1. Find general antiderivative for the given functions. Assume that all functions are defined for all positive real number.

(a) \( f(x) = 4x^5 \)

Ans:
\[
\int f(x) \, dx = \int 4x^5 \, dx = 4 \int x^5 \, dx = \frac{4}{5}x^5 + C.
\]

(b) \( f(x) = 5x^4 \)

Ans:
\[
\int f(x) \, dx = \int 5x^4 \, dx = 5 \int x^4 \, dx = \frac{5}{5}x^5 + C = x^5 + C.
\]

(c) \( f(x) = x^{-1} \)

Ans:
\[
\int f(x) \, dx = \int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln x + C.
\]

(d) \( f(x) = \sin(3x) \)

Ans:
\[
\int f(x) \, dx = \int \sin(3x) \, dx = -\frac{1}{3} \cos(3x) + C.
\]

(e) \( f(x) = \cos(5x + 1) \)

Ans:
\[
\int f(x) \, dx = \int \cos(5x + 1) \, dx = \frac{1}{5} \sin(5x + 1) + C.
\]

(f) \( f(x) = 5 \sec^2(2x - 1) \)

Ans:
\[
\int f(x) \, dx = \int 5 \sec^2(2x - 1) \, dx = \frac{5}{2} \tan(2x - 1) + C.
\]

(g) \( f(x) = \frac{2\sqrt{x} + x^3}{x^2} \)

Ans:
\[
\int f(x) \, dx = \int \frac{2\sqrt{x} + x^3}{x^2} \, dx = 2 \int x^{-3/2} \, dx + \int x \, dx = -4x^{-1/2} + \frac{x^2}{2} + C
\]

(h) \( f(x) = e^{3x} \)

Ans:
\[
\int f(x) \, dx = \int e^{3x} \, dx = \frac{1}{3}e^{3x} + C.
\]

(i) \( f(x) = 2xe^{x^2} \)

Ans: Let \( u = x^2 \), then \( du = 2x \, dx \). Therefore,
\[
\int f(x) \, dx = \int 2xe^{x^2} \, dx = \int e^u \, du = e^u + C = e^{x^2} + C.
\]
Solutions to problems 2 and 3 from discussion sheet #0

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2.

• a) $y = e^{\tan(3x)} \Rightarrow \frac{dy}{dx} = e^{\tan(3x)} \frac{d}{dx} \tan(3x) = e^{\tan(3x)} \sec^2(3x)\cdot 3$

• b) $f(x) = e^{\tan(5x)} \sec^2(5x) \Rightarrow F(x) = \int e^{\tan(5x)} \sec^2(5x)dx$. Using "u-substitution" with $u = \tan(5x)$ we obtain: $F(x) = \int e^{u} \frac{du}{5} = \frac{e^{u}}{5} + C = \frac{e^{\tan(5x)}}{5} + C$.

3.

• a) $y = \sin^2(3x) \Rightarrow \frac{dy}{dx} = 2 \sin(3x) \cos(3x)\cdot 3 = 6 \sin(3x) \cos(3x)$. $y = \sin^5(3x) \Rightarrow \frac{dy}{dx} = 5 \sin^4(3x) \cos(3x)\cdot 3 = 15 \sin^4(3x) \cos(3x)$.

• b) $f(x) = 15 \cos(5x) \sin(5x) \Rightarrow F(x) = \int 15 \cos(5x) \sin(5x)dx = \frac{3}{2} \sin^2(5x) + C$.

• c) $f(x) = 15 \cos(5x) \sin^2(5x) \Rightarrow F(x) = \int 15 \cos(5x) \sin^2(5x)dx = \sin^3(5x) + C$. 
Problem 4

(a) Take derivative by Chain rule to get:
\[
\frac{dy}{dx} = \frac{3x^2 + 3}{x^3 + 3x + 6}
\]

(b) We can get a clue from problem (a), where the numerator of the rational function is related to the derivative of its denominator. In particular, \((x^4 + 8x^2 + 16)' = 4(x^3 + 4x)\), so we have the antiderivative:
\[
F(x) = \frac{1}{4} \ln(x^4 + 8x^2 + 16)
\]

(c) Similarly with (b), since \(\tan x = \frac{\sin x}{\cos x}\), where \((\cos x)' = -\sin x\), we have:
\[
F(x) = -\ln(\cos x)
\]

(d) we can use the table to get:
\[
F(x) = -\frac{1}{7} \ln(\cos 7x)
\]

(e) Similarly with (d):
\[
F(x) = -\frac{1}{7} \ln(\cos(7x + 9))
\]
5. Find a function $f(x)$ whose antiderivative is given as $y$.

Solution: It suffices to find the derivatives for the given functions.

(a) $y = x^{\sin x}$

Note that $\ln y = \sin x \ln x$. So, taking derivative with respect to $x$ at each side gives
\[
\frac{1}{y}y' = \cos x \ln x + \frac{\sin x}{x}.
\]
So,
\[
y' = \left(\cos x \ln x + \frac{\sin x}{x}\right) x^{\sin x}.
\]

(b) $y = (x^3) e^{e^x^2}$

Applying product rule and chain rule gives
\[
y' = 3x^2 e^{e^x^2} + 2x^4 e^{x^2} e^{e^x^2}.
\]

(c) $y = \frac{1}{2} e^x \cos x$

Applying product rule gives
\[
y' = \frac{1}{2} e^x (\cos x - \sin x).
\]

(d) $y = \ln (\cos x)$

Applying chain rule gives
\[
y' = -\tan x.
\]

(e) $y = \sin(g(x))$

Applying chain rule gives
\[
y' = \cos(g(x))g'(x).
\]

(f) $y = \ln(\ln x)$

Applying chain rule gives
\[
y' = \frac{1}{x \ln x}.
\]
(g) \( y = e^{x^2+x} \)

Applying chain rule gives
\[ y' = (2x + 1)e^{x^2+x}. \]

(g) \( y = \ln(x + 1) + \frac{1}{x+1} \)

Applying chain rule gives
\[ y' = \frac{x}{(x + 1)^2}. \]