Math 21B
Vogler
Worksheet 2

1.) Compute the following indefinite integrals.

a.) $\int 2xe^{x^2}\,dx$ b.) $\int \cos 5x\,dx$ c.) $\int \sin x \cos x\,dx$ (HINT: $D\{\sin x\} = \cos x$.)

d.) $\int \left(\frac{3}{x} - \frac{1}{1+x}\right)dx$ e.) $\int \frac{x(x-1)^2}{x+1}\,dx$ (HINT: Use polynomial division.)

f.) $\int \frac{1}{1+x^2}\,dx$ g.) $\int \frac{e^x}{1+e^{2x}}\,dx$ (HINT: See f.)

2.) Determine the function $y = f(x)$ which satisfies the given conditions for each of the following.

a.) $\frac{dy}{dx} = \frac{1}{\sqrt{x}} + \sqrt{x}$ and $f(1) = 0$

b.) $\frac{d^2y}{dx^2} = e^x - 1$ and $f'(0) = 0, f(0) = -1$

3.) Evaluate the following sums.

a.) $\sum_{i=1}^{n} 9$ b.) $\sum_{i=1}^{1053} 9$ c.) $\sum_{i=34}^{867} 9$ d.) $\sum_{i=1}^{n} i(2i+3)$

e.) $\sum_{i=1}^{60} (5i - i^2)$ f.) $\sum_{i=26}^{62} (5i - i^2)$ g.) $\sum_{i=1}^{n} (\ \ln(i+2) - \ln(i+1)\ )$

h.) $\sum_{i=1}^{4} \cos \pi i$ i.) $\sum_{i=1}^{17} \cos \pi i$ j.) $\sum_{i=1}^{n} \cos \pi i$

4.) Consider the following sequence of numbers : 1, 5, 9, 13, 17, 21, 25, ... Find the sum of the 61st through the 127th numbers in this sequence.

5.) Prove that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

6.) The density at a point along a thin rod two feet long is given by $x^2 + 1$ (gm./ft.), where x is the distance (ft.) from the point to the left end of the rod. Use four equal subdivisions and midpoints (sampling points) to estimate the total mass of the rod.

7.) a.) Sketch the graph of $y = x^3$ on the interval $[0, 1]$.

b.) Estimate the area of the region below the graph of $y = x^3$ and above $[0, 1]$ using rectangles above three equal subdivisions and

i.) left endpoints of the subdivisions.
ii.) right endpoints of the subdivisions.
iii.) midpoints of the subdivisions.

8.) Differentiate each : a.) $F(x) = \int_{-100}^{\sin 3x} \sqrt{1+t^2}\, dt$ b.) $F(x) = \int_{\tan x}^{\arctan x} 5^{t^2}\, dt$

9.) Determine an equation of the line tangent to the graph of $F(x) = 3 + 2x + x \int_1^x \arctan t\, dt$ at $x = 1$.

10.) Use the limit definition of a definite integral to evaluate $\int_{-1}^{1} (x^2 - 2x + 3)\, dx$.

11.) Use the limit definition of the definite integral to evaluate $\int_1^5 \frac{1}{x^2}\, dx$; use an arbitrary partition $1 = x_0 < x_1 < x_2 < ... < x_{n-1} < x_n = 5$ of the interval [1, 5] and use sampling points $c_i = \sqrt{x_{i-1}x_i}$ for $i = 1, 2, 3, ..., n$.

12.) A thin rod is 16 cm. long. Its density x cm. from its left end is given by $2 + (x)^{-1/2}$ gm./cm.

a.) Estimate the rod's total mass by using four equal subdivisions and midpoints to estimate densities.
b.) Set up a definite integral and compute the exact mass of the rod.

13.) The following limit is equal to a definite integral : $\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 + \frac{3i}{n}\right)^5 \left(\frac{6}{n}\right)$. Determine a definite integral by starting with the suggested formula for the right-hand endpoint, x_i. Do not evaluate the definite integral.

a.) $x_i = 4 + \frac{3i}{n}$ b.) $x_i = \frac{3i}{n}$ c.) $x_i = \frac{i}{n}$

14.) Each of the following limits is equal to a definite integral. Determine a definite integral for each. Do not evaluate the definite integral.

a.) $\lim_{n\to\infty} \sum_{i=1}^{n} 3\left(1 + \frac{2i}{n}\right)^{-4} \left(\frac{2}{n}\right)$ b.) $\lim_{n\to\infty} \sum_{i=1}^{n} \ln\left(3 + \frac{2i}{n}\right)\left(\frac{8}{n}\right)$

c.) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{in + n^2}$ d.) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{(n+3i)^2}{n^3}$

15.) Evaluate the following definite integrals.

a.) $\int_0^1 (x + e^x)\, dx$ a.) $\int_0^{\frac{\pi}{2}} \cos x\, e^{\sin x}\, dx$ b.) $\int_{-1}^{1} 5^x\, dx$ c.) $\int_0^{\pi/4} 5\sec^2 3x\, dx$

16.) Find the average value of each of the following functions over the given interval. Draw a sketch showing the connection between your answer and the definite integral.

a.) $f(x) = x^3 + 1$ on $[-1, 1]$ b.) $f(x) = 5 + \sqrt{x}$ on $[0, 4]$

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

17.) Write a formula for the nth term in the following sequence : 1, 3, 1, 3, 1, 3, 1, 3, ...

Thanks to Dr. Kouba