Defn. An equation in \( x \) & \( y \) is a function if each \( x \)-value has a unique \( y \)-value.

Note: The graph of a function passes the vertical line test.

Notation: If \( y \) is a function of \( x \), we write \( y = f(x) \) or \( y(x) \).

Defn. Assume \( y = f(x) \) is a function.

I) The domain of function \( f \) is set of all admissible \( x \)-values.

II) The range of function \( f \) is set of all corresponding \( y \)-values.

Defn. Assume \( y = f(x) \) & \( y = g(x) \) are functions. The composition of functions \( f \) & \( g \) is \((f \circ g)(x) = f(g(x))\).

Defn. A function \( y = f(x) \) is one-to-one (or 1-1) if each \( y \)-value has exactly one \( x \)-value. More precisely, a one-to-one function has the property that if \( f(x_1) = f(x_2) \) (\( y \)-values are equal), then \( x_1 = x_2 \) (\( x \)-values are equal).

Defn. The inverse function of function \( y = f(x) \) is the function \( y = f^{-1}(x) \) for which \( f(f^{-1}(x)) = x = f^{-1}(f(x)) \).

Defn. Let \( y = f(x) \) be a function.

I) \( y = f(x) \) is even if \( f(x) = f(-x) \).

II) \( y = f(x) \) is odd if \( f(-x) = -f(x) \).

Ex. \( |x|, \cos x, \cosh x, x^2, x^4, \ldots \)

Ex. \( \sin x, \sinh x, x, x^3, x^5, \ldots \)
Basic Translation Rules For Graphs

Assume \( f(x) \) is a function.

1) \( f(-x) \) is \( f(x) \) reflected about \( y \)-axis.
2) \(-f(x)\) is \( f(x) \) reflected about \( x \)-axis.
3) \( f(x-a) \) is \( f(x) \) shifted right \( a \) units \((a>0)\)
4) \( f(x+a) \) is \( f(x) \) shifted left \( a \) units \((a>0)\)
5) \( f(x)+a \) is \( f(x) \) shifted up \( a \) units \((a>0)\)
6) \( f(x)-a \) is \( f(x) \) shifted down \( a \) units \((a>0)\)
7) \( a \cdot f(x) \) is \( f(x) \) stretched by a factor of \( a \) \((a>1)\)
8) \( \frac{1}{a} \cdot f(x) \) is \( f(x) \) contracted by a factor of \( a \) \((a>1)\)

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Basic Graphs You Should Know & Love

I) \( y = x \)

II) \( y = x^2, x^3, \ldots \)

III) \( y = x^3, x^5, \ldots \)

IV) \( y = |x| \)

V) \( \frac{1}{x} = y \)

VI) \( y = \frac{1}{x^2} \)

VII) \( y = b^x \) (for some \( b \))

VIII) \( y = \log_b x \) (for some \( b \))

Note: Make sure you are also familiar with the trigonometric graphs, which is depicted on 'trigonometry' online handout.