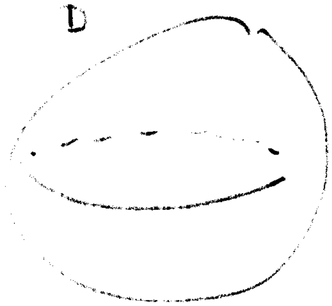
HW 5

Q1 D



∂D is $x^2+y^2+z^2=1$

<u>Fix x, y</u> $\quad -\sqrt{1-x^2-y^2} < z < \sqrt{1-x^2-y^2}$

<u>Fix x</u> $\quad -\sqrt{1-x^2} < y < \sqrt{1-x^2}$

$$\Rightarrow \quad = \int_{-1}^{1} dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} dz$$

$$= 2\int_{-1}^{1} dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy \sqrt{1-x^2-y^2}$$

$$= 2\int_{-1}^{1} dx \sqrt{1-x^2} \int_{-\pi/2}^{\pi/2} d(\cos\theta) \sqrt{1-x^2} \sqrt{1-\cos^2\theta}$$

call $y = \sqrt{1-x^2}\cos\theta$

$$= +2\int_{-1}^{1} dx \left(1-x^2\right) \underbrace{\int_{-\pi/2}^{\pi/2} d\theta \sin^2\theta}_{\pi/2}$$

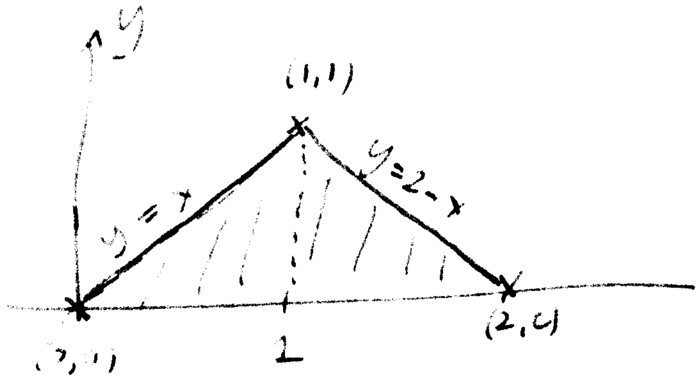
$$= \pi \underbrace{\left[x - \tfrac{1}{3}x^3\right]_{-1}^{1}}_{4/3}$$

$$= 4/3\, \pi$$

Q2 See lectures

Q3 §15.2

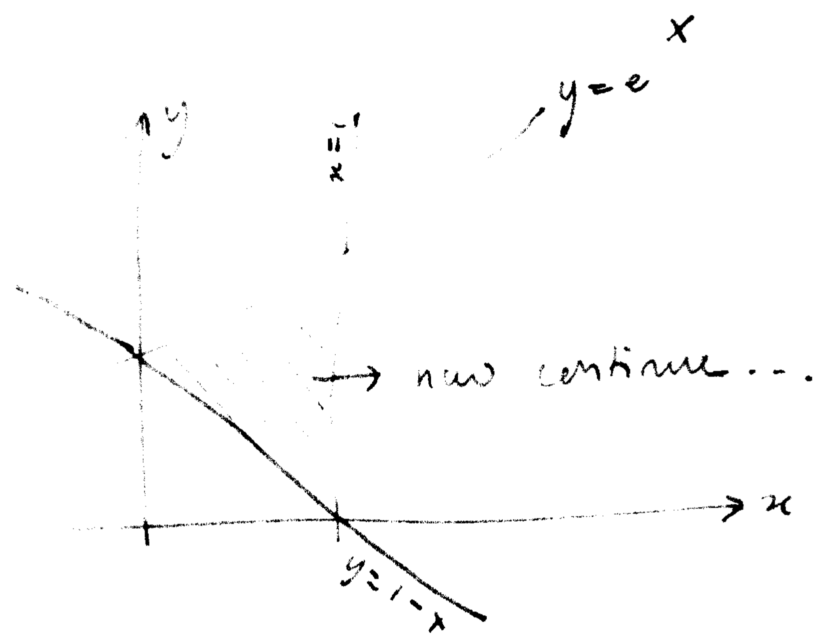
9)



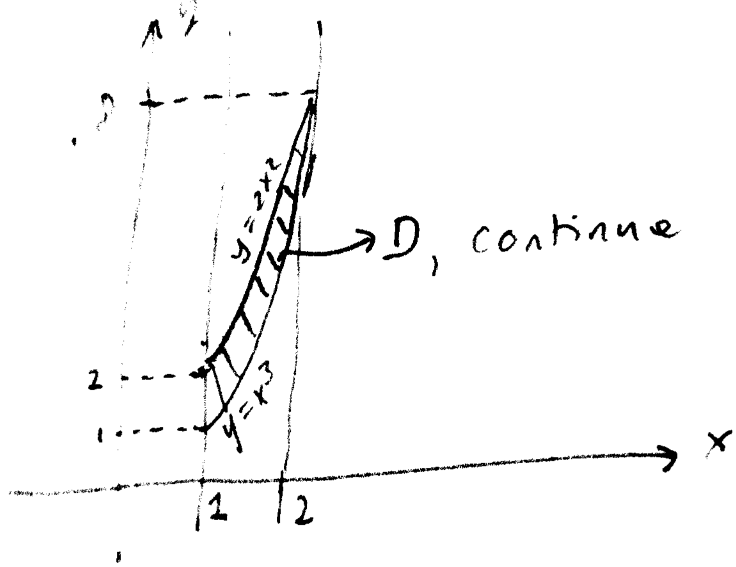
i) fix $\begin{cases} 0 < x < 1 \Rightarrow 0 < y < x \\ 1 < x < 2 \Rightarrow 0 < y < 2 - x \end{cases}$

ii) fix $0 < y < 1 \Rightarrow y < x < 2 - y$

10)



q 14

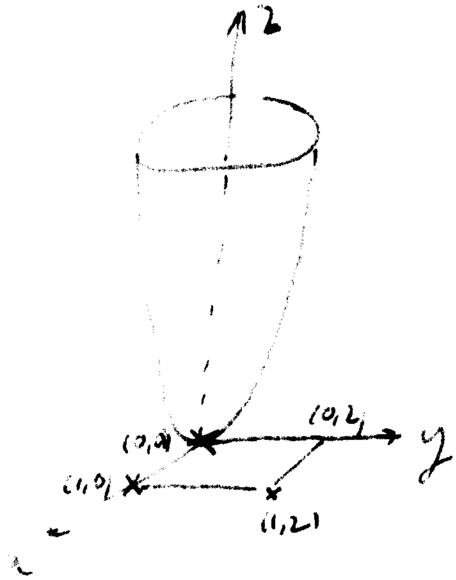


q 18

$$\int_1^2 dx \int_x^{2x} dy = \int_1^2 dx \left[y \right]_x^{2x} = \int_1^2 dx \left[2x - x \right] = \int_1^2 dx$$

$$= 2 - 1 = 1$$

q 24



$$\int_0^1 dx \int_0^2 dy \, (x^2 + 2y^2)$$

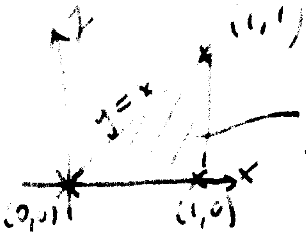
$$= \int_0^1 dx \left[x^2 y + \frac{2}{3} y^3 \right]_0^2$$

$$= \int_0^1 dx \left[2x^2 + \frac{16}{3} \right]$$

$$= \left[\frac{2}{3} x^3 + \frac{16}{3} \right]_0^1$$

$$= \frac{2}{3} + \frac{16}{3} = \frac{18}{3} = 6$$

92.8



$$\rho = \frac{1}{1+x^2}$$

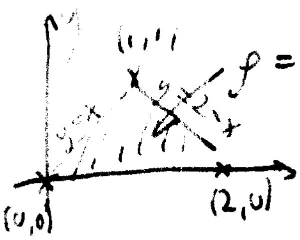
$$m = \int_0^1 dx \int_0^x dy \, \frac{1}{1+x^2}$$

$$= \int_0^1 \frac{dx \cdot x}{1+x^2} = \frac{1}{2}\int_0^1 \frac{d(x^2)}{1+x^2}$$

$$= \frac{1}{2}\Big[\log(1+x^2)\Big]_0^1 = \frac{1}{2}\log 2.$$

Q4 § 15.3

§ 7



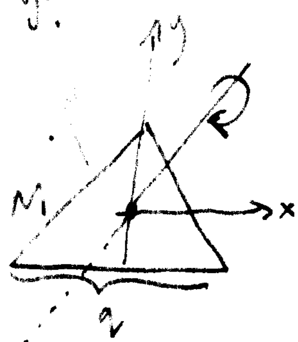
$\rho = y$

$$(\bar{x},\bar{y}) = \left(\frac{\int_0^1 dy \int_y^{2-y} dx \, x \cdot y}{\int_0^1 dy \int_y^{2-y} dx \cdot y} \; , \; \frac{\int_0^1 dy \int_y^{2-y} dx \, y^2}{\int_0^1 dy \int_y^{2-y} dx \, y} \right)$$

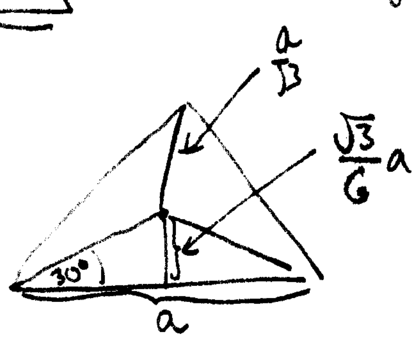
$$= \left(1, \frac{1}{2}\right)$$

$y = \sqrt{3}x + \frac{\sqrt{3}}{3}$

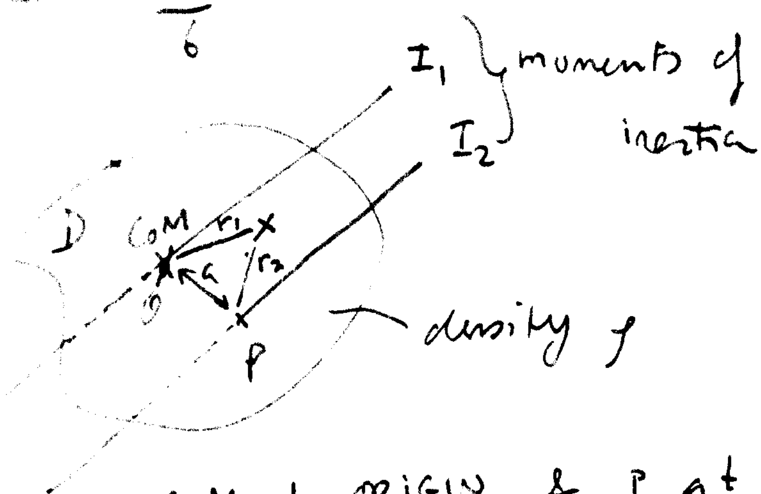
9 10



$$I = M \int_{-\frac{\sqrt{3}a}{6}}^{\frac{\sqrt{3}a}{3}} dy \int_{\frac{\sqrt{3}y}{3} - \frac{a}{3}}^{-\frac{\sqrt{3}y}{3} + \frac{a}{3}} dx \, (x^2 + y^2) = \frac{Ma^2}{12}$$



§16 $I = \frac{Ma^2}{6}$



put CoM at ORIGIN & P at $(0,a)$ say

$$I_2 = \int_D dx\,dy\,\left(x^2+(y-a)^2\right)\rho(x,y)$$

$$I_1 = \int_D dx\,dy\,(x^2+y^2)\,\rho(x,y)$$

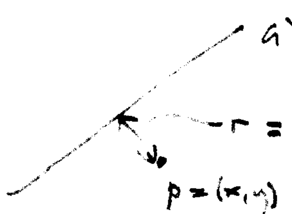
But $I_2 = \int_D dx\,dy\,(x^2+y^2-2ay+a^2)\rho$

$$= I_1 - 2a\underbrace{\int_D dx\,dy\,y\rho}_{0} + a^2\underbrace{\int_D dx\,dy\,\rho}_{M}$$

b/c center of mass is at $(0,0)$

$\Rightarrow \quad I_2 = I_1 + a^2 M$

§20 b)



$$r = \frac{|ax+by|}{\sqrt{a^2+b^2}}$$ (see §12.4)

Q5 $I_{sphere} = \frac{2}{5} M R^2$ $M =$ Mass

$R =$ Radius

$M = 6 \times 10^{24}$ kg

$R = 6 \times 10^6$ m

$\omega = 2\pi / 24.60.60 \doteqdot 7 \times 10^{-5}$ rad/sec

$E = \frac{1}{2} \left(\frac{2}{5} M R^2 \right) \omega^2$

$= \frac{1}{2} \frac{2}{5} \cdot 6 \times 10^{24} (6 \times 10^6)^2 (7 \times 10^{-5})^2$

$= \frac{1}{2} \frac{2}{5} 6^3 7^2 \times 10^{22} \doteqdot 2 \times 10^{25}$ JOULES

$E_{nuclear} \doteqdot 100 \times 10^9$ Joules

$\rightsquigarrow$ would be pretty hard to stop the earth spinning!