1. Pr. 19.6
(a) Let \( f(x) = \sqrt{x} \) for \( x \leq 0 \). Show \( f' \) is unbounded on \((0,1]\) but \( f \) is uniformly continuous on \((0,1]\). Does this contradict to Theorem 19.6?
(b) Show \( f \) is uniformly continuous on \((1,\infty)\).

2. Pr. 19.7
(a) Let \( f \) be a continuous function on \([0,\infty)\). Prove that if \( f \) is uniformly continuous on \([k,\infty)\)
for some \( k \), then \( f \) is uniformly continuous on \([0,\infty)\).
(b) Use (a) and Exercise 19.6(b) to prove \( f \) is uniformly continuous on \([0,\infty)\).

3. Pr. 19.9 Let \( f(x) = x \sin(\frac{1}{x}) \) for \( x \neq 0 \) and \( f(0) = 0 \).
(a) Show that \( f \) is continuous on \( \mathbb{R} \).
(b) Why is \( f \) uniformly continuous on any bounded subset of \( \mathbb{R} \)?
(c) Is \( f \) uniformly continuous on \( \mathbb{R} \)?

4. Pr. 13.4 Let \((S,d)\) be a metric space. What is the definition for a subset \( E \subset S \) to be open in \( S \). Show that
(a) The union of any collection of open sets is open.
(b) The intersection of finitely many open sets is open.

5. Pr. 13.7 Show that every open set in \( \mathbb{R} \) is the disjoint union of a finite or countably infinite sequence of open intervals.