Last Initial C

FULL Name Captain Conundrum

Student ID π

22A MIDTERM I

Friday February 19, 2012

PLEASE DISPLAY YOUR STUDENT ID CARD

Declaration of honesty: I, the undersigned, do hereby swear to uphold the VERY highest standards of academic honesty, including, but not limited to, submitting work that is original, my own and unaided by notes, books, calculators, mobile phones, immobile phones, fluffy toys, pet rocks or any other device, electronic or otherwise.

Signature CC **Date** TODAY

Q1	___
Q2	___
Q3	___
Q4	___
Σ	___

Question 1 *(30 points)*

Let

$$M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

(i) Compute the matrices M^2, M^3 and M^4.

(ii) Once you see a pattern, give the formula for M^k for any integer k.

Now, let the matrix

$$A = \begin{pmatrix} 1 & 0 & a & b \\ 0 & 1 & c & d \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

(iii) Find a formula for A^k.

i) $M^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, $M^3 = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$, $M^4 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

$\ldots \quad M^k = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$

ii) $A = \left(\begin{array}{c|c} I & M \\ \hline 0 & I \end{array}\right) \quad \bar{c} \quad M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\Rightarrow A^2 = \left(\begin{array}{c|c} I & 2M \\ \hline 0 & I \end{array}\right), \ldots, A^k = \left(\begin{array}{c|c} I & kM \\ \hline 0 & I \end{array}\right)$

$= \left(\begin{array}{cc|cc} 1 & 0 & ka & kb \\ 0 & 1 & kc & kd \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$

Question 2 *(40 points)*

Consider the system

$$\begin{cases} x + 2y = 1 \\ x + 2y + 2z + 6w = 3 \\ -x - 2y - z - 3w = -2 \\ 2x + 4y + z + 3z = 3 \end{cases}$$

(i) Write an augmented matrix for this system.

(ii) Use elementary row operations to find its reduced row echelon form.

(iii) Write the solution set for the system in the form

$$S = \{X_0 + \sum_i \mu_i Y_i : \mu_i \in \mathbb{R}\}.$$

(iv) What are the vectors X_0 and Y_i called *and* which matrix equations do they solve?

(v) Check separately that X_0 and each Y_i solve the matrix systems you claimed they solved in part (iv).

(i) $\left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 1 \\ 1 & 2 & 2 & 6 & 3 \\ -1 & -2 & -1 & -3 & -2 \\ 2 & 4 & 1 & 3 & 3 \end{array}\right) \underset{R_4-2R_1}{\overset{R_2-R_1,\ R_3+R_1}{\sim}} \left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 2 & 6 & 2 \\ 0 & 0 & -1 & -3 & -1 \\ 0 & 0 & 1 & 3 & 1 \end{array}\right)$

(ii) $\underset{R_4-R_2/2,\ \frac{R_2}{2}}{\overset{R_3+R_4}{\sim}} \left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

GRADERS, PROCEDURE IS NOT UNIQUE GIVE FULL CREDIT IF ARRIVES @ RREF (CORRECT).

$\Rightarrow y = \mu_1,\ w = \mu_2$ arbitrary $x = 1 - 2\mu_1,\ z = 1 - 3\mu_2$

iii) $\Rightarrow S = \left\{ \underset{X_0}{\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}} + \mu_1 \underset{Y_1}{\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}} + \mu_2 \underset{Y_2}{\begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \end{pmatrix}} \,\middle|\, \mu_1, \mu_2 \in \mathbb{R} \right\}$

(iv) X_0 particular sol$^{\underline{n}}$, Y_1, Y_2 homogeneous sol$^{\underline{n}}$

$M = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 1 & 2 & 2 & 6 \\ -1 & -2 & -1 & -3 \\ 2 & 4 & 1 & 3 \end{pmatrix}$ $MX_0 = \begin{pmatrix} 1 \\ 3 \\ -2 \\ 3 \end{pmatrix}$, $MY_1 = 0 = MY_2$ associated homogeneous system

v) Matrix multiply $MX_0 = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 1 & 2 & 2 & 6 \\ -1 & -2 & -1 & -3 \\ 2 & 4 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -2 \\ 3 \end{pmatrix}$ ✓, $MY_1 = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 1 & 2 & 6 & 6 \\ -1 & -2 & -1 & -3 \\ 2 & 4 & 1 & 3 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ ✓, $MY_2 = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 1 & 2 & 2 & 6 \\ -1 & -2 & -1 & -3 \\ 2 & 4 & 1 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ ✓

Question 3 *(30 points)*

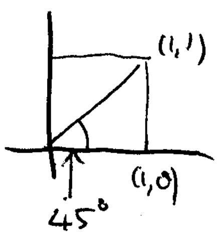
(2) Find the angle between the diagonal of the unit square in $\mathbb{R}^2$ and one of the coordinate axes.

(3) Find the angle between the diagonal of the unit cube in $\mathbb{R}^3$ and one of the coordinate axes.

(n) Find the angle between the diagonal of the unit (hyper)-cube in $\mathbb{R}^n$ and one of the coordinate axes.

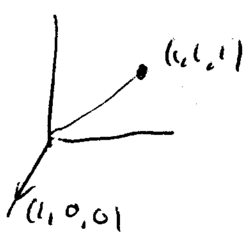
(∞) What is the limit as $n \to \infty$ of the angle between the diagonal of the unit (hyper)-cube in $\mathbb{R}^n$ and one of the coordinate axes?

2)



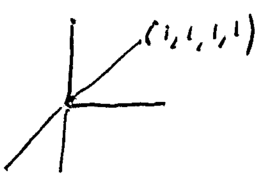
$$\cos^{-1}\frac{(1,0)\cdot(1,1)}{\|(1,0)\|\,\|(1,1)\|} = \cos^{-1}\frac{1}{1\cdot\sqrt{2}} = 45^\circ$$

3)



$$\cos^{-1}\frac{(1,0,0)\cdot(1,1,1)}{\|(1,0,0)\|\,\|(1,1,1)\|} = \cos^{-1}\frac{1}{\sqrt{3}}$$

4)



$$\cos^{-1}\frac{(1,0,0,0)\cdot(1,1,1,1)}{\|(1,0,0,0)\|\,\|(1,1,1,1)\|} = \cos^{-1}\frac{1}{\sqrt{4}} = \cos^{-1}\frac{1}{2} = 30^\circ$$

n)

$$\cos^{-1}\frac{(1,0,\dots,0)\cdot(1,1,1,\dots)}{\|(1,0,\dots)\|\,\|(1,1,\dots)\|} = \cos^{-1}\frac{1}{\sqrt{n}}$$

$n \to \infty$)

$$\cos^{-1}\frac{1}{\sqrt{n}} \xrightarrow{n\to\infty} 0^\circ$$

GRADERS ACCEPT ANSWERS IN DEGREES OR RADIANS

Question 4 *(5 points Extra Credit)*

There are NO wrong answers to the following questions (brief answers preferred, you may leave blanks). Give your opinions and suggestions for improving:

(i) The typeset lecture notes.

Fabulous

(ii) The webwork homework system.

Neat

(iii) The written homeworks.

Fun

(iv) The online office hours.

Mindblowing

(v) The homework collection system.

Efficient

(vi) The clickers and clicker questions.

T-riffic

(vii) The lectures.

Spellbinding

(viii) The course in general.

Teach Conundrum to write neater!