Math 21D
Constructing Bounds of Integration
for \( \iint_R f(x,y) \, dA \)

a) vertical cross-sections \( (dA = dy \, dx) \)
- Region \( R \) is bounded by graphs
  - \( x = a \), \( x = b \), \( y = g(x) \), \( y = h(x) \)
  - (Graph)

\[
\begin{align*}
\text{Described region } R &: \quad dx: \ a \leq x \leq b \\
& \quad g(x) \leq y \leq h(x)
\end{align*}
\]

Integral of \( f(x,y) \) over \( R \):
\[
\iint_R f(x,y) \, dA = \int_a^b \int_{g(x)}^{h(x)} f(x,y) \, dy \, dx
\]

b) horizontal cross-sections \( (dA = dx \, dy) \)
- Region \( R \) is bounded by graphs
  - \( y = c \), \( y = d \), \( x = g(y) \), \( x = h(y) \)
  - (Graph)

\[
\begin{align*}
\text{Described region } R &: \quad dy: \ c \leq y \leq d \\
& \quad g(y) \leq x \leq h(y)
\end{align*}
\]

Integral of \( f(x,y) \) over \( R \):
\[
\iint_R f(x,y) \, dA = \int_c^d \int_{g(y)}^{h(y)} f(x,y) \, dx \, dy
\]

Notes:
1) Outside bounds of integration **MUST** be numbers.
2) Inside bounds of integration **MUST** be functions of outside variables (Numbers are OK).