HW #7

Q1 §10.1

2 $\frac{1}{e} \doteq .3678794412\ldots$

$1 + (-1) + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \frac{(-1)^4}{4!} = 1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} = .375$

$1 + (-1) + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \frac{(-1)^4}{4!} + \frac{(-1)^5}{5!} = .375 - \frac{1}{120} = .366666\ldots$

$\doteq .367$

10 $\sum \frac{1}{n^{\frac{1}{3}}}$ is a divergent p-series.

12 See derivation in class

14 Use $\int_0^x \frac{dt}{1+t^2} = \tan^{-1} x$

§ 2 , § 10.2

q 2 $\lim_{n\to\infty} (1.01)^n$ does not exist

q 10 $\lim_{n\to\infty} \frac{e^n}{n!} = 0$

q 11 $\lim_{n\to\infty} \frac{(-1)^n}{n} = 0$

q 12 $\lim_{n\to\infty} \left(\frac{n-1}{n}\right)^n = \cancel{\lim \left(\frac{n}{n+1}\right) = \lim \frac{1}{\ldots}}$

$= \lim_{n\to\infty} \left(1 - \frac{1}{(-n)}\right)^n$

$= \lim_{n\to\infty} \frac{1}{\left(1 - \frac{1}{(-n)}\right)^{-n}} = \frac{1}{e}$

q 14 $\dfrac{\frac{11.8^{n+1}}{(n+1)!}}{\frac{11.8^n}{n!}} = \frac{11.8}{n+1} \begin{cases} > 1 & n > 10 \\ < 1 & n \leq 10 \end{cases}$

$\Rightarrow$ maximum is $\frac{11.8^{11}}{11!} \approx 15472$

2a) $\lim_{n\to\infty} \frac{\sin n}{n} = 0$

Proof Let $\varepsilon > 0$. Choose $N > \frac{1}{\varepsilon}$.

since $\left|\frac{\sin n}{n} - 0\right| \leq \frac{|\sin n|}{|n|} \leq \frac{1}{n} < \frac{1}{N}$ when

$n > N$ so $\left|\frac{\sin n}{n}\right| < \varepsilon$ QED.

2b) $\lim_{n\to\infty} \frac{3}{n^2} = 0$ Choose $N > \sqrt{3/\varepsilon}$

since $\left|\frac{3}{n^2}\right| < \frac{3}{N^2} < \frac{3}{(\sqrt{3/\varepsilon})^2} = \varepsilon$ QED

for $n > N$

3) Choose $\varepsilon = \frac{1}{2}$ Then $|(-1)^n - 0| = 1 > \varepsilon \quad \forall n$

QED

Q.3 , p. 3

1. 2 $\lim_{n\to\infty} a_n = 6$

a) FALSE , $\sum_{n=1}^{\infty} a_n$ diverges by n^{th} term test

b) FALSE

c) TRUE see a

d) FALSE

Q. 4 $\sum_{n=1}^{\infty} \frac{(-1)^n 5}{2^n}$

a) $\frac{(-1)^4 5}{2^4} = \frac{5}{16} = .3125$

b) $-\frac{5}{2} + \frac{5}{4} - \frac{5}{8} + \frac{5}{16} = -1.5625$

c) $\lim_{n\to\infty} \frac{(-1)^n 5}{2^n} = 0$

d) $\sum_{n=1}^{\infty} \frac{(-1)^n 5}{2^n} = 5\left(\frac{1}{1+\frac{1}{2}} - 1\right) = -\frac{5}{3}$

geometric series

9. $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} \cdots = \frac{1}{1+\frac{1}{3}} = \frac{3}{4}$

21st $\sum \frac{1}{(1+\frac{1}{n})^n}$ diverges by

n^{th} Term Test since $\lim_{n\to\infty} \frac{1}{(1+\frac{1}{n})^n} = \frac{1}{e} \neq 0$

22nd $\sum \left(\frac{1}{4^n} + \frac{1}{n}\right)$ diverges since $\sum \frac{1}{n}$ diverges.

23rd

transaction #	\$
1	1
2	$1 + .8$
3	$1 + .8 + .8 \times .8$
$\vdots$	
n	$1 + .8 \cdots + .8^{n-1}$

Geometric series, sum is $\frac{1}{1-.8} = 5$

2+

$$\int_0^\infty \frac{dx\, e^{-x/R}}{1+x} = \int_0^\infty dx\, e^{-x/R}\left(1 - x + x^2 - x^3 \cdots\right)$$

illegal b/c $\frac{1}{1+x} = 1 - x + x^2 - x^3 \cdots$ valid only for $|x|<1$ but we integrate over $0 \leq x < \infty$

$$= \sum_{n=0}^{\infty} \int_0^\infty dx\, e^{-x/R} (-x)^n$$

$$= \sum_{n=0}^{\infty} n!\,(-)^n R^{n+1}$$

diverges b/c $\lim_{n\to\infty} n!\,R^{n+1} = \infty$ nth term test

But approximation

$$\int_0^\infty \frac{dx\, e^{-x/R}}{1+x} \doteq R - R + 2!\,R^2 - 3!\,R^3 + 4!\,R^4 + \cdots$$

$= .0099019424$ VERY ACCURATE!

25 ... Lectures