The Derivative of a Function

\[ y = f(x) \]

Secant line with slope \( \frac{f(x+\Delta x)-f(x)}{\Delta x} = \text{ARC on } [x, x+\Delta x] \)

\[ \Delta f = f(x+\Delta x)-f(x) \text{ rise} \]

Tangent line with slope \( f'(x) = \text{IRC at } x \)

Note that the slope of the secant line is

\[ \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{f(x+h)-f(x)}{\Delta x} \]

We can obtain slope of tangent line by letting \( h = \Delta x \) get `small' (i.e. \( \Delta x \to 0 \)) which leads to the following:

\text{Defn} The derivative of } f \text{ at } x \text{ is

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \]

Notes: 1) \( f'(x) \) is the slope of tangent line through point \((x, f(x))\).

2) \( \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} \approx \frac{\Delta f}{\Delta x} \) can be considered infinitesimal division (i.e. division by two `small' numbers).

3) The derivative is sometimes referred to as the Instantaneous Rate of Change (IRC).

\text{Defn} The Average Rate of Change (ARC) of a function \( y = f(x) \) on interval \([a, b]\) is

\[ \text{A.R.C.} = \frac{f(b)-f(a)}{b-a} \]

Note: ARC is slope of secant line between \((a, f(a)) \& (b, f(b))\).