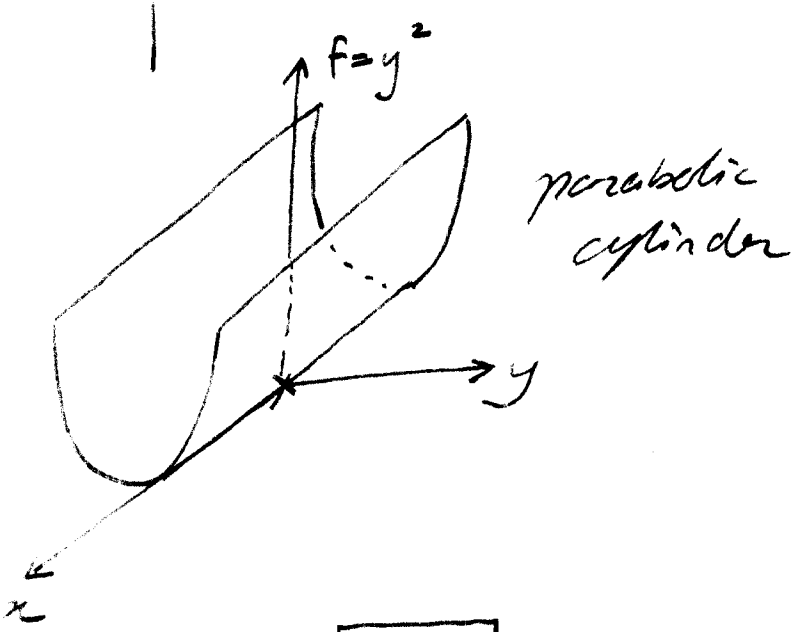
Q1 p 14. 3

2a



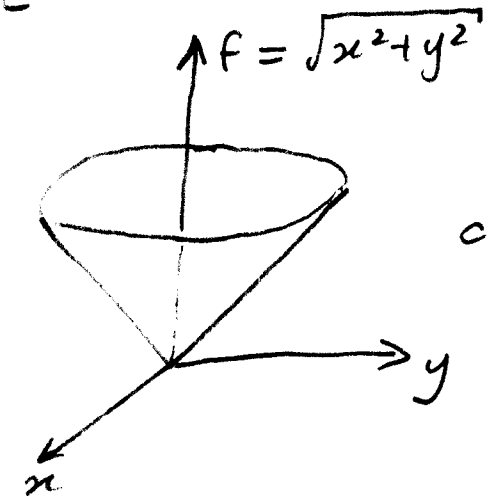
plane

2b



parabolic cylinder

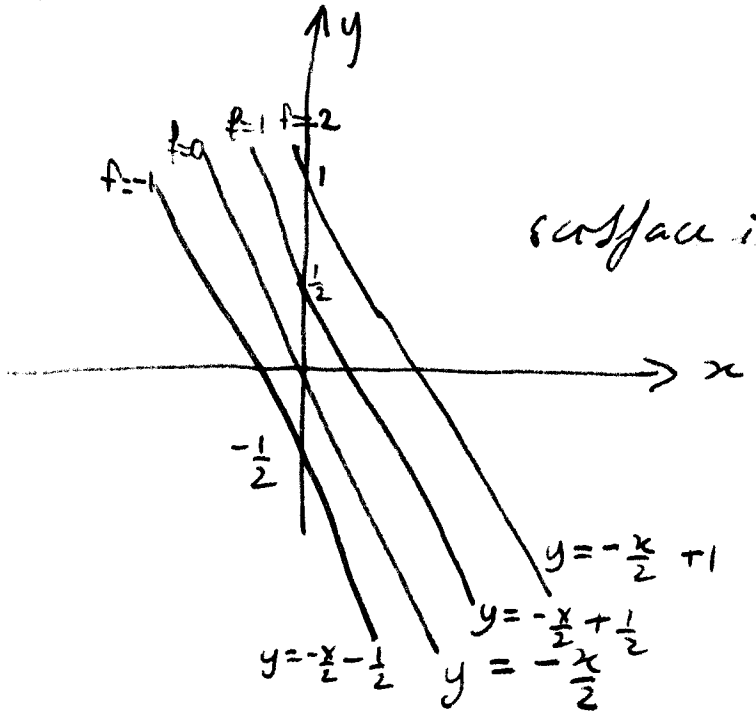
2c



cone - note that the height f is the radius of a circle $x^2+y^2 = f^2$!

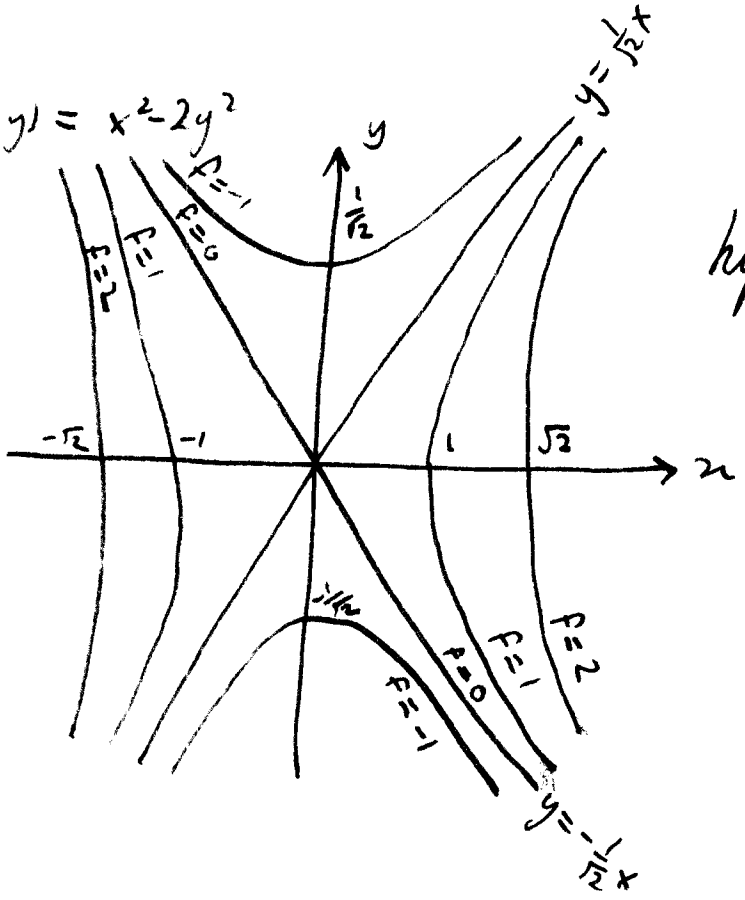
2.12 $f(x,y) = x + 2y$

$f = -1, 0, 1, 2$ all give straight lines

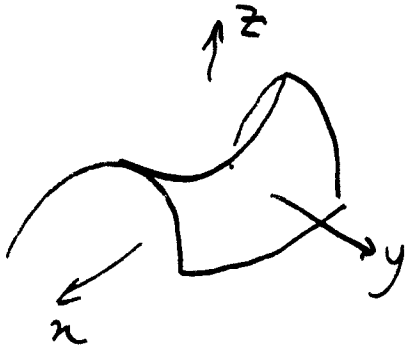


surface is a plane

2.14 $f(x,y) = x^2 - 2y^2$

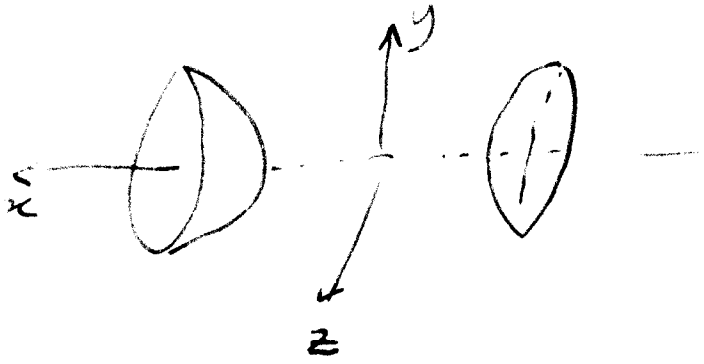


hyperbolas, surface is a saddle



q 20 b) As you get higher, there is less atmosphere to protect you from the "sun's harmful rays"! Hence expect more radiation on the Rocky & Sierra Nevada mountains which explains the "bends".

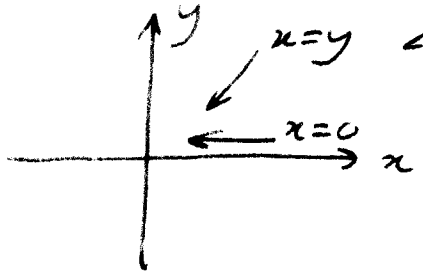
q 22 d) $x^2 - y^2 - z^2 = 1$ is one of our standard forms — a 2-sheeted hyperboloid



Q2 §14.4

g2) $\lim_{(x,y)\to(1,1)} \frac{x^2}{x^2+y^2} = \frac{1}{2}$ (expression is non-singular)

g4) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist because



try approaching along different lines

$x=y \quad \lim_{z\to 0} \frac{x^2}{2x^2} = \frac{1}{2}$

$x=0 \quad \lim_{y\to 0} \frac{0}{y^2} = \lim_{y\to 0} 0 = 0$

g6) $\lim_{(x,y)\to(0,0)} x^{2y}$ does not exist because

$\lim_{(x,0)\to(0,0)} x^{2y} = \lim_{x\to 0} x^0 = \lim_{x\to 0} 1 = 1$

along $y=0$

$\lim_{(0,y)\to(0,0)} x^{2y} = \lim_{y\to 0} 0^y = \lim_{y\to 0} 0 = 0$

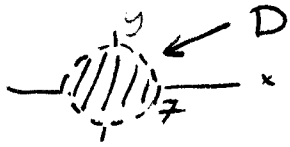
along $x=0$

g8) recall $\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$ (per definition)

q14 $f(x,y) = \dfrac{1}{\sqrt{49 - x^2 - y^2}}$

a) Domain is disc $x^2 + y^2 < 49$

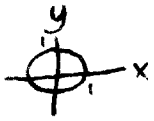
radius 7



b) continuous, $\forall\ (x_0, y_0) \in D$ $\lim_{(y,x)\to(y_0,x_0)} \dfrac{1}{\sqrt{49 - x^2 - y^2}}$

exists (even though it doesn't on the boundary).

q16 ∂R is the unit circle $x^2 + y^2 = 1$



q21 See Steinellas / student manual

q24 $f(x,y) = \dfrac{5x^2 y}{2x^4 + 3y^2}$

a) Domain is $\mathbb{R} \setminus \{0\}$ (i.e. plane excluding the origin where f is singular)

b) $\dfrac{5x^2 \cdot 2x}{2x^4 + 12x^2} = \dfrac{10x}{2x^2 + 12} \xrightarrow[x\to 0]{} 0$

c) $\dfrac{5x^2 3x}{2x^4 + 27x^2} = \dfrac{15x}{2x^2 + 27} \xrightarrow[x\to 0]{} 0$

d) $\dfrac{5x^2 \cdot x^2}{2x^4 + 3x^4} = \dfrac{5}{5} = 1 \xrightarrow[x\to 0]{} 1$ ↯ c) LIMIT DOES NOT EXIST.

Q3 $\frac{d}{dx}(x^x) = \frac{d}{dx}(e^{x \log x})$

since $x = e^{\log x}$

$= e^{x \log x} \frac{d}{dx}(x \log x)$

$= e^{x \log x}(\log x + 1)$

Q4 See lectures!