1. (*pts*) Prove that $2^n > (n+1)^2$ for all integers $n \geq 6$ using induction.

2. (*pts*) Prove that $\sqrt[3]{6-\sqrt{3}}$ is irrational using the Rational Zero's Theorem.

3. (*pts*) Use the definition (i.e. ϵN-property) to prove that

$$\lim_{n\to\infty} \frac{5n}{n+2} = 5$$

4. (*pts*) For the following sets, determine its maximum, minimum, supremum, and infimum.

   (a) $A = \cap_{n=1}^{\infty} \left[-\frac{1}{n}, 1 + \frac{1}{n}\right]$

   (b) $B = \left\{\frac{1}{n} : n \in \mathbb{N} \text{ and } n \text{ is prime}\right\}$

   (c) $C = \left\{\cos\left(\frac{n\pi}{3}\right) : n \in \mathbb{N}\right\}$

5. (*pts*) For the next two problems, use the Ordered Field Axioms and Theorems 3.1 and 3.2 (These will be given to you on the exam).

   (a) Prove if $ab = 0$, then $a = 0$ or $b = 0$.

   (b) Prove if $a \leq b$ and $c \leq d$, then $a + c \leq b + d$.

   (c) (*pts*) Let $x \in \mathbb{R}$. Prove that if x^3 is irrational, then x is irrational.

6. (*pts*) Consider the sequence $s_n = (-1)^n$. Use the definition (i.e. ϵN-property) to prove that

$$\lim_{n\to\infty} s_n \neq -1$$

7. (*pts*) For the following sequences, determine whether it converges and, if so, give its limit. Otherwise, state it diverges. You may use any method you wish to determine this.

   (a) $a_n = \dfrac{7n^4 + 8n}{2n^3 - 31}$

   (b) $b_n = 51 + \dfrac{\sin n}{n}$

   (c) $c_n = \left(1 + \dfrac{5}{n}\right)^n$

8. (*pts*) Prove that for all $x \in \mathbb{I}$ you can construct a sequence of rational numbers $\{x_n\}$ which converges to x. Note: Just show how to build the sequence. You DO NOT have to prove the convergence.

The following extra credit problems are OPTIONAL and you are advised to finish the rest of the test before trying these problems.

1. (*pts*) Prove that $\sup \{r \in \mathbb{Q} : r < a\} = a, \forall a \in \mathbb{R}$.