Math 25
Vogler

Induction Proof Example

Ex Using induction, prove

$$1^2+3^2+5^2+\dots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3} \quad \forall n \in \mathbb{N}.$$

Pf i) This is true for $n=1$ since $1 = 1^2 = \frac{1\cdot 1\cdot 3}{3} = 1$.

ii) Let $n \in \mathbb{N}$. § $1^2+3^2+\dots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$ is true.

Then, we have

$$1^2+3^2+5^2+\dots+(2n-1)^2+(2(n+1)-1)^2 = 1^2+3^2+\dots+(2n-1)^2+(2n+1)^2$$

$$= \frac{n(2n-1)(2n+1)}{3} + (2n+1)^2\cdot\frac{3}{3} = \frac{n(2n-1)(2n+1)+3(2n+1)^2}{3}$$

$$= \frac{[2n^2-n+6n+3](2n+1)}{3} = \frac{(2n^2+5n+3)(2n+1)}{3} = \frac{(n+1)(2n+1)(2n+3)}{3}$$

$$= \frac{(n+1)[2(n+1)-1][2(n+1)+1]}{3}.$$

So the statement is true for $n+1$.

Therefore, by the Principle of Mathematical Induction (PMI), we conclude

$$1^2+3^2+5^2+\dots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3} \quad \forall n \in \mathbb{N}. \blacksquare$$