Discussion Problems 5 (Tue., Nov. 3)

1. Find the equation of the tangent line to the graph of the function \( y = f(x) \) at the point \((1,0)\) if the function satisfies the equation
\[
x \cdot f(x)^2 + \ln(x + f(x)) = x^3 - \cos(xf(x))
\]

2. Differentiate: (a) \( f(x) = \frac{\arctan x}{\ln(3x + 1)} \), (b) \( f(x) = \arctan((\sin x)^{1/3}) \).

3. A function \( y = f(x) \) satisfies
\[
(x - y)^3 = x^2 - y^2 - 2
\]
Find the equation of the tangent to the graph of this function at the point \((2,1)\). At which point does the tangent cross the \(x\)-axis and at what angle?

4. A function \( y = f(x) \) satisfies
\[
xy = y^2 - 1
\]
Determine the first derivative \( y' \) and the second derivative \( y'' \) of this function at the point \((0,1)\).

5. Assume that a baseball is projected directly upward from the ground with an initial velocity of 112 ft/s. Assuming acceleration due to gravity is \(-32 \text{ ft/sec}^2\), derive the equation for the height of the ball above ground after \( t \) seconds. In how many seconds does the ball strike the gound?