Math 17A
Vogler
Discussion Sheet 2

1.) Two enterprising college women decide to start there own nationwide service sorority called Alpha Beta Zeta. Assume that each year the number of women in Alpha Beta Zeta triples. Let t be the number of years and let N_t be the number of women in Alpha Beta Zeta at time t.

a.) State the initial value and a recursion for N_t.
b.) Determine an exponential growth formula for N_t for $t = 0, 1, 2, 3, 4, \cdots$.
c.) How many women are in Alpha Beta Zeta after 5 years ? after 10 years ?
d.) How long does it take for the number of members to reach 3,188,646 ?

2.) There are 2,000,000,000 people worldwide watching the Super Bowl on TV. Every 10 minutes, 2% of the viewers turn the game off. Let t be the number of 10 minute intervals and let N_t be the number of viewers at time t.

a.) State the initial value and a recursion for N_t.
b.) Determine an exponential decay formula for N_t for $t = 0, 1, 2, 3, 4, \cdots$.
c.) How many people are watching the Super Bowl after 40 minutes ? after 3 hours ?
d.) How long does it take for the number of viewers to reach 1,634,145,614 ?

3.) Compute the first five terms (starting with n=0) of each sequence. Determine whether each sequence converges or diverges.

a.) $a_n = 3$ b.) $a_n = 3^n$ c.) $a_n = \frac{3}{n}$ d.) $a_n = \left(\frac{1}{3}\right)^n$ e.) $a_n = 3^{1/n}$

f.) $a_n = \frac{n+5}{n+2}$ g.) $a_n = n(3-n)$ h.) $a_n = \frac{n^3+n^2-n+7}{4n^3+5n^2-2}$ i.) $a_n = (0.9999)^n$

j.) $a_n = (1.00001)^n$ k.) $a_n = \left(\frac{-2}{3}\right)^n$ l.) $14/3, 15/5, 16/7, 17/9, \ldots$

m.) $a_n = \sin n\pi$ n.) $a_n = \left(\frac{\sqrt{7}}{\ln 14}\right)^n$ o.) $a_n = \cos(2n\pi)$ p.) $a_n = (1 + 1/n)^3$

q.) $a_n = n(n-1)(n-2)(n-3)(n-4)$ r.) $a_n = \sin(\pi/2 + n\pi)$

s.) $a_n = 3 + (-1)^n$

4.) Find a formula a_n, where $n = 0, 1, 2, 3, 4, \cdots$, for each of the following sequences.

a.) $1, 3, 5, 7, 9, \cdots$ b.) $2, 4, 6, 8, 10, \cdots$ c.) $1, 4, 9, 16, 25, \cdots$
d.) $3, 7, 11, 15, 19, \cdots$ e.) $12, 36, 108, 324, 972, \cdots$ f.) $1, -1, 1, -1, 1, \cdots$
g.) $4, 8, 4, 8, 4, \cdots$ h.) $0, 0, 2, 6, 12, 20, 30, \cdots$
i.) $4/3, 7/7, 10/11, 13/15, 16/19, 19, 23 \cdots$ j.) $-1/9, 1/3, -1, 3, -9, 27, -81, \cdots$
(FACT : $1 + 2 + 3 + \cdots + n = (1/2)n(n+1)$)
k.) $1, 3, 6, 10, 15, 21, 28, \cdots$ l.) $4, 8, 13, 19, 26, 34, 43, \cdots$

5.) Determine how many number are in each finite list.

a.) $2, 4, 6, 8, 10, \cdots, 9864$ b.) $55, 57, 59, 61, 63, \cdots, 637$
c.) $3^7, 3^{11}, 3^{15}, 3^{19}, \cdots, 3^{203}$ d.) $3, 6, 10, 15, 21, \cdots, 1891$

e.) $1, 2, 4, 7, 8, 10, 13, 14, 16, 19, 20, 22, \cdots, 601, 602, 604$

6.) Find the 15th number in the following sequence : $1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$

7.) Find the 200th number in the following sequence : $3, 7, 11, 15, 19, 23, \ldots$

8.) A super ball bearing is dropped from a building 1000 feet high. Each time the ball bearing rebounds to 75% of its falling distance. How high does the ball bounce on its 20th rebound ?

10.) Determine the limit of the following sequence :

$$2, \quad 2-\frac{1}{2}, \quad 2-\frac{1}{2-\frac{1}{2}}, \quad 2-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}, \quad 2-\frac{1}{2-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}} \quad \cdots$$

11.) Use algebra to evaluate the following limits.

a.) $\displaystyle\lim_{x\to 2} \frac{x^2+3x}{x^2-16}$

b.) $\displaystyle\lim_{x\to 1} \frac{x^2-1}{x^2+x-2}$

c.) $\displaystyle\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$

d.) $\displaystyle\lim_{x\to -1} \frac{\frac{1}{x}+1}{x+1}$

e.) $\displaystyle\lim_{x\to 2} \cos\frac{\pi}{3}x$

f.) $\displaystyle\lim_{x\to -1} \sin\frac{\pi}{4}x$

g.) $\displaystyle\lim_{x\to 0} \frac{\sqrt{x^2+16}-4}{x^2}$

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The following problem is for recreational purposes only.

12.) Plant 10 trees in 5 straight rows of four trees each.

Thanks to Dr. Kouba