Math 210

Vogler

**Average Value of** $f(x,y)$
**over** $2D$ **region** $R$

Consider a solid $D$ beneath the surface $z = f(x,y)$ (i.e. height function) & above $2D$ region $R$.

To find average value/height, we find a flat-topped cylinder whose volume equals that of original solid $D$

$$
\iiint_R f(x,y) \, dA = \text{Vol. of } D = \text{Vol. of Cylinder} = (\text{Ave. Height})(\text{Area of } R)
$$

Algebraic manipulation leads to following:

**Defn** The **Average Value of** $f(x,y)$ on region $R$ is

$$
\text{Ave} := \frac{1}{\text{Area of } R} \iiint_R f(x,y) \, dA = \frac{1}{\iint_R 1 \, dA} \iint_R f(x,y) \, dA.
$$

(*) One can imagine solid $D$ as a block of ice in a cylindrical cup & melting, resulting flat-topped cylindrical cup of water.