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FULL Name CAPTAIN CONUNDRUM

Student ID π

22A MIDTERM II

Wednesday February 29, 2012

Declaration of honesty: I, the undersigned, do hereby swear to uphold the VERY highest standards of academic honesty, including, but not limited to, submitting work that is original, my own and unaided by notes, books, calculators, mobile phones, pet rocks , slabs of granite with formulas inscribed upon them or any other device (beyond a pen(cil) and eraser), electronic or otherwise.

Signature C^2 Date TODAY

Q1	
Q2	
Q3	
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Question 1

Let M be a square $n \times n$ matrix. For each of the following situations, give a formula for $\det M'$ in terms of $\det M$

(i) M' equals M save that the third and fourth rows have been swapped.

$$\det M' = - \det M$$

(ii) $M' = M^T$.

$$\det M' = \det M$$

(iii) M' is the exactly same as M except that the last column has been replaced by the first column.

$\det M' = 0$ (2 columns ~~row~~ the same |

(iv) $M' = \lambda M$.

$$\det M' = \lambda^n \det M$$

(v) $M' = MN$ where N is some $n \times n$ matrix.

$$\det M' = (\det N) . \det M$$

(vi) M' is obtained from M by applying the row operation $R_3 \rightarrow R_3 + 13\, R_2$.

$$\det M' = \det M$$

To be continued...

Now compute the following determinants: *Hint: Think before you leap!*

$$\det\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}, \qquad \det\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{pmatrix},$$

$$\det\begin{pmatrix}1 & 2 & 3 & 4\\ 5 & 6 & 7 & 8\\ 9 & 10 & 11 & 12\\ 13 & 14 & 15 & 16\end{pmatrix}, \qquad \det\begin{pmatrix}1 & 2 & 3 & & n\\ n+1 & n+2 & n+3 & \cdots & 2n\\ 2n+1 & 2n+2 & 2n+3 & & 3n\\ & \vdots & & \ddots & \vdots\\ n^2-n+1 & n^2-n+2 & n^2-n+3 & \cdots & n^2\end{pmatrix}.$$

$$\det\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix} = 4-6 = \underline{-2}$$

$$\det\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{pmatrix} \underset{\substack{\uparrow\\ R_3-2R_2}}{=} \det\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ -1 & -2 & -3\end{pmatrix} = \underline{0}$$

2 rows the same up to a $-$

$$\det\begin{pmatrix}1 & 2 & 3 & 4\\ 5 & 6 & 7 & 8\\ 9 & 10 & 11 & 12\\ 13 & 14 & 15 & 16\end{pmatrix} \underset{R_3-2R_2}{=} \det\begin{pmatrix}1 & 2 & 3 & 4\\ 5 & 6 & 7 & 8\\ -1 & -2 & -3 & -4\\ 13 & 14 & 15 & 16\end{pmatrix} = \underline{0}$$

$$\det\begin{pmatrix}1 & 2 & 3 & & n\\ n+1 & n+2 & \cdots & & \\ 2n+1 & 2n+2 & \cdots & & \\ \vdots & & & & \end{pmatrix} \underset{R_3-2R_1}{=} \det\begin{pmatrix}1 & 2 & 3 & \cdots & \\ n+1 & n+2 & \cdots & & \\ -1 & -2 & \cdots & & \\ & & & & \end{pmatrix} = \underline{0}$$

Question 2

Define what it means for:

(i) Vectors $v_1, v_2, \ldots, v_n$ to be linearly independent.

The only solution to $\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = 0$ is $\alpha_1 = \alpha_2 = \cdots = \alpha_n = 0$

(ii) Vectors $v_1, v_2, \ldots, v_m$ to span a vector space V.

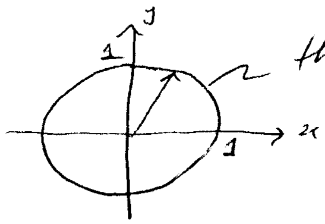
$V = \{\alpha_1 v_1 + \cdots + \alpha_m v_m \mid \alpha_i \in \mathbb{R}\}$

(iii) Vectors $v_1, v_2, \ldots, v_q$ to be a basis for a vector space V.

$v_1, \ldots, v_q$ are linearly independent & span V.

Now try this problem:

(a) Draw the collection of all unit vectors in $\mathbb{R}^2$.



they all lie on the unit circle.

(b) Let $S_x = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, x \right\}$ where x is a unit vector (*i.e.*, $||x|| = 1$) in $\mathbb{R}^2$. For which unit vectors x is S_x a basis for $\mathbb{R}^2$? *Briefly explain your answer*

Any unit vector save for $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$, because we need 2 linearly independent vectors so choices $\parallel$ to $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ are illegal.

Question 3

Let

$$M = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}.$$

Compute the eigenvalues and associated eigenvectors of M.

$$\det \begin{pmatrix} 1-\lambda & 2 \\ 3 & 6-\lambda \end{pmatrix} = (\lambda-1)(\lambda-6) - 6 = \lambda(\lambda-7)$$

$$\lambda = 0, 7$$

$\underline{\lambda = 0}$ $\quad \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \Rightarrow y = 1, \ x = -2$

$$\underline{\lambda = 0}, \quad \underline{\begin{pmatrix} -2 \\ 1 \end{pmatrix}}$$

$\underline{\lambda = 7}$ $\quad \begin{pmatrix} -6 & 2 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \Rightarrow y = 1, \ x = \frac{1}{3}$

$$\underline{\lambda = 7}, \quad \begin{pmatrix} \frac{1}{3} \\ 1 \end{pmatrix}$$

To be continued...

Let k be any positive integer. What are the eigenvalues and associated eigenvectors of M^k? *Include a brief explanation of your result.*

$0, \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ & $7^k, \begin{pmatrix} \frac{1}{3} \\ 1 \end{pmatrix}$

because of $Mv = \lambda v$, $M^k v = M^{k-1} M v$
$= M^{k-1} \lambda v = \cdots = \lambda^k v$

so keep the same eigenvectors and set eigenvalue λ^k.

Now suppose that the matrix N is nilpotent. *I.e.*

$$N^k = 0$$

for some positive integer $k \geq 2$. Show that zero is the only possible eigenvalue for N.

Now suppose $Nv = \lambda v$

$\Rightarrow N^k v = \lambda^k v = 0$

Thus $\lambda^k v = 0$ $v \neq 0$ (always for an eigenvector.)

$\Rightarrow \lambda^k = 0 \Rightarrow \lambda = 0$ QED

Poll:

Would you prefer online lecture notes or a commercial textbook for this course?
(Circle one, your choice will not affect your grade for this test/course in any way.)

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Comments:

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