HW #3

§1 §10.4

Q2 $\sum_{n=1}^{\infty} \frac{1}{n^{.9}}$ $\quad f(x) = \frac{1}{x^{.9}}$

but $\int_1^{\infty} dx\, f(x) = \left. \frac{x^{1-.9}}{1-.9} \right|_1^{\infty}$ diverges.

integral diverges so $\sum_{n=1}^{\infty} \frac{1}{n^{.9}}$ DIVERGES

Q6 $\sum \frac{1}{n+1000} \Rightarrow f(x) = \frac{1}{x+1000}$

but $\int_1^{\infty} \frac{dx}{x+1000} = \left. \log(x+1000) \right|_1^{\infty}$ diverges

$\Rightarrow \sum \frac{1}{n+1000}$ DIVERGES

Q10 $\sum \frac{1}{n^p}$ converges for $p > 1$

$\Rightarrow \sum \frac{1}{n^3}$ has $p = 3 > 1$ CONVERGENT

Q12 $p = .9992 < 1 \Rightarrow \sum \frac{1}{n^{.9992}}$ is DIVERGENT

2.16 $\underset{=}{\text{i)}}$ $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90} = 1.082323234\ldots$

ii) $1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} \doteq 1.0788$

iii) $\int_5^{\infty} \frac{dx}{x^4} < R_4 < \int_4^{\infty} \frac{dx}{x^4}$

$\Rightarrow \frac{1}{3.5^3} < R_4 < \frac{1}{3.4^3}$

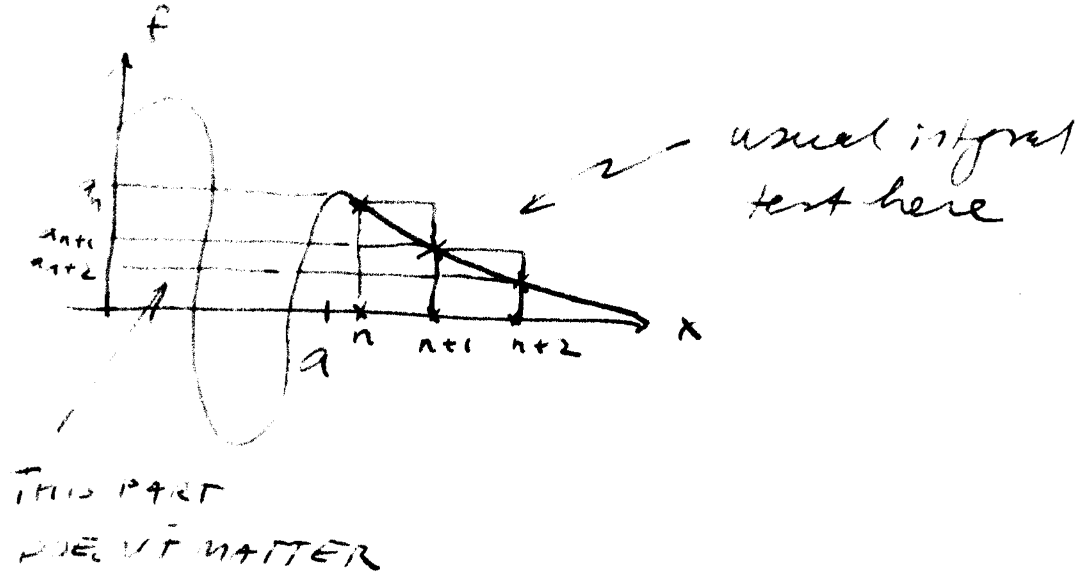
$\Rightarrow 0.0027 < R_4 < 0.0052$

$\Rightarrow 1.0788 + .0027 < \sum_{n=1}^{\infty} \frac{1}{n^4} < 1.0788 + .0052$

$\Rightarrow 1.0815 < \sum_1 \frac{1}{n^4} < 1.0840$

✓ WORKS WELL

9.24



9.3) a)

$$\prod_{i=1}^{n} (1+a_i) = (1+a_1)(1+a_2)\cdots(1+a_n)$$

$$= (1+a_1+a_2+a_1a_2)(1+a_3)\cdots(1+a_n)$$

$$\overset{a_1,a_2>0}{>} (1+[a_1+a_2])(1+a_3)\cdots(1+a_n)$$

$$= (1+[a_1+a_2]+a_3+[a_1+a_2]a_3)(1+a_4)\cdots(1+a_n)$$

$$> (1+[a_1+a_2+a_3])(1+a_4)\cdots(1+a_n)$$

can write out, careful proof by induction here

$$\vdots$$

$$> (1+a_1+a_2+\cdots+a_n)$$

$$> a_1+\cdots+a_n = \sum_{i=1}^{n} a_i$$

b) Obvious :: (use theorem on increasing sequences with upper bounds)

? 32 Let $p_1, \dots, p_m$ be the only primes.

a) $$\frac{1}{1-\frac{1}{p_i}} = 1 + \frac{1}{p_i} + \left(\frac{1}{p_i}\right)^2 + \left(\frac{1}{p_i}\right)^3 + \cdots$$

geometric series

b) $$\frac{1}{1-\frac{1}{p_1}} \, \frac{1}{1-\frac{1}{p_2}} \, \frac{1}{1-\frac{1}{p_3}} \cdots \frac{1}{1-\frac{1}{p_m}}$$

$$= \left(1 + \frac{1}{p_1} + \frac{1}{p_1^2} + \cdots\right)\left(1 + \frac{1}{p_2} + \frac{1}{p_2^2} + \cdots\right) \times \left(1 + \frac{1}{p_3} + \frac{1}{p_3^2} + \cdots\right) \cdots \left(1 + \frac{1}{p_m} + \frac{1}{p_m^2} \cdots\right)$$

$$\left.\begin{aligned} = 1 &+ \frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_m} \\ &+ \frac{1}{p_1^2} + \frac{1}{p_1 p_2} + \frac{1}{p_1 p_3} + \cdots + \frac{1}{p_1 p_m} \\ &\cdots + \frac{1}{p_1 \cdots p_m} \end{aligned}\right\} \text{sum over all prime factorizations}$$

$$= \sum_{n=1} \frac{1}{n} \quad \text{diverges.}$$

$\therefore$ LHS of b) is clearly finite $\lightning$ QED

Q2 §11.5

q1 $\sum_{n=1}^{\infty} \frac{1}{n^2+3}$, $\frac{1}{n^2+3} < \frac{1}{n^2}$ and $\sum \frac{1}{n^2}$

converges $\Rightarrow \sum \frac{1}{n^2+3}$ CONVERGES

q3 $\sum_{n=1}^{\infty} \frac{1}{n2^n}$ $\quad \frac{1}{n2^n} < \left(\frac{1}{2}\right)^n$ and $\sum \frac{1}{2^n}$

converges $\Rightarrow \sum \frac{1}{n2^n}$ CONVERGES

q2 $\sum \frac{n+2}{(n+1)\sqrt{n}}$, $\frac{n+2}{(n+1)\sqrt{n}} > \frac{n+1}{(n+1)\sqrt{n}} = \frac{1}{\sqrt{n}}$

and $\sum \frac{1}{\sqrt{n}}$ diverges $\Rightarrow \sum \frac{n+2}{(n+1)\sqrt{n}}$ DIVERGES

q $\sum \frac{\left(1+\frac{1}{n}\right)^n}{n^2}$. compare to $\sum \frac{1}{n^2}$

$$\lim_{n\to\infty} \frac{\frac{\left(1+\frac{1}{n}\right)^n}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e \quad \underline{\text{exists}}$$

$\Rightarrow \sum \frac{\left(1+\frac{1}{n}\right)^n}{n^2}$ CONVERGES

15. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2}$ $\displaystyle\lim_{n\to\infty} \frac{2^n}{n^2} = \infty$

$\Rightarrow$ DIVERGES by nth Term test.

18. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n^n}$ CONVERGES by comparison with geometric series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5}{6}\right)^n$

30. <u>NOTHING</u> Play with (counter-)examples

$\sum \frac{1}{n}$, $\sum \frac{1}{n^{3/2}}$, $\sum \frac{1}{n^2}$

34. <u>NOTHING</u> $\sum \frac{1}{3n}$ diverges as does $\sum \frac{1}{2n}$

while $\sum \frac{1}{n}$ diverges but $\sum \frac{1}{2n^2}$ converges

38 $\sum_{n=1}^{\infty} n^{P} e^{-n}$

convergent via limit comparison test with $\sum_{n=1}^{\infty} e^{-n/2}$, a convergent geometric series.

42 use $\int dx \frac{\log x}{x^2} = -\frac{\log x}{x} - \frac{1}{x}$

to approximate the sum.

The actual answer is $\approx .94$.

q^3 $f(r, \theta) = r$

$$\lim_{p \to 0} f = \lim_{r \to 0} r = 0 \implies f(0) = 0$$

Proof Let $\varepsilon > 0$ and suppose

$$\sqrt{(x-0)^2 + (y-0)^2} < \varepsilon$$

$$\implies r < \varepsilon$$

$$\implies |f(r, \theta) - 0| < \varepsilon \quad \text{QED}$$