Divergence Theorem

Definition: Let \( \mathbf{F}(x,y,z) = M(x,y,z) \mathbf{i} + N(x,y,z) \mathbf{j} + P(x,y,z) \mathbf{k} \) be a vector field defined on solid \( D \). The divergence of \( \mathbf{F} \) at \( (x,y,z) \) is the scalar function

\[
\text{div} \, \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} = M_x + N_y + P_z
\]

Note: Divergence can be shown to be a measure of expansion (\( \text{div} \, \mathbf{F} > 0 \)) or compression (\( \text{div} \, \mathbf{F} < 0 \)) of a fluid or gas at a point \( (x,y,z) \).

Divergence Theorem

Let \( \mathbf{F}(x,y,z) = M(x,y,z) \mathbf{i} + N(x,y,z) \mathbf{j} + P(x,y,z) \mathbf{k} \) be a vector field defined on a solid \( D \), enclosed by surface \( S \) with outward-pointing unit normal vector \( \mathbf{n} \).

\[
\Rightarrow \quad \iiint_D \text{div} \, \mathbf{F} \, dV = \oiint_S \mathbf{F} \cdot \mathbf{n} \, dS
\]

Outward Flux across Boundary

Divergence Integral or Expansion Rates across Interior

Surface

Solid \( D \)