Math 21B
Vogler
Worksheet 9

1.) Use any method to determine the following indefinite integrals (antiderivatives).

a.) $\int \frac{x+2}{x^2+4x+5}\,dx$ b.) $\int \frac{x+1}{x^2+4x+5}\,dx$ c.) $\int \frac{x+4}{x^2+4x+3}\,dx$

f.) $\int \frac{x}{x^2+4x+13}\,dx$ g.) $\int \frac{1}{25x^2+9}\,dx$

2.) Use partial fractions to integrate the following.

a.) $\int \frac{x^2}{x^2-1}\,dx$ b.) $\int \frac{x+3}{(x-1)^2(x+2)}\,dx$

c.) $\int \frac{7-x^2}{(x^2+4)(x+4)^2}\,dx$ d.) $\int \frac{1}{x^3+1}\,dx$

3.) Write the partial fractions decomposition for each. DO NOT SOLVE FOR THE UNKNOWN CONSTANTS !

a.) $\frac{x^2+7x-5}{(7x^2+3)^2\; x^2\; (x+3)^3}$ b.) $\frac{1}{x^4+x^2+1}$ c.) (challenging) $\frac{1}{x^4+1}$

4.) Integrate $\int \frac{1}{x(x^2+1)^2}\,dx$ using

a.) trig substitution. b.) partial fractions.

5.) Assume that $f''(x) = x^2 e^{3x}$. Find a number M so that $\max_{0 \le x \le 2} |f''(x)| \le M$.

6.) Assume that $f''(x) = \frac{7}{2x+3}$. Find a number M so that $\max_{-1 \le x \le 1} |f''(x)| \le M$.

7.) Assume that $f^{(4)}(x) = \frac{x-3}{5-x}$. Find a number M so that $\max_{-2 \le x \le 3} |f^{(4)}(x)| \le M$.

8.) Consider the definite integral $\int_0^4 \ln(x^2+1)\,dx$.

a.) Find T_4 , the Trapezoidal Estimate using $n = 4$.
b.) What should n be in order that the Trapezoidal Estimate T_n estimate the exact value of this definte integral with absolute error at most 0.0001 ?

9.) Consider the definite integral $\int_0^2 \frac{x+1}{x+3}\,dx$.

a.) Find S_4 , the Simpson Estimate using $n = 4$.
b.) What should n be in order that the Simpson Estimate S_n estimate the exact value of this definte integral with absolute error at most 0.0001 ?

10.) Consider the region R lying below the graph of $y = \frac{1}{x}$ and above the x-axis on the interval $[1, \infty)$.

a.) Determine if R has finite or infinite area.

b.) Form a solid by revolving R about the x-axis. Determine if the resulting volume is finite or infinite.

c.) Form a solid by revolving R about the y-axis. Determine if the resulting volume is finite or infinite.

11.) Compute the following improper integrals.

a.) $\int_1^\infty \frac{1}{x(x+4)}\,dx$ b.) $\int_{-\infty}^0 e^{3x}\,dx$ c.) $\int_{-1}^\infty \frac{1}{\sqrt{x+1}}\,dx$

d.) $\int_{-\infty}^{\sqrt{3}} \frac{1}{x^2+9}\,dx$ e.) $\int_1^{e^2+1} \frac{7}{x-1}\,dx$ f.) $\int_0^e x^2 \ln x\,dx$

g.) $\int_2^\infty \frac{1}{x(\ln x)^2}\,dx$ h.) $\int_0^5 \frac{1}{\sqrt{25-x^2}}\,dx$ i.) $\int_1^\infty \frac{24}{2x^2+5x+2}\,dx$

j.) $\int_0^5 \frac{8x}{x^2-9}\,dx$ k.) $\int_{-\infty}^\infty x^2 e^{x^3}\,dx$ l.) $\int_0^{\pi/2} \csc x \cot x\,dx$

m.) $\int_0^\infty x e^{-5x}\,dx$ n.) $\int_0^\infty \frac{1}{x^2}\,dx$ o.) $\int_0^1 \ln x\,dx$ p.) $\int_0^\infty \frac{e^{-1/x}}{x^2}\,dx$

12.) Use the Comparison Test to show that the following improper integral converges, i.e., is finite : $\int_1^\infty \frac{1}{\sqrt{x^3+16}}\,dx$

13.) Use the Comparison Test to show that the following improper integral diverges, i.e., is infinite : $\int_1^\infty \frac{x+4}{\sqrt{x^3+16}}\,dx$

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" What you have been obliged to discover by yourself leaves a path in your mind which you can use again when the need arises." – G. Lichtenberg

" I hear and I forget. I see and I remember. I do and I understand." – Chinese Proverb

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