Homework 2

Due Friday, October 14, 2022 at 11:59PM on Gradescope

Each homework is worth 30 points. Some problems will be graded for completion, and others will be graded for accuracy and quality of explanations. You must show work to receive full credit. Challenge problems are **optional** and go beyond what you are expected to know for exams. They may be completed for an additional 2 points each, and partial credit will be awarded.

Required Problems:

- 5.3: 71,72,81
- 4.8: 14th/15th edition: 91,97,105,114,128 ; 13th edition: 71,77,83,92,102
- 5.4: 1,2,3,4,7,8,9,10,11,15,21,22,25,27,28, 14th/15th edition: 45,46,52 ; 13th edition: 39,40,46
- 5.5: 1,4,18,19,25,31

1. Let $f(x) = \lceil x \rceil$ denote the *ceiling function*, or the least integer greater than or equal to x. In other words, you round x up to the nearest integer. For example, $f(1) = 1$, $f(1.0001) = 2$, $f(1.9) = 2$, $f(2.4) = 3$, etc.

   Find $\int_0^n \lceil x \rceil \, dx$. You may assume $n \geq 0$, and your final answer may involve floor and/or ceiling functions.

2. Recall that a function $f(x)$ is *even* if $f(x) = f(-x)$ for all x, and $f(x)$ is *odd* if $f(x) = -f(-x)$ for all x.

   (a) Show that if $f(x)$ is even, then $\int_{-a}^{a} f(x) \, dx = 2\int_0^a f(x) \, dx$. Hint: split the first integral into two integrals, then use a substitution on one of them.

   (b) Show that if $f(x)$ is odd, then $\int_{-a}^{a} f(x) \, dx = 0$.

Challenge problems:

A) Use integrals to compute

$$\left\lceil \sum_{n=1}^{10000} \frac{1}{n^{\frac{1}{4}}} \right\rceil.$$

You can check your answer with a computer, but this problem should be done entirely by hand.

B) (Sorry, *one* more Riemann sum)

Let $f(x)$ be the function

$$\begin{cases} 1 & \text{if } x = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of the lower sums L_n for all n.

(b) Calculate the upper sums U_8, U_{16}, U_{32}, and U_{64}. Hint: it may help to count the number of "empty" rectangles in each case.

(c) Find a formula for U_{2^n}.

(d) Use WolframAlpha or ask your instructor to compute $\lim_{n\to\infty} U_{2^n}$.

C) In physics, a classical kinematics equation says $\Delta x = v_0 t + \frac{1}{2}at^2$, where Δx is the change in an object's position, v_0 is the initial velocity, and a is the (constant) acceleration. Rewriting $\Delta x = s(t) - s(0)$ and $v_0 = v(0)$, we obtain

$$s(t) = s(0) + v(0)t + \frac{1}{2}at^2.$$

(a) Rederive this equation from the differential equation $s''(t) = a$.

(b) The third derivative $s'''(t)$ of position is called *jerk*. If the jerk of an object is equal to some constant j, find an equation for the object's position $s(t)$ in terms of its initial position $s(0)$, initial velocity $v(0)$, initial acceleration $a(0)$, the time t, and j.

(c) Generalize this equation for the case where the n-th derivative $s^{(n)}(t)$ is equal to some constant r. Write your answer in sigma notation. (You don't have to formally "prove" anything, just find the pattern.)