Recall: I) Discrete Compound Interest: \( A = P \left(1 + \frac{r}{n}\right)^{nt} \)

II) \( e := \lim_{k \to \infty} \left(1 + \frac{1}{k}\right)^k \)

To obtain formula for continuous compound interest, the interest must be compounded an \( \infty \) number of times, which is equivalent to letting \( n \to \infty \) in above formula.

\[
\lim_{n \to \infty} P \left(1 + \frac{r}{n}\right)^{nt} = \lim_{n \to \infty} P \left( \left(1 + \frac{1}{\left(n\right)} \right)^{\left(n\right)} \right)^{rt} = P e^{rt}
\]

By II) = \( e \)

Hence, continuous compound interest formula is \( A = P e^{rt} \).

**Note:** You don’t need to know this argument for the exams, but it could be an extra credit question.

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**Ex 2** You wish an initial deposit of \( \$800 \) to grow to \( \$1800 \) in 8 years. If interest is compounded daily, what should the yearly interest rate \( r \) be?

Given: \( P = \$800, \quad A = \$1800, \quad t = 8 \text{ yrs}, \quad n = 365 \)

Plugging into \( A = P \left(1 + \frac{r}{n}\right)^{nt} \), we need to solve for \( r \)

\[
1800 = 800 \left(1 + \frac{r}{365}\right)^{365 \times 8}
\]

\[
\Rightarrow \left(\frac{18}{8}\right)^{365 \times 8} = 1 + \frac{r}{365} \Rightarrow 1 + \frac{r}{365} \approx 1.000277254
\]

\[
\Rightarrow r = 0.1013 \quad \text{or} \quad r = 10.13\%
\]