1. (*pts*) Use the definition (i.e. MN property) to prove that

$$\lim_{n\to\infty} \frac{n^3+4n+6}{3n^2+5n+7} = \infty.$$

2. (*pts*) Prove $\lim_{n\to\infty} \frac{3n+7}{6n-5} = \frac{1}{2}$ using Theorems 9.2-9.7. These Theorems (or any other necessary ones) will be provided on the exam.

3. (*pts*) For the following sequence, determine its set of subsequential limits, its $\limsup_{n\to\infty}$ and $\liminf_{n\to\infty}$, whether it converges or diverges, whether it is monotonically non-increasing or non-decreasing, and whether it is bounded.

   (a) $a_n = (-2)^n$

   (b) $b_n = 5^{(-1)^n}$

   (c) $c_n = -\frac{n}{4} + \left\lceil \frac{n}{4} \right\rceil + (-1)^n$

4. (*pts*) Let $t_1 = 1$ and $t_{n+1} = \left(1 - \dfrac{1}{4n^2}\right) t_n$ for $n \geq 1$. Prove this sequence $\{t_n\}$ converges.

5. (*pts*) Prove if $\lim\limits_{n\to\infty} s_n = s$ and $\lim\limits_{n\to\infty} t_n = t$, then $\lim\limits_{n\to\infty} s_n + t_n = s + t$ using the definition (i.e. ϵN property).

6. (*pts*) Consider the sequence $s_n = (-1)^n + \cos\left(\frac{n\pi}{2}\right)$. Find the set of subsequential limits S by providing a subsequence that converges to each subsequential limit point.

7. (*pts*) Let $\{s_n\}$ and $\{t_n\}$ be bounded sequences with $s_n \le t_n \ \forall n \in \mathbb{N}$. Prove that $\limsup_{n\to\infty} \ s_n \le \limsup_{n\to\infty} \ t_n$. (Note: Since the sequences are bounded, their lim sup's must be finite).

8. (*pts*) Determine whether the following statements are TRUE or FALSE. If true, BRIEFLY justify the given statement; if false, give a counterexample to the statement.

   (a) For $a, b \in \mathbb{R}$, there exists a sequence where the set of subsequential limits is $[a, b]$.

   (b) If $\{s_n\}$ is a Cauchy sequence, then $\{s_n\}$ is a monotone sequence.

   (c) For a given sequence $\{t_n\}$, if the corresponding sequence $\{\sin(t_n)\}$ converges, then $\{t_n\}$ converges.

9. (*pts*)

   (a) Let $\{s_n\}$ be sequence such that $|s_{n+1} - s_n| < \frac{1}{7^n}$ $\forall n \in \mathbb{N}$. Prove that

$$|s_m - s_n| < \frac{1}{7^{n-1}} \quad \forall n, m \in \mathbb{N} \text{ with } m > n. \tag{1}$$

   (b) Use inequality (1) found in part (a) to prove the sequence $\{s_n\}$ converges.

The following extra credit problems are OPTIONAL and you are advised to finish the rest of the test before trying these problems.

1. (*pts*) Give an example of a sequence that has subsequences converging to every real number in $[0, 1]$ and no other real numbers.