Undergraduate Thesis in Applied Mathematics

University of California, Davis

Modeling a Multipolar Arms Race

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Abstract

Motivated by the Cold War of the 20th Century, mathematical models of arms races focused on bipolar conflicts. These models do not fit to todays multipolar world consisting of the US with NATO, Russia, and China. In this paper, we extend the bipolar arms race model of Richardson (1960) to a multipolar arms race model and relate stability in a bipolar world to stability in a multipolar world. We show that every bipolar stable solution can be extended to a multipolar stable solution. Moreover, there are multipolar stable solutions that when restricted to the biopolar case cease to be stable. We also discuss an alternative interpretation of the model as arms race with an additional novel weapon system.

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1 Acknowledgements

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2 Introduction

Prior to the conclusion of the Cold War, the world had been contemplated as only bipolar - having only two major superpowers that drove world politics and warfare. Previous mathematical modeling of warfare has followed this trend, seen in Richardson (1960) and further applications of his work, such as Smoker (1964). There is substantial literature devoted to discussing the bipolar world, surveyed in O'Neill (1994) and Brito and Intriligator (1995). However, current policy demonstrates that we no longer reside in a bipolar world, especially not bipolar warfare. Biden's (2021) "Interim National Security Strategic Guidance" mentions several countries deemed as aggressive, paying special attention to Russia and China, saying "We face a world of rising nationalism, receding democracy, growing rivalry with China, Russia, and other authoritarian states, and a technological revolution that is reshaping every aspect of our lives." Additionally, the prevalence international organizations such as United Nations and NATO guarantee that a multi-polar scope is needed for future mathematical modeling. Current events also demonstrate the ongoing prevalence of warfare, particularly discussing the War on Ukraine. This motivation prompts us to look at an arms race through this multi-polar lens, and to see how the stability of the arms race is impacted, as well as look at the transition periods when nations are joining an pre-existing arms race.

3 Model

3.1 Original Richardson Equations

We first will discuss the Richardson (1960) modeling of an arms race. These equations account for two nations represented by x and y respectively. They assume that growth rate of arms depends on the amount of armament (eg. bombs, guns, nuclear weapons) possessed by the opposing sides, the cost of the armaments, as well as the attitudes towards the other side. These assumptions give us the following two coupled ordinary differential equations:

$$\frac{dx}{dt} = ky - \alpha x + g$$
$$\frac{dy}{dt} = \ell x - \beta y + h$$

Here, $\frac{dx}{dt}$ and $\frac{dy}{dt}$ represent the rate at which nations began to arm themselves over time, with x representing the country A's armament (number of weapons), and y representing country B's. The constants k and ℓ represent each country's defense coefficient, which is essentially the inclination of each nation to arm defensively arm itself due to a large amount of weaponry possessed by the other side. Additionally α and β represent each nation's expense coefficient, which is defined by how much the government could fiscally afford as well as the fatigue they would acquire from sustaining such a large arsenal. This coefficient is negative as it is an depreciation factor, one that decreases that nation's spending capabilities. Finally, g and h are the attitudes of each nation to the opposing nation, such as how they would interact in the context of treaties and trade.

3.2 Discrete Time Version of Richardson Model

To make our model, we assume time is discrete, as opposed to Richardson's continuous model. We make this adjustment due to the nature of arms races, and how they tend to have periods within them, down to a fiscal spending budget. Additionally, we allow multiple countries to join this arms race. Our arms race begins with country A and country B, but over time can be expanded to any number of countries.

3.2.1 Bipolar Model

At $t = 1$, we have two countries, each of whom is in possession of a powerful weapon, presumably nuclear technology. Now, before we are able to add in more countries, we must adapt our Richardson equations to discrete time.

We say that our Richardson equation can be represented as the following

$$\Delta w^t = \begin{pmatrix} -\alpha & \kappa \\ \ell & -\beta \end{pmatrix} w^t + \begin{pmatrix} g \\ h \end{pmatrix}$$

This allows us to present our difference equation in its standard form

$$\boldsymbol{w}^{t+1} = \boldsymbol{C}\boldsymbol{w}^t + \boldsymbol{k}$$

where

$$\boldsymbol{w}^t = \begin{pmatrix} w_A^t \\ w_B^t \end{pmatrix}$$

w_A^t denotes country A's weapons stock in period t and

$$\boldsymbol{k} = \begin{pmatrix} k_A \\ k_B \end{pmatrix} \qquad \boldsymbol{C}_1 = \begin{pmatrix} c_A^A & c_B^A \\ c_A^B & c_B^B \end{pmatrix}$$

To reiterate from our derivation, here c_B^A and c_A^B represent each country's defense coefficient, or their inclination to arm themselves with weapons due to the opposing country's stock of weapons. c_B^B and c_B^B represent each country's expense coefficient, or their fatigue coefficient to maintain their stock of weapons. Finally, k is equivalent to the nations feelings towards each other at time t.

3.2.2 Multipolar Model

This system will continue, until a new country is able to innovate the same technology as countries A and B. We expand our difference equations to accommodate for these countries, allowing them to join the arms race by doing the following:

$$\boldsymbol{w}^t = \begin{pmatrix} w_A^t \\ w_B \\ \vdots \\ w_Z^t \end{pmatrix}.$$

So our system in its entirety is given by a linear system of inhomogenous difference equation

$$\boldsymbol{w}^{t+1} = \boldsymbol{C}\boldsymbol{w}^t + \boldsymbol{k}$$

where

$$\boldsymbol{k} = \begin{pmatrix} k_A \\ k_B \\ \vdots \\ k_Z \end{pmatrix}$$

$$\boldsymbol{C}_i = \begin{pmatrix} c_A^A & c_B^A & \dots & c_Z^A \\ c_A^B & c_B^B & \dots & c_Z^B \\ \vdots & \vdots & \ddots & \vdots \\ c_A^Z & c_B^Z & \dots & c_Z^Z \end{pmatrix}$$

For our paper, we will focus on the tripolar model, which describes countries A, B and D

3.3 Restrictions

While analyzing our model, we will apply some restrictions for the sake of simplicity. All diagonal elements will be negative, because these are fatigue coefficients, the weight of maintaining ones own armory. As we are only looking at the bipolar and tripolar case, this simply means $c_A^A < 0, c_B^B < 0$ and $c_D^D < 0$. We will assume symmetry, where $c_B^A = c_A^B$, as country's responses should be proportional to each other. Additionally, we will assume within our matrix $\boldsymbol{k}$, $k_A = k_B$, and $k_A = k_B = \cdots = k_Z$. Hence, we will be assuming that each county has equivalent attitudes towards each other, hostile or amiable. For our analysis, we will be looking at the following two cases, discussing solutions and understanding the dynamics of the shift from t_0 to t_1.

$$C_2 = \begin{pmatrix} c_A^A & c_B^A \\ c_B^A & c_B^B \end{pmatrix}$$

at T = t_0 and

$$C_3 = \begin{pmatrix} c_A^A & c_B^A & c_D^A \\ c_B^A & c_B^B & c_D^B \\ c_D^A & c_D^B & c_D^D \end{pmatrix}$$

at T = t_1

4 Analysis

4.1 Bipolar Model

We begin by discussing our simpler case. Additionally, please note that all solutions must be real numbers, as we are working with a real-world system. Hence, we have the following case:

$$C_2 = \begin{pmatrix} c_A^A & c_B^A \\ c_B^A & c_B^B \end{pmatrix}$$

We are able to find stable solutions when Determinant(C_2) > 0 and Trace(C_2) < 0. This occurs at several conditions, which describes a stable state for our system.

We denote this stable set as follows:

Say

$$S_{2x2} = \left\{ (c_A^A, c_B^A, c_B^B) \in \mathbb{R} : \begin{pmatrix} c_B^B > 0, 0 < c_A^A < c_B^B, -\sqrt{c_A^A c_B^B} < c_B^A < \sqrt{c_A^A c_B^B} \vee \\ c_B^B > 0, c_A^A = c_B^B, -\sqrt{c_A^A c_B^B} < c_B^A < 0 \vee \\ c_B^B > 0, c_A^A = c_B^B, 0 < c_B^A < \sqrt{c_A^A c_B^B} \vee \\ c_B^B > 0, c_A^A < c_B^B, -\sqrt{c_A^A c_B^B} < c_B^A < \sqrt{c_A^A c_B^B} \end{pmatrix} \right\}$$

We also can conclude that we will have a unstable solutions when these conditions are not met, and that we have both unstable nodes and saddle nodes as possible solutions. This set will be described as U_{2x2}.

We can also rewrite our system in such a way that allows us to use Rsolve and LinearSolve in Mathematica to solve the equation recursively. To do this, we do the following

$$\begin{aligned} \boldsymbol{w}^{t+1} &= \boldsymbol{C}\boldsymbol{w}^t + \boldsymbol{k} \\ \boldsymbol{w}^{t+1} &= (\boldsymbol{B} + \mathbb{1})\boldsymbol{w}^t + \boldsymbol{k}\boldsymbol{w}^{t+1} - \boldsymbol{w}^t = \boldsymbol{B} + \boldsymbol{k} \end{aligned}$$

Here, $\boldsymbol{B} = C_2 - \mathbb{1}$, where $\mathbb{1}$ is the Identity Matrix. Our our new equation reads as

$$\boldsymbol{w}_2^t = \begin{pmatrix} w_A^t \\ w_B \end{pmatrix} = \begin{pmatrix} 1 - c_A^A & c_B^A \\ c_B^A & 1 - c_B^B \end{pmatrix} + \begin{pmatrix} k \\ k \end{pmatrix}$$

By using Linearsolve on $\boldsymbol{w}_2^t$, we find a steady state emerges (given that $c_A^A, c_B^A, c_B^B \in S_{2x2}$) at

$$w_A^t = \frac{-c_B^A k - c_B^B k}{c_B^{A2} - c_A^A c_B^B} \qquad w_B^t = \frac{-c_A^A k - c_B^A k}{c_B^{A2} - c_A^A c_B^B}$$

Here, we show an example for stable parameters

$$c_A^A = .4, c_B^A = .15, c_B^B = .5, k = 100$$

. We can graph this using Mathematica to show it reach the steady state of $w_A^t = 366.197$ and $w_B^t = 309.859$, which reaches relatively quickly over time.

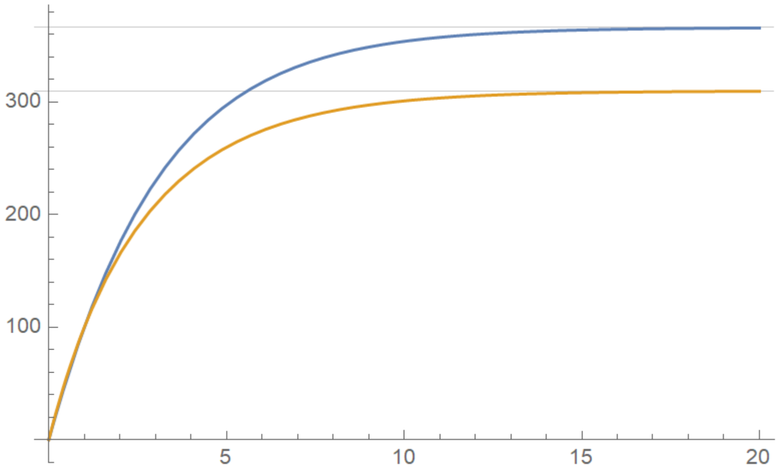

Figure 1: Stead State Described Above

The real world understanding of the stable solutions makes sense as well. As long as both country A and B are producing relatively similar, smaller amounts of armaments and the impact of weapons being made by both countries is below a certain threshold, then the arms race should not proliferate, and instead reach a stable-state. However, if we do not have this pattern of behavior, instead we will see a spike in armaments,resulting in an instability between the two counties and spiraling into proliferation.

4.2 Tripolar Model

We will now be working with the following matrix, which occurs after another country, country D has entered the arms race.

$$C_2 = \begin{pmatrix} c_A^A & c_B^A & c_D^A \\ c_B^A & c_B^B & c_D^B \\ c_D^A & c_D^B & c_D^D \end{pmatrix} \tag{1}$$

From this form, we will again focus on our stable solutions, here dubbed S_{3x3}, which occur when $c_A^A, c_B^A, c_B^B, c_D^A, c_D^B, c_D^D$ lead to both a stable 3x3 system. As this matrix has significantly more variables, the solutions are more complex. We show the conditions fully described in the Appendix, let us call this set of conditions $\mathbb{Y}$ and our stable set S_{3x3}, where

$$S_{3x3} := \left\{(c_A^A, c_B^A, c_D^A, c_B^B, c_D^B, c_D^D,) \in \mathbb{R} : \mathbb{Y}\right\}$$

We denote the $proj_{\mathbb{R}^3}\mathbf{S}_{3x3}$ as the projection of stable solutions of the 3x3 matrix onto parameter of the 2x2 matrix, $((c_A^A, c_B^A, c_B^B)$, which allows us to state the following theorem.

Theorem 1. *We can say* $S_{2x2} \subset proj_{\mathbb{R}^3}\mathbf{S}_{3x3}$ *but* $proj_{\mathbb{R}^3}\mathbf{S}_{3x3} \not\subset S_{2x2}$.

Proof. To show the first part, we can show that there are conditions S_{2x2} that exist within $proj_{\mathbb{R}^3}\mathbf{S}_{3x3}$. We can solve these conditions easily in Mathematica, which tells us as long as one of the following conditions is met, both our 3x3 and 2x2 matrix will be stable. These conditions are the following:

$$\begin{aligned}
&c_B^B > 0, 0 < c_A^A < c_B^B, c_A^A, -\sqrt{c_A^A c_B^B} < c_B^A < \sqrt{c_A^A c_B^B}, -c_A^A - c_B^B < c_D^D < \mathbb{X} \\
&c_B^B > 0, c_A^A = c_B^B, \sqrt{c_A^A c_B^B} < c_B^A < 0, -c_A^A - c_B^B < c_D^D < \mathbb{X} \\
&c_B^B > 0, c_B^A < \sqrt{c_A^A c_B^B}, -c_A^A - c_B^B < c_D^D < \mathbb{X} \\
&c_B^B > 0, c_A^A > c_B^B, -\sqrt{c_A^A c_B^B} < c_B^A < \sqrt{c_A^A c_B^B}, -c_A^A - c_B^B < c_D^D < \mathbb{X}
\end{aligned}$$

note: $\mathbb{X} = \frac{c_B^B c_D^{A^2} - 2c_B^A c_D^A c_D^B + c_A^A c_D^{B^2}}{-c_B^{A^2} + c_A^A c_B^B}$

To prove the second part, we show one of our stability conditions for S_{3x3}. In this case, we will use the condition that states

$$c_A^A = 0, c_B^B \leq 0, c_D^D > -c_B^B, c_B^A < 0, c_D^A = 0, c_A^A < 0, \frac{c_B^{A^2}}{c_A^A} < c_B^B < \frac{-c_A^{A^2} - c_D^{A^2}}{c_A^A}$$

However, we see for this condition, that $c_B^B < 0$, which violates our any stability condition required for the 2x2. ■

We will also see that provided stable conditions, this system will reach a steady state as well at

$$w_A^t = \frac{-c_B^B c_D^A k - c_B^A c_D^B k + c_D^A c_D^B k + c_D^{B^2} k - c_B^A c_D^D k - c_B^B c_D^D k}{c_B^B c_D^{A^2} - 2c_B^A c_D^A c_D^B + c_A^A c_D^{B^2} + c_B^{A^2} c_D^D - c_A^A c_B^B c_D^D}$$

$$w_B^t = \frac{c_B^A c_D^A k + c_D^{A2} k - c_A^A c_D^B k + c_D^A c_D^B k - c_A^A c_D^D k - c_B^A c_D^D k}{c_B^B c_D^{A^2} - 2c_B^A c_D^A c_D^B + c_A^A c_D^{B^2} + c_B^{A^2} c_D^D - c_A^A c_B^B c_D^D}$$

$$w_D^t = \frac{c_B^{A^2} k - c_A^A c_B^B k + c_B^A c_D^A k + c_B^B c_D^A k - c_A^A c_D^B k - c_B^A c_D^B k}{c_B^B c_D^{A^2} - 2c_B^A c_D^A c_D^B + c_A^A c_D^{B^2} + c_B^{A^2} c_D^D - c_A^A c_B^B c_D^D}$$

There remains a lot to be understood about our tripolar perspective, our Theorem has interesting implications for the real world. This means that a third country can enter an arms race while maintaining stability, though just because we have a stable system when the third country enters does not imply that the arms race had previously been stable. This tracks historically, as we have nuclear deterrence as a common threat as more countries can become nuclear capable, hence by the addition of another county, while armaments are still growing there still can be stability within the system.

To examine the shift of our system from 2x2 to 3x3, we look at the following. This has conditions

$$c_A^A = .4, c_B^A = .35, c_B^B = .5, c_A^D = .2, c_B^D = .6, c_D^D = .55, k = 10$$

For the 2x2 system, we can see the dynamics in Figure 2. This shows an arms race that is stable, but both sides are decreasing their armory, indicating lacking hostility towards each other. However, we can see in Figure 2 that after this shift takes place, both county A and B have begun producing more weapons, though they have initially been outpaced country D. While this system still tends to stability, it is surprising to see the apparent advantage country D is able to have, simply by entering the arms race. This shift seems relatively smooth, with nothing to indicate any sort of huge incident between the two, which also is reassuring: countries can come enter the system while the outbreak war is still avoided.

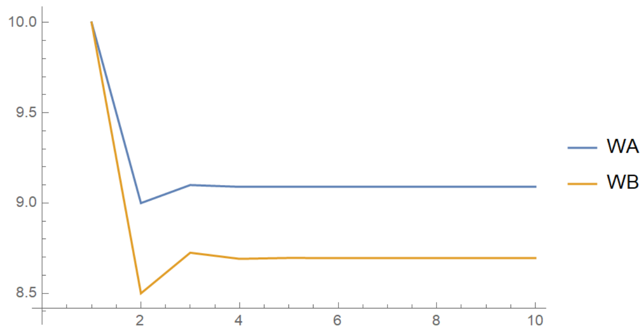

Figure 2: A Stable 2x2 System showing Country A and B's weaponry before Country D enters the Arms Race

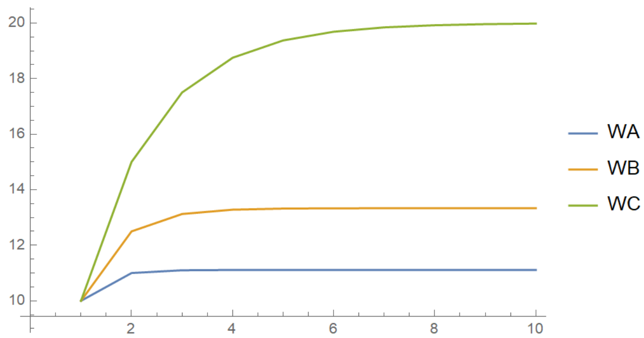

Figure 3: A Stable 3x3 System showing Country A, B and D's weaponry as Country D enters the Arms Race

One interesting thing to note is how the variability of the 'friendliness coefficient' k will impact the arsenals of each country. Take the following case, where we have $c_A^A = .4, c_B^A = .15, c_B^B = .5, c_A^D = .9, c_B^D = .9, c_D^D = 1$. This is shown in Figure 4, where we can see that as friendliness increases, the armaments decrease, and vice versa, which makes sense with the terms of our model.

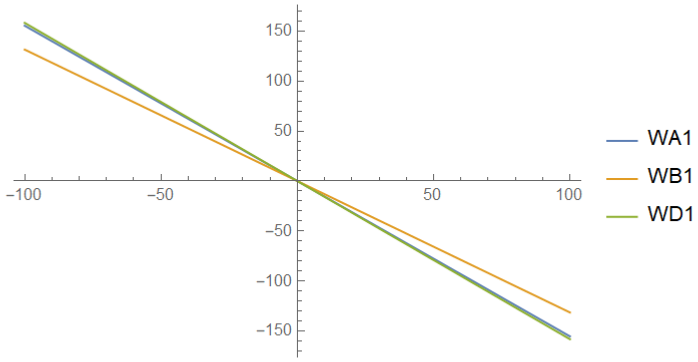

Figure 4: The Impact of Varying k on Weapon Stocks

4.3 A Novel Weapons Perspective

While this paper has mostly discussed our model in the context of countries entering an arms race, we could also see this system as a race between two countries, each of which are adding weapons to their arsenals over time. We could do this very easily, simply rewriting our system C_3', and adding a subscript on each country to indicate which weapon is being discussed. This gives us

$$C_3' = \begin{pmatrix} c_{A1}^{A1} & c_{B1}^{A1} & c_{A2}^{A1} \\ c_{B1}^{A1} & c_{B1}^{B1} & c_{A2}^{B1} \\ c_{A2}^{A1} & c_{A2}^{B1} & c_{A2}^{A2} \end{pmatrix} \tag{2}$$

Here, our new system examines the impact of country A inventing a new weapon, weapon 2, and accounts for the reactions within the arms race. We will still require the diagonal signs to be negative, and additionally require $c_{A2}^{A1} < 0$ and $c_{A2}^{A2} < 0$. This is due to the same country having to maintain an arsenal filled with both weapons. This system has very similar to our previous case, only varying in a few signs, but an interesting question can emerge: how does the difference between the new weapon 2 and the preexisting weapon 1 impact the stability of the system?

As it turns out, this difference does impact the dynamics of the weaponry, as seen in Figure 5, which takes place at stable conditions $c_{A1}^{A1} = .4, c_{B1}^{A1} = .15, c_{B1}^{B1} = .5, c_{A2}^{B2} = .9, c_{A2}^{A2} = 1, k = 100$. We don't know precisely how a new weapon system would impact the system as we don't have empirical data to calculate the parameter. Here we conduct a sensitivity analysis on the parameter c_{A1}^{A2}, by varying it and seeing how this changes the general weapons stock and steady state of the system. While we can see the general weapon stocks are impacted, the long term stability of the system is unaffected by dynamics are impacted by varying c_{A1}^{A2}. This is unexpected. We consider c_{A1}^{A2} the difference between our first and second weapon. For example, a sword advancing to a gun would be one transition, but advancing from an arrowhead to a nuclear warhead differs quite drastically. It's surprising that we don't see this impact the stability of the system, despite how one would intuitively think. This perspective could offer an interesting model to learn more about.

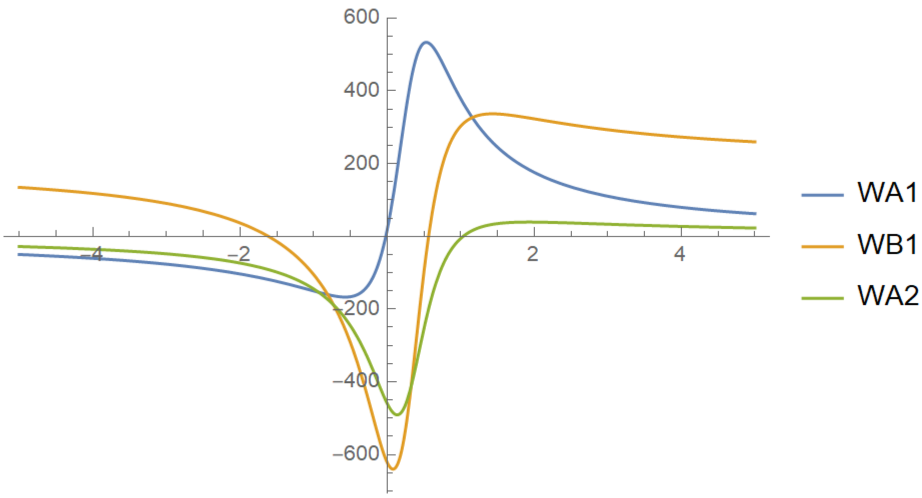

Figure 5: The Impact of Varying c_{A1}^{A2} on Weapon Stocks

5 Discussion

This model provides a different perspective on long existing arms race modeling. Like Richardson, this model doesn't specifically engage in meticulous plotting of past spending, but aims to give a general look into a discrete model of arms races. As technology advances, so does warfare, and gaining insight on the mathematical modeling allows us to learn. A clear question remains about our multipolar system - specifically regarding the transition period. While we know that it is possible to transition between a stable bipolar to a stable tripolar, it would be interesting to learn how the system would adapt in cases where many other countries joined, and some would drop out over time. Additionally, we also can explore this model in terms of inventing new weapons, and perhaps explore how the differences in weaponry in arsenals can impact each other. This is especially pertinent in the style of modern warfare, where armories are becoming more optimized.

While we aimed to address critiques that had been given to previous work in modeling of warfare, this interpretation is not without its weaknesses. In this paper, we were only able to thoroughly examine one of our base cases, and while we were able to draw some other conclusions from looking at more complex examples there remains much to be gleaned from them. More thorough numerical analysis or further applications can explore adding more nations to the arms race, and also look at the impact of them leaving. More work could also be done on the novel weapons perspective, to see if specific parameters could optimize a country's arsenal. Future iterations could proceed from several perspectives, and possibly create their own. All in all, our model is a versatile tool that can allow many interpretations of modern warfare that could use much more fine tuning and analysis.

References

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Appendix

The full solution for our tripolar 3x3 matrix can be seen below:

In[1109]:=
```
C3 = {{-c1, c2, c4}, {c2, -c3, c5}, {c4, c5, -c6}};
C3 // MatrixForm;
stablesoln2 = Reduce[ Det[C3] > 0 && Tr[C3] < 0 ];
stablesoln2
```

Out[1112]=

$$(c3 \mid c5) \in \mathbb{R} \;\&\&\; \Bigg(\Bigg(c2 < 0 \;\&\&\; \Bigg(\Bigg(c4 < 0 \;\&\&\; \Bigg(\Bigg(c1 < 0 \;\&\&\; \Bigg(\Bigg(\frac{c2^2}{c1} < c3 < \frac{-c1^2 - c4^2}{c1} \;\&\&$$

$$\Bigg(\Bigg(c5 < -\frac{c2\,c4}{c1} - \sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \;\&\&$$

$$c6 > \frac{c3\,c4^2 + 2\,c2\,c4\,c5 + c1\,c5^2}{-c2^2 + c1\,c3}\Bigg) \;||$$

$$\Bigg(-\frac{c2\,c4}{c1} - \sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \le c5 \le$$

$$-\frac{c2\,c4}{c1} + \sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \;\&\&\; c6 >$$

$$-c1 - c3\Bigg) \;||\; \Bigg(c5 > -\frac{c2\,c4}{c1} +$$

$$\sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \;\&\&$$

$$c6 > \frac{c3\,c4^2 + 2\,c2\,c4\,c5 + c1\,c5^2}{-c2^2 + c1\,c3}\Bigg)\Bigg)\Bigg) \;||\; \Bigg(c3 == \frac{-c1^2 - c4^2}{c1} \;\&\&$$

$$\Bigg(\Bigg(c5 < -\frac{c2\,c4}{c1} + \sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \;\&\&$$

$$c6 > \frac{c3\,c4^2 + 2\,c2\,c4\,c5 + c1\,c5^2}{-c2^2 + c1\,c3}\Bigg) \;||$$

$$\Bigg(c5 == -\frac{c2\,c4}{c1} + \frac{\sqrt{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}}{\mathrm{Abs}[c1]} \;\&\&$$

$$c6 > -c1 - c3\Bigg) \;||\; \Bigg(c5 > -\frac{c2\,c4}{c1} +$$

$$\sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \;\&\&$$

$$c6 > \frac{c3\,c4^2 + 2\,c2\,c4\,c5 + c1\,c5^2}{-c2^2 + c1\,c3}\Bigg)\Bigg)\Bigg) \;||$$

$$\Bigg(c3 > \frac{-c1^2 - c4^2}{c1} \;\&\&\; c6 > \frac{c3\,c4^2 + 2\,c2\,c4\,c5 + c1\,c5^2}{-c2^2 + c1\,c3}\Bigg)\Bigg)\Bigg) \;||$$

$$\left(c1 == 0 \,\&\&\, \left(\left(c5 < \frac{c2^2\, c3 - c3\, c4^2}{2\, c2\, c4} \,\&\&\, c6 > -\frac{c3\, c4^2 + 2\, c2\, c4\, c5}{c2^2}\right) \,||\right.\right.$$

$$\left.\left.\left(c5 \geq \frac{c2^2\, c3 - c3\, c4^2}{2\, c2\, c4} \,\&\&\, c6 > -c3\right)\right)\right) \,||$$

$$\left(c1 > 0 \,\&\&\, \left(\left(c3 \leq \frac{-c1^2 - c4^2}{c1} \,\&\&\, c6 > -c1 - c3\right) \,||\, \left(\frac{-c1^2 - c4^2}{c1} < c3 < \frac{c2^2}{c1} \,\&\&\right.\right.\right.$$

$$\left(\left(c5 \leq -\frac{c2\, c4}{c1} - \sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2 + c2^2\, c4^2 - c1\, c3\, c4^2}{c1^2}} \,\&\&\right.\right.$$

$$\left. c6 > -c1 - c3\right) \,||$$

$$\left(-\frac{c2\, c4}{c1} - \sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2 + c2^2\, c4^2 - c1\, c3\, c4^2}{c1^2}} < c5 <\right.$$

$$-\frac{c2\, c4}{c1} + \sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2 + c2^2\, c4^2 - c1\, c3\, c4^2}{c1^2}} \,\&\&$$

$$\left. c6 > \frac{c3\, c4^2 + 2\, c2\, c4\, c5 + c1\, c5^2}{-c2^2 + c1\, c3}\right) \,||$$

$$\left(c5 \geq -\frac{c2\, c4}{c1} + \sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2 + c2^2\, c4^2 - c1\, c3\, c4^2}{c1^2}} \,\&\&\right.$$

$$\left.\left.\left. c6 > -c1 - c3\right)\right)\right) \,||\, \left(c3 == \frac{c2^2}{c1} \,\&\&\right.$$

$$\left(\left(c5 < -\frac{c2\, c4}{c1} + \sqrt{\frac{c2^2\, c4^2 - c1\, c3\, c4^2}{c1^2}} \,\&\&\, c6 > -c1 - c3\right) \,||\right.$$

$$\left.\left.\left(c5 > -\frac{c2\, c4}{c1} + \sqrt{\frac{c2^2\, c4^2 - c1\, c3\, c4^2}{c1^2}} \,\&\&\, c6 > -c1 - c3\right)\right)\right) \,||$$

$$\left.\left.\left.\left(c3 > \frac{c2^2}{c1} \,\&\&\, -c1 - c3 < c6 < \frac{c3\, c4^2 + 2\, c2\, c4\, c5 + c1\, c5^2}{-c2^2 + c1\, c3}\right)\right)\right)\right) \,||$$

$$\left(c4 == 0 \,\&\&\, \left(\left(c1 < 0 \,\&\&\, \left(\left(\frac{c2^2}{c1} < c3 < -c1 \,\&\&\, \left(\left(c5 < -\sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2}{c1^2}} \,\&\&\right.\right.\right.\right.\right.\right.$$

$$\left. c6 > \frac{c1\, c5^2}{-c2^2 + c1\, c3}\right) \,||\, \left(-\sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2}{c1^2}} \leq\right.$$

$$\left. c5 \leq \sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2}{c1^2}} \,\&\&\, c6 > -c1 - c3\right) \,||$$

$$\left(c5 > \sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2}{c1^2}} \;\&\&\; c6 > \frac{c1\, c5^2}{-c2^2 + c1\, c3}\right)\Bigg)\Bigg)\Bigg) \;||$$

$$\Bigg(c3 == -c1 \;\&\&\; \Bigg(\Bigg(c5 < \sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2}{c1^2}} \;\&\&$$

$$c6 > \frac{c1\, c5^2}{-c2^2 + c1\, c3}\Bigg) \;||\; \left(c5 == \frac{\sqrt{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2}}{\mathrm{Abs}[c1]} \;\&\&\right.$$

$$\left. c6 > -c1 - c3\right) \;||\; \left(c5 > \sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2}{c1^2}} \;\&\&\right.$$

$$\left.c6 > \frac{c1\, c5^2}{-c2^2 + c1\, c3}\right)\Bigg)\Bigg) \;||\; \left(c3 > -c1 \;\&\&\; c6 > \frac{c1\, c5^2}{-c2^2 + c1\, c3}\right)\Bigg)\Bigg) \;||$$

$$\left(c1 == 0 \;\&\&\; \left(\left(c3 \le 0 \;\&\&\; c6 > -c3\right) \;||\; \left(c3 > 0 \;\&\&\; c6 > 0\right)\right)\right) \;||$$

$$\Bigg(c1 > 0 \;\&\&\; \Bigg(\left(c3 \le -c1 \;\&\&\; c6 > -c1 - c3\right) \;||\; \Bigg(-c1 < c3 < \frac{c2^2}{c1} \;\&\&$$

$$\Bigg(\left(c5 \le -\sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2}{c1^2}} \;\&\&\; c6 > -c1 - c3\right) \;||$$

$$\Bigg(-\sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2}{c1^2}} < c5 <$$

$$\sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2}{c1^2}} \;\&\&\; c6 > \frac{c1\, c5^2}{-c2^2 + c1\, c3}\Bigg) \;||$$

$$\left(c5 \ge \sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2}{c1^2}} \;\&\&\; c6 > -c1 - c3\right)\Bigg)\Bigg) \;||$$

$$\left(c3 == \frac{c2^2}{c1} \;\&\&\; \left(\left(c5 < 0 \;\&\&\; c6 > -c1 - c3\right) \;||\; \left(c5 > 0 \;\&\&\; c6 > -c1 - c3\right)\right)\right) \;||$$

$$\left(c3 > \frac{c2^2}{c1} \;\&\&\; -c1 - c3 < c6 < \frac{c1\, c5^2}{-c2^2 + c1\, c3}\right)\Bigg)\Bigg)\Bigg)\Bigg) \;||$$

$$\Bigg(c4 > 0 \;\&\&\; \Bigg(\Bigg(c1 < 0 \;\&\&\; \Bigg(\Bigg(\frac{c2^2}{c1} < c3 < \frac{-c1^2 - c4^2}{c1} \;\&\&\; \Bigg(\Bigg(c5 < -\frac{c2\, c4}{c1} -$$

$$\sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2 + c2^2\, c4^2 - c1\, c3\, c4^2}{c1^2}} \;\&\&$$

$$c6 > \frac{c3\, c4^2 + 2\, c2\, c4\, c5 + c1\, c5^2}{-c2^2 + c1\, c3}\Bigg) \;||\; \Bigg(-\frac{c2\, c4}{c1} -$$

$$\sqrt{\frac{c1^2\, c2^2 - c1^3\, c3 + c1\, c2^2\, c3 - c1^2\, c3^2 + c2^2\, c4^2 - c1\, c3\, c4^2}{c1^2}} \le c5 \le$$

$$-\frac{c2\ c4}{c1}+\sqrt{\frac{c1^2\ c2^2-c1^3\ c3+c1\ c2^2\ c3-c1^2\ c3^2+c2^2\ c4^2-c1\ c3\ c4^2}{c1^2}}\ \&\&$$

$$c6>-c1-c3)\ ||\ (c5>-\frac{c2\ c4}{c1}+$$

$$\sqrt{\frac{c1^2\ c2^2-c1^3\ c3+c1\ c2^2\ c3-c1^2\ c3^2+c2^2\ c4^2-c1\ c3\ c4^2}{c1^2}}\ \&\&$$

$$c6>\frac{c3\ c4^2+2\ c2\ c4\ c5+c1\ c5^2}{-c2^2+c1\ c3})))\ ||\ (c3==\frac{-c1^2-c4^2}{c1}\ \&\&$$

$$((c5<-\frac{c2\ c4}{c1}+\sqrt{\frac{c1^2\ c2^2-c1^3\ c3+c1\ c2^2\ c3-c1^2\ c3^2+c2^2\ c4^2-c1\ c3\ c4^2}{c1^2}}\ \&\&$$

$$c6>\frac{c3\ c4^2+2\ c2\ c4\ c5+c1\ c5^2}{-c2^2+c1\ c3})\ ||$$

$$(c5==-\frac{c2\ c4}{c1}+\frac{\sqrt{c1^2\ c2^2-c1^3\ c3+c1\ c2^2\ c3-c1^2\ c3^2+c2^2\ c4^2-c1\ c3\ c4^2}}{\mathrm{Abs}[c1]}\ \&\&$$

$$c6>-c1-c3)\ ||\ (c5>-\frac{c2\ c4}{c1}+$$

$$\sqrt{\frac{c1^2\ c2^2-c1^3\ c3+c1\ c2^2\ c3-c1^2\ c3^2+c2^2\ c4^2-c1\ c3\ c4^2}{c1^2}}\ \&\&$$

$$c6>\frac{c3\ c4^2+2\ c2\ c4\ c5+c1\ c5^2}{-c2^2+c1\ c3})))\ ||$$

$$(c3>\frac{-c1^2-c4^2}{c1}\ \&\&\ c6>\frac{c3\ c4^2+2\ c2\ c4\ c5+c1\ c5^2}{-c2^2+c1\ c3})))\ ||$$

$$(c1==0\ \&\&\ ((c5\le\frac{c2^2\ c3-c3\ c4^2}{2\ c2\ c4}\ \&\&\ c6>-c3)\ ||\ (c5>\frac{c2^2\ c3-c3\ c4^2}{2\ c2\ c4}\ \&\&$$

$$c6>-\frac{c3\ c4^2+2\ c2\ c4\ c5}{c2^2})))\ ||$$

$$(c1>0\ \&\&\ ((c3\le\frac{-c1^2-c4^2}{c1}\ \&\&\ c6>-c1-c3)\ ||\ (\frac{-c1^2-c4^2}{c1}<c3<\frac{c2^2}{c1}\ \&\&$$

$$((c5\le-\frac{c2\ c4}{c1}-\sqrt{\frac{c1^2\ c2^2-c1^3\ c3+c1\ c2^2\ c3-c1^2\ c3^2+c2^2\ c4^2-c1\ c3\ c4^2}{c1^2}}\ \&\&$$

$$c6>-c1-c3)\ ||$$

$$(-\frac{c2\ c4}{c1}-\sqrt{\frac{c1^2\ c2^2-c1^3\ c3+c1\ c2^2\ c3-c1^2\ c3^2+c2^2\ c4^2-c1\ c3\ c4^2}{c1^2}}<c5<$$

$$-\frac{c2\,c4}{c1}+\sqrt{\frac{c1^2\,c2^2-c1^3\,c3+c1\,c2^2\,c3-c1^2\,c3^2+c2^2\,c4^2-c1\,c3\,c4^2}{c1^2}}\ \&\&$$

$$c6>\frac{c3\,c4^2+2\,c2\,c4\,c5+c1\,c5^2}{-c2^2+c1\,c3}\Bigg)\ ||$$

$$\Bigg(c5\geq-\frac{c2\,c4}{c1}+\sqrt{\frac{c1^2\,c2^2-c1^3\,c3+c1\,c2^2\,c3-c1^2\,c3^2+c2^2\,c4^2-c1\,c3\,c4^2}{c1^2}}\ \&\&$$

$$c6>-c1-c3\Bigg)\Bigg)\Bigg)\ ||\ \Bigg(c3==\frac{c2^2}{c1}\ \&\&$$

$$\Bigg(\Bigg(c5<-\frac{c2\,c4}{c1}+\sqrt{\frac{c2^2\,c4^2-c1\,c3\,c4^2}{c1^2}}\ \&\&\ c6>-c1-c3\Bigg)\ ||$$

$$\Bigg(c5>-\frac{c2\,c4}{c1}+\sqrt{\frac{c2^2\,c4^2-c1\,c3\,c4^2}{c1^2}}\ \&\&\ c6>-c1-c3\Bigg)\Bigg)\Bigg)\ ||$$

$$\Bigg(c3>\frac{c2^2}{c1}\ \&\&\ -c1-c3<c6<\frac{c3\,c4^2+2\,c2\,c4\,c5+c1\,c5^2}{-c2^2+c1\,c3}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\ ||$$

$$\Bigg(c2==0\ \&\&\ \Bigg(\Bigg(c4<0\ \&\&\ \Bigg(\Bigg(c1<0\ \&\&\ \Bigg(\Bigg(0<c3<\frac{-c1^2-c4^2}{c1}\ \&\&$$

$$\Bigg(\Bigg(c5<-\sqrt{\frac{-c1^3\,c3-c1^2\,c3^2-c1\,c3\,c4^2}{c1^2}}\ \&\&\ c6>\frac{c3\,c4^2+c1\,c5^2}{c1\,c3}\Bigg)\ ||$$

$$\Bigg(-\sqrt{\frac{-c1^3\,c3-c1^2\,c3^2-c1\,c3\,c4^2}{c1^2}}\leq c5\leq\sqrt{\frac{-c1^3\,c3-c1^2\,c3^2-c1\,c3\,c4^2}{c1^2}}\ \&\&$$

$$c6>-c1-c3\Bigg)\ ||\ \Bigg(c5>\sqrt{\frac{-c1^3\,c3-c1^2\,c3^2-c1\,c3\,c4^2}{c1^2}}\ \&\&$$

$$c6>\frac{c3\,c4^2+c1\,c5^2}{c1\,c3}\Bigg)\Bigg)\Bigg)\ ||\ \Bigg(c3==\frac{-c1^2-c4^2}{c1}\ \&\&$$

$$\Bigg(\Bigg(c5<\sqrt{\frac{-c1^3\,c3-c1^2\,c3^2-c1\,c3\,c4^2}{c1^2}}\ \&\&\ c6>\frac{c3\,c4^2+c1\,c5^2}{c1\,c3}\Bigg)\ ||$$

$$\Bigg(c5==\frac{\sqrt{-c1^3\,c3-c1^2\,c3^2-c1\,c3\,c4^2}}{\mathrm{Abs}[c1]}\ \&\&\ c6>-c1-c3\Bigg)\ ||$$

$$\Bigg(c5>\sqrt{\frac{-c1^3\,c3-c1^2\,c3^2-c1\,c3\,c4^2}{c1^2}}\ \&\&\ c6>\frac{c3\,c4^2+c1\,c5^2}{c1\,c3}\Bigg)\Bigg)\Bigg)\ ||$$

$$\Bigg(c3>\frac{-c1^2-c4^2}{c1}\ \&\&\ c6>\frac{c3\,c4^2+c1\,c5^2}{c1\,c3}\Bigg)\Bigg)\Bigg)\ ||\ (c1==0\ \&\&\ c3>0\ \&\&\ c6>-c3)\ ||$$

$$\Big(c1 > 0 \;\&\&\; \Big(\Big(c3 \le \frac{-c1^2 - c4^2}{c1} \;\&\&\; c6 > -c1 - c3\Big) \;||\; \Big(\frac{-c1^2 - c4^2}{c1} < c3 < 0 \;\&\&$$

$$\Big(\Big(c5 \le -\sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} \;\&\&\; c6 > -c1 - c3\Big) \;||$$

$$\Big(-\sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} < c5 <$$

$$\sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} \;\&\&\; c6 > \frac{c3\,c4^2 + c1\,c5^2}{c1\,c3}\Big) \;||$$

$$\Big(c5 \ge \sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} \;\&\&\; c6 > -c1 - c3\Big)\Big)\Big) \;||$$

$$\big(c3 == 0 \;\&\&\; \big(\big(c5 < 0 \;\&\&\; c6 > -c1\big) \;||\; \big(c5 > 0 \;\&\&\; c6 > -c1\big)\big)\big) \;||$$

$$\Big(c3 > 0 \;\&\&\; -c1 - c3 < c6 < \frac{c3\,c4^2 + c1\,c5^2}{c1\,c3}\Big)\Big)\Big)\Big)\Big) \;||$$

$$\Big(c4 == 0 \;\&\&\; \Big(\Big(c1 < 0 \;\&\&\; \Big(\Big(0 < c3 < -c1 \;\&\&\; \Big(\Big(c5 < -\sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2}{c1^2}} \;\&\&\; c6 > \frac{c5^2}{c3}\Big) \;||$$

$$\Big(-\sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2}{c1^2}} \le c5 \le \sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2}{c1^2}} \;\&\&\; c6 > -c1 - c3\Big) \;||$$

$$\Big(c5 > \sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2}{c1^2}} \;\&\&\; c6 > \frac{c5^2}{c3}\Big)\Big)\Big) \;||$$

$$\Big(c3 == -c1 \;\&\&\; \Big(\Big(c5 < \sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2}{c1^2}} \;\&\&\; c6 > \frac{c5^2}{c3}\Big) \;||$$

$$\Big(c5 == \frac{\sqrt{-c1^3\,c3 - c1^2\,c3^2}}{\mathrm{Abs}[c1]} \;\&\&\; c6 > -c1 - c3\Big) \;||$$

$$\Big(c5 > \sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2}{c1^2}} \;\&\&\; c6 > \frac{c5^2}{c3}\Big)\Big)\Big) \;||\; \Big(c3 > -c1 \;\&\&\; c6 > \frac{c5^2}{c3}\Big)\Big)\Big) \;||$$

$$\Big(c1 > 0 \;\&\&\; \Big(\big(c3 \le -c1 \;\&\&\; c6 > -c1 - c3\big) \;||\; \Big(-c1 < c3 < 0 \;\&\&$$

$$\Big(\Big(c5 \le -\sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2}{c1^2}} \;\&\&\; c6 > -c1 - c3\Big) \;||$$

$$\Big(-\sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2}{c1^2}} < c5 < \sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2}{c1^2}} \;\&\&\; c6 > \frac{c5^2}{c3}\Big) \;||$$

$$\Big(c5 \ge \sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2}{c1^2}} \;\&\&\; c6 > -c1 - c3\Big)\Big)\Big) \;||$$

$$\left(c3 == 0 \,\&\&\, \left(\left(c5 < 0 \,\&\&\, c6 > -c1\right) \,||\, \left(c5 > 0 \,\&\&\, c6 > -c1\right)\right)\right) \,||$$

$$\left(c3 > 0 \,\&\&\, -c1 - c3 < c6 < \frac{c5^2}{c3}\right)\Bigg)\Bigg)\Bigg)\Bigg) \,||$$

$$\Bigg(c4 > 0 \,\&\&\, \Bigg(\Bigg(c1 < 0 \,\&\&\, \Bigg(\Bigg(0 < c3 < \frac{-c1^2 - c4^2}{c1} \,\&\&\, \Bigg(\Bigg(c5 < -\sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} \,\&\&$$

$$c6 > \frac{c3\,c4^2 + c1\,c5^2}{c1\,c3}\Bigg) \,||\, \Bigg(-\sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} \leq$$

$$c5 \leq \sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} \,\&\&\, c6 > -c1 - c3\Bigg) \,||$$

$$\Bigg(c5 > \sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} \,\&\&\, c6 > \frac{c3\,c4^2 + c1\,c5^2}{c1\,c3}\Bigg)\Bigg)\Bigg) \,||\, \Bigg(c3 ==$$

$$\frac{-c1^2 - c4^2}{c1} \,\&\&\, \Bigg(\Bigg(c5 < \sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} \,\&\&\, c6 > \frac{c3\,c4^2 + c1\,c5^2}{c1\,c3}\Bigg) \,||$$

$$\Bigg(c5 == \frac{\sqrt{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}}{\mathrm{Abs}[c1]} \,\&\&\, c6 > -c1 - c3\Bigg) \,||$$

$$\Bigg(c5 > \sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} \,\&\&\, c6 > \frac{c3\,c4^2 + c1\,c5^2}{c1\,c3}\Bigg)\Bigg)\Bigg) \,||$$

$$\Bigg(c3 > \frac{-c1^2 - c4^2}{c1} \,\&\&\, c6 > \frac{c3\,c4^2 + c1\,c5^2}{c1\,c3}\Bigg)\Bigg)\Bigg) \,||\, \left(c1 == 0 \,\&\&\, c3 > 0 \,\&\&\, c6 > -c3\right) \,||$$

$$\Bigg(c1 > 0 \,\&\&\, \Bigg(\Bigg(c3 \leq \frac{-c1^2 - c4^2}{c1} \,\&\&\, c6 > -c1 - c3\Bigg) \,||\, \Bigg(\frac{-c1^2 - c4^2}{c1} < c3 < 0 \,\&\&$$

$$\Bigg(\Bigg(c5 \leq -\sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} \,\&\&\, c6 > -c1 - c3\Bigg) \,||$$

$$\Bigg(-\sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} < c5 <$$

$$\sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} \,\&\&\, c6 > \frac{c3\,c4^2 + c1\,c5^2}{c1\,c3}\Bigg) \,||$$

$$\Bigg(c5 \geq \sqrt{\frac{-c1^3\,c3 - c1^2\,c3^2 - c1\,c3\,c4^2}{c1^2}} \,\&\&\, c6 > -c1 - c3\Bigg)\Bigg)\Bigg) \,||$$

$$\left(c3 == 0 \,\&\&\, \left(\left(c5 < 0 \,\&\&\, c6 > -c1\right) \,||\, \left(c5 > 0 \,\&\&\, c6 > -c1\right)\right)\right) \,||$$

$$\left(c3 > 0 \,\&\&\, -c1 - c3 < c6 < \frac{c3\,c4^2 + c1\,c5^2}{c1\,c3}\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg) \,||$$

$$\left(c2 > 0 \,\&\&\, \left(\left(c4 < 0 \,\&\&\, \left(\left(c1 < 0 \,\&\&\, \left(\left(\frac{c2^2}{c1} < c3 < \frac{-c1^2 - c4^2}{c1} \,\&\&\, \left(\left(c5 < -\frac{c2\,c4}{c1} - \sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \,\&\&\, c6 > \frac{c3\,c4^2 + 2\,c2\,c4\,c5 + c1\,c5^2}{-c2^2 + c1\,c3}\right) \,||\, \left(-\frac{c2\,c4}{c1} - \sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \le c5 \le -\frac{c2\,c4}{c1} + \sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \,\&\&\, c6 > -c1 - c3\right) \,||\, \left(c5 > -\frac{c2\,c4}{c1} + \sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \,\&\&\, c6 > \frac{c3\,c4^2 + 2\,c2\,c4\,c5 + c1\,c5^2}{-c2^2 + c1\,c3}\right)\right)\right) \,||\, \left(c3 == \frac{-c1^2 - c4^2}{c1} \,\&\&\, \left(\left(c5 < -\frac{c2\,c4}{c1} + \sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \,\&\&\, c6 > \frac{c3\,c4^2 + 2\,c2\,c4\,c5 + c1\,c5^2}{-c2^2 + c1\,c3}\right) \,||\, \left(c5 == -\frac{c2\,c4}{c1} + \frac{\sqrt{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}}{\mathrm{Abs}[c1]} \,\&\&\, c6 > -c1 - c3\right) \,||\, \left(c5 > -\frac{c2\,c4}{c1} + \sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \,\&\&\, c6 > \frac{c3\,c4^2 + 2\,c2\,c4\,c5 + c1\,c5^2}{-c2^2 + c1\,c3}\right)\right)\right) \,||\, \left(c3 > \frac{-c1^2 - c4^2}{c1} \,\&\&\, c6 > \frac{c3\,c4^2 + 2\,c2\,c4\,c5 + c1\,c5^2}{-c2^2 + c1\,c3}\right)\right)\right) \,||\, \left(c1 == 0 \,\&\&\, \left(\left(c5 \le \frac{c2^2\,c3 - c3\,c4^2}{2\,c2\,c4} \,\&\&\, c6 > -c3\right) \,||\, \left(c5 > \frac{c2^2\,c3 - c3\,c4^2}{2\,c2\,c4} \,\&\&\, c6 > -\frac{c3\,c4^2 + 2\,c2\,c4\,c5}{c2^2}\right)\right)\right) \,||$$

$$\Bigg(\text{c1} > 0\ \&\&\ \Bigg(\Bigg(\text{c3} \le \frac{-\text{c1}^2-\text{c4}^2}{\text{c1}}\ \&\&\ \text{c6} > -\text{c1}-\text{c3}\Bigg)\ ||\ \Bigg(\frac{-\text{c1}^2-\text{c4}^2}{\text{c1}} < \text{c3} < \frac{\text{c2}^2}{\text{c1}}\ \&\&$$

$$\Bigg(\Bigg(\text{c5} \le -\frac{\text{c2 c4}}{\text{c1}} - \sqrt{\frac{\text{c1}^2\,\text{c2}^2 - \text{c1}^3\,\text{c3} + \text{c1 c2}^2\,\text{c3} - \text{c1}^2\,\text{c3}^2 + \text{c2}^2\,\text{c4}^2 - \text{c1 c3 c4}^2}{\text{c1}^2}}\ \&\&$$

$$\text{c6} > -\text{c1}-\text{c3}\Bigg)\ ||$$

$$\Bigg(-\frac{\text{c2 c4}}{\text{c1}} - \sqrt{\frac{\text{c1}^2\,\text{c2}^2 - \text{c1}^3\,\text{c3} + \text{c1 c2}^2\,\text{c3} - \text{c1}^2\,\text{c3}^2 + \text{c2}^2\,\text{c4}^2 - \text{c1 c3 c4}^2}{\text{c1}^2}} < \text{c5} <$$

$$-\frac{\text{c2 c4}}{\text{c1}} + \sqrt{\frac{\text{c1}^2\,\text{c2}^2 - \text{c1}^3\,\text{c3} + \text{c1 c2}^2\,\text{c3} - \text{c1}^2\,\text{c3}^2 + \text{c2}^2\,\text{c4}^2 - \text{c1 c3 c4}^2}{\text{c1}^2}}\ \&\&$$

$$\text{c6} > \frac{\text{c3 c4}^2 + 2\,\text{c2 c4 c5} + \text{c1 c5}^2}{-\text{c2}^2 + \text{c1 c3}}\Bigg)\ ||$$

$$\Bigg(\text{c5} \ge -\frac{\text{c2 c4}}{\text{c1}} + \sqrt{\frac{\text{c1}^2\,\text{c2}^2 - \text{c1}^3\,\text{c3} + \text{c1 c2}^2\,\text{c3} - \text{c1}^2\,\text{c3}^2 + \text{c2}^2\,\text{c4}^2 - \text{c1 c3 c4}^2}{\text{c1}^2}}\ \&\&$$

$$\text{c6} > -\text{c1}-\text{c3}\Bigg)\Bigg)\Bigg)\ ||\ \Bigg(\text{c3} == \frac{\text{c2}^2}{\text{c1}}\ \&\&$$

$$\Bigg(\Bigg(\text{c5} < -\frac{\text{c2 c4}}{\text{c1}} + \sqrt{\frac{\text{c2}^2\,\text{c4}^2 - \text{c1 c3 c4}^2}{\text{c1}^2}}\ \&\&\ \text{c6} > -\text{c1}-\text{c3}\Bigg)\ ||$$

$$\Bigg(\text{c5} > -\frac{\text{c2 c4}}{\text{c1}} + \sqrt{\frac{\text{c2}^2\,\text{c4}^2 - \text{c1 c3 c4}^2}{\text{c1}^2}}\ \&\&\ \text{c6} > -\text{c1}-\text{c3}\Bigg)\Bigg)\Bigg)\ ||$$

$$\Bigg(\text{c3} > \frac{\text{c2}^2}{\text{c1}}\ \&\&\ -\text{c1}-\text{c3} < \text{c6} < \frac{\text{c3 c4}^2 + 2\,\text{c2 c4 c5} + \text{c1 c5}^2}{-\text{c2}^2 + \text{c1 c3}}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\ ||$$

$$\Bigg(\text{c4} == 0\ \&\&\ \Bigg(\Bigg(\text{c1} < 0\ \&\&\ \Bigg(\Bigg(\frac{\text{c2}^2}{\text{c1}} < \text{c3} < -\text{c1}\ \&\&\ \Bigg(\Bigg(\text{c5} < -\sqrt{\frac{\text{c1}^2\,\text{c2}^2 - \text{c1}^3\,\text{c3} + \text{c1 c2}^2\,\text{c3} - \text{c1}^2\,\text{c3}^2}{\text{c1}^2}}\ \&\&$$

$$\text{c6} > \frac{\text{c1 c5}^2}{-\text{c2}^2 + \text{c1 c3}}\Bigg)\ ||\ \Bigg(-\sqrt{\frac{\text{c1}^2\,\text{c2}^2 - \text{c1}^3\,\text{c3} + \text{c1 c2}^2\,\text{c3} - \text{c1}^2\,\text{c3}^2}{\text{c1}^2}} \le$$

$$\text{c5} \le \sqrt{\frac{\text{c1}^2\,\text{c2}^2 - \text{c1}^3\,\text{c3} + \text{c1 c2}^2\,\text{c3} - \text{c1}^2\,\text{c3}^2}{\text{c1}^2}}\ \&\&\ \text{c6} > -\text{c1}-\text{c3}\Bigg)\ ||$$

$$\Bigg(\text{c5} > \sqrt{\frac{\text{c1}^2\,\text{c2}^2 - \text{c1}^3\,\text{c3} + \text{c1 c2}^2\,\text{c3} - \text{c1}^2\,\text{c3}^2}{\text{c1}^2}}\ \&\&\ \text{c6} > \frac{\text{c1 c5}^2}{-\text{c2}^2 + \text{c1 c3}}\Bigg)\Bigg)\Bigg)\ ||$$

$$\Bigg(\text{c3} == -\text{c1}\ \&\&\ \Bigg(\Bigg(\text{c5} < \sqrt{\frac{\text{c1}^2\,\text{c2}^2 - \text{c1}^3\,\text{c3} + \text{c1 c2}^2\,\text{c3} - \text{c1}^2\,\text{c3}^2}{\text{c1}^2}}\ \&\&$$

$$
\begin{aligned}
&\left.\left. c6 > \frac{c1\,c5^2}{-c2^2 + c1\,c3}\right) \;||\; \left(c5 == \frac{\sqrt{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2}}{\text{Abs}[c1]} \;\&\&\right.\right.\\
&\left. c6 > -c1 - c3\right) \;||\; \left(c5 > \sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2}{c1^2}} \;\&\&\right.\\
&\left.\left.\left.\left. c6 > \frac{c1\,c5^2}{-c2^2 + c1\,c3}\right)\right)\right)\right) \;||\; \left.\left.\left(c3 > -c1 \;\&\&\; c6 > \frac{c1\,c5^2}{-c2^2 + c1\,c3}\right)\right)\right) \;||\\
&\left(c1 == 0 \;\&\&\; \left(\left(c3 \le 0 \;\&\&\; c6 > -c3\right) \;||\; \left(c3 > 0 \;\&\&\; c6 > 0\right)\right)\right) \;||\\
&\left(c1 > 0 \;\&\&\; \left(\left(c3 \le -c1 \;\&\&\; c6 > -c1 - c3\right) \;||\; \left(-c1 < c3 < \frac{c2^2}{c1} \;\&\&\right.\right.\right.\\
&\quad\left(\left(c5 \le -\sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2}{c1^2}} \;\&\&\; c6 > -c1 - c3\right) \;||\right.\\
&\quad\left(-\sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2}{c1^2}} < c5 <\right.\\
&\quad\left.\sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2}{c1^2}} \;\&\&\; c6 > \frac{c1\,c5^2}{-c2^2 + c1\,c3}\right) \;||\\
&\quad\left.\left.\left.\left(c5 \ge \sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2}{c1^2}} \;\&\&\; c6 > -c1 - c3\right)\right)\right)\right) \;||\\
&\quad\left(c3 == \frac{c2^2}{c1} \;\&\&\; \left(\left(c5 < 0 \;\&\&\; c6 > -c1 - c3\right) \;||\; \left(c5 > 0 \;\&\&\; c6 > -c1 - c3\right)\right)\right) \;||\\
&\quad\left.\left.\left.\left.\left.\left(c3 > \frac{c2^2}{c1} \;\&\&\; -c1 - c3 < c6 < \frac{c1\,c5^2}{-c2^2 + c1\,c3}\right)\right)\right)\right)\right)\right) \;||\\
&\left(c4 > 0 \;\&\&\; \left(\left(c1 < 0 \;\&\&\; \left(\left(\frac{c2^2}{c1} < c3 < \frac{-c1^2 - c4^2}{c1} \;\&\&\; \left(\left(c5 < -\frac{c2\,c4}{c1} -\right.\right.\right.\right.\right.\right.\right.\\
&\quad\sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \;\&\&\\
&\quad\left. c6 > \frac{c3\,c4^2 + 2\,c2\,c4\,c5 + c1\,c5^2}{-c2^2 + c1\,c3}\right) \;||\; \left(-\frac{c2\,c4}{c1} -\right.\\
&\quad\sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \le c5 \le\\
&\quad -\frac{c2\,c4}{c1} + \sqrt{\frac{c1^2\,c2^2 - c1^3\,c3 + c1\,c2^2\,c3 - c1^2\,c3^2 + c2^2\,c4^2 - c1\,c3\,c4^2}{c1^2}} \;\&\&\\
&\quad\left. c6 > -c1 - c3\right) \;||\; \left(c5 > -\frac{c2\,c4}{c1} +\right.
\end{aligned}
$$

$$\sqrt{\frac{c1^2\ c2^2 - c1^3\ c3 + c1\ c2^2\ c3 - c1^2\ c3^2 + c2^2\ c4^2 - c1\ c3\ c4^2}{c1^2}}\ \&\&$$

$$c6 > \frac{c3\ c4^2 + 2\ c2\ c4\ c5 + c1\ c5^2}{-c2^2 + c1\ c3}\Big)\Big)\Big)\ ||\ \Big(c3 == \frac{-c1^2 - c4^2}{c1}\ \&\&$$

$$\Big(\Big(c5 < -\frac{c2\ c4}{c1} + \sqrt{\frac{c1^2\ c2^2 - c1^3\ c3 + c1\ c2^2\ c3 - c1^2\ c3^2 + c2^2\ c4^2 - c1\ c3\ c4^2}{c1^2}}\ \&\&$$

$$c6 > \frac{c3\ c4^2 + 2\ c2\ c4\ c5 + c1\ c5^2}{-c2^2 + c1\ c3}\Big)\ ||$$

$$\Big(c5 == -\frac{c2\ c4}{c1} + \frac{\sqrt{c1^2\ c2^2 - c1^3\ c3 + c1\ c2^2\ c3 - c1^2\ c3^2 + c2^2\ c4^2 - c1\ c3\ c4^2}}{\mathrm{Abs}[c1]}\ \&\&$$

$$c6 > -c1 - c3\Big)\ ||\ \Big(c5 > -\frac{c2\ c4}{c1} +$$

$$\sqrt{\frac{c1^2\ c2^2 - c1^3\ c3 + c1\ c2^2\ c3 - c1^2\ c3^2 + c2^2\ c4^2 - c1\ c3\ c4^2}{c1^2}}\ \&\&$$

$$c6 > \frac{c3\ c4^2 + 2\ c2\ c4\ c5 + c1\ c5^2}{-c2^2 + c1\ c3}\Big)\Big)\Big)\ ||$$

$$\Big(c3 > \frac{-c1^2 - c4^2}{c1}\ \&\&\ c6 > \frac{c3\ c4^2 + 2\ c2\ c4\ c5 + c1\ c5^2}{-c2^2 + c1\ c3}\Big)\Big)\Big)\ ||$$

$$\Big(c1 == 0\ \&\&\ \Big(\Big(c5 < \frac{c2^2\ c3 - c3\ c4^2}{2\ c2\ c4}\ \&\&\ c6 > -\frac{c3\ c4^2 + 2\ c2\ c4\ c5}{c2^2}\Big)\ ||$$

$$\Big(c5 \geq \frac{c2^2\ c3 - c3\ c4^2}{2\ c2\ c4}\ \&\&\ c6 > -c3\Big)\Big)\Big)\ ||$$

$$\Big(c1 > 0\ \&\&\ \Big(\Big(c3 \leq \frac{-c1^2 - c4^2}{c1}\ \&\&\ c6 > -c1 - c3\Big)\ ||\ \Big(\frac{-c1^2 - c4^2}{c1} < c3 < \frac{c2^2}{c1}\ \&\&$$

$$\Big(\Big(c5 \leq -\frac{c2\ c4}{c1} - \sqrt{\frac{c1^2\ c2^2 - c1^3\ c3 + c1\ c2^2\ c3 - c1^2\ c3^2 + c2^2\ c4^2 - c1\ c3\ c4^2}{c1^2}}\ \&\&$$

$$c6 > -c1 - c3\Big)\ ||$$

$$\Big(-\frac{c2\ c4}{c1} - \sqrt{\frac{c1^2\ c2^2 - c1^3\ c3 + c1\ c2^2\ c3 - c1^2\ c3^2 + c2^2\ c4^2 - c1\ c3\ c4^2}{c1^2}} < c5 <$$

$$-\frac{c2\ c4}{c1} + \sqrt{\frac{c1^2\ c2^2 - c1^3\ c3 + c1\ c2^2\ c3 - c1^2\ c3^2 + c2^2\ c4^2 - c1\ c3\ c4^2}{c1^2}}\ \&\&$$

$$c6 > \frac{c3\ c4^2 + 2\ c2\ c4\ c5 + c1\ c5^2}{-c2^2 + c1\ c3}\Big)\ ||$$

$$\left(c5 \geq -\frac{c2\ c4}{c1} + \sqrt{\frac{c1^2\ c2^2 - c1^3\ c3 + c1\ c2^2\ c3 - c1^2\ c3^2 + c2^2\ c4^2 - c1\ c3\ c4^2}{c1^2}}\ \&\&\right.$$

$$\left.c6 > -c1 - c3\right)\Big)\Big)\ ||\ \left(c3 == \frac{c2^2}{c1}\ \&\&\right.$$

$$\left(\left(c5 < -\frac{c2\ c4}{c1} + \sqrt{\frac{c2^2\ c4^2 - c1\ c3\ c4^2}{c1^2}}\ \&\&\ c6 > -c1 - c3\right)\ ||\right.$$

$$\left.\left(c5 > -\frac{c2\ c4}{c1} + \sqrt{\frac{c2^2\ c4^2 - c1\ c3\ c4^2}{c1^2}}\ \&\&\ c6 > -c1 - c3\right)\right)\Big)\ ||$$

$$\left(c3 > \frac{c2^2}{c1}\ \&\& -c1 - c3 < c6 < \frac{c3\ c4^2 + 2\ c2\ c4\ c5 + c1\ c5^2}{-c2^2 + c1\ c3}\right)\Big)\Big)\Big)\Big)\Big)\Big)\Big)$$