Section 3.9

2.) a.) $\arctan(-1) = -\frac{\pi}{4}$

b.) $\arctan\sqrt{3} = \pi/3$

c.) $\arctan\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

3.) a.) $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

b.) $\arcsin\left(1/\sqrt{2}\right) = \frac{\pi}{4}$

c.) $\arcsin\left(-\sqrt{3}/2\right) = -\frac{\pi}{3}$

5.) a.) $\arccos\left(1/2\right) = \pi/3$

b.) $\arccos\left(-1/\sqrt{2}\right) = \frac{3\pi}{4}$

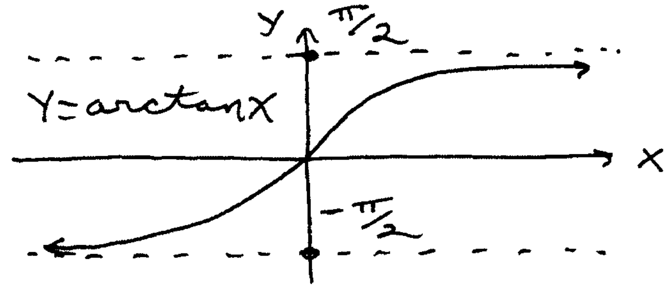
c.) $\arccos\left(\sqrt{3}/2\right) = \frac{\pi}{6}$

11.) $\tan\left(\arcsin\left(-1/2\right)\right) = \tan\left(-\pi/6\right)$

$= \frac{-1/2}{\sqrt{3}/2} = \frac{-1}{\sqrt{3}}$

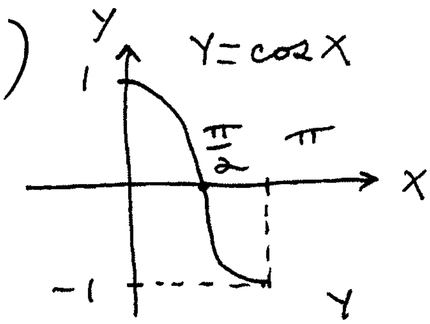
15.) (SEE graph.)

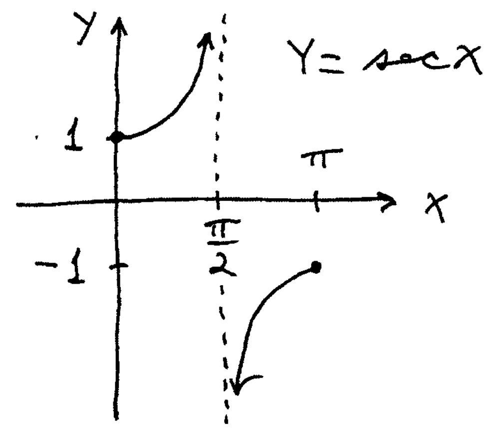
$$\lim_{x \to \infty} \arctan x = \frac{\pi}{2}$$

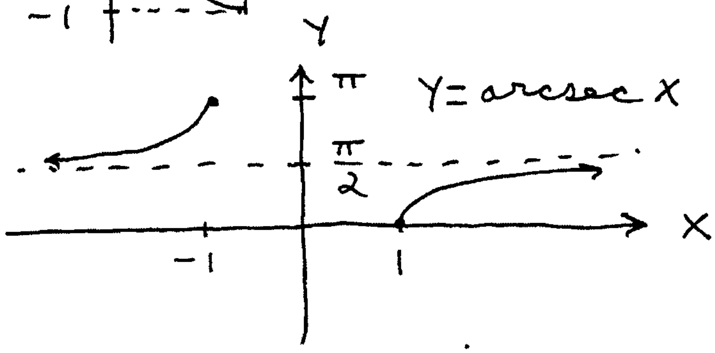


16.) (SEE graph.) $\lim_{x \to -\infty} \arctan x = -\frac{\pi}{2}$

17.)







(SEE graph.) $\lim_{x \to \infty} \operatorname{arcsec} x = \frac{\pi}{2}$

18.) (SEE graph.) $\lim_{x \to -\infty} \operatorname{arcsec} x = \frac{\pi}{2}$

21.) $y = \arccos(x^2) \xrightarrow{D} y' = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot (2x)$

24.) $Y = \arcsin(1-t) \xrightarrow{D} Y' = \dfrac{1}{\sqrt{1-(1-t)^2}} \cdot (-1)$

27.) $Y = \operatorname{arccsc}(x^2+1) \xrightarrow{D}$

$Y' = \dfrac{-1}{|x^2+1|\sqrt{(x^2+1)^2-1}} \cdot 2x$

29.) $Y = \operatorname{arcsec}(1/t) \longrightarrow$

$Y' = \dfrac{1}{\left|\frac{1}{t}\right| \cdot \sqrt{\left(\frac{1}{t}\right)^2 - 1}} \cdot \dfrac{-1}{t^2}$

34.) $Y = \arctan(\ln x) \xrightarrow{D}$

$Y' = \dfrac{1}{1+(\ln x)^2} \cdot \dfrac{1}{x}$

36.) $Y = \arccos(e^{-t}) \xrightarrow{D}$

$Y' = \dfrac{-1}{\sqrt{1-(e^{-t})^2}} \cdot e^{-t} \cdot (-1)$

39.) $Y = \arctan\sqrt{x^2-1} + \operatorname{arccsc} x \xrightarrow{D}$

$Y' = \dfrac{1}{1+(\sqrt{x^2-1})^2} \cdot \dfrac{1}{\cancel{2}}(x^2-1)^{-1/2} \cdot \cancel{2}x + \dfrac{-1}{|x|\sqrt{x^2-1}}$

$= \dfrac{x}{(\cancel{1}+x^2-\cancel{1})\sqrt{x^2-1}} - \dfrac{1}{x\sqrt{x^2-1}} \quad (x > 1)$

$= \dfrac{1}{x\sqrt{x^2-1}} - \dfrac{1}{x\sqrt{x^2-1}} = 0$

41.) $Y = x \cdot \arcsin x + \sqrt{1-x^2} \xrightarrow{D}$

$$Y' = x \cdot \frac{1}{\sqrt{1-x^2}} + (1) \arcsin x + \frac{1}{\cancel{2}}(1-x^2)^{-1/2} \cdot (-\cancel{2}x)$$

$$= \cancel{\frac{x}{\sqrt{1-x^2}}} + \arcsin x + \cancel{\frac{-x}{\sqrt{1-x^2}}}$$

$$= \arcsin x$$

42.) $Y = \ln(x^2+4) - x \cdot \arctan\left(\frac{x}{2}\right) \xrightarrow{D}$

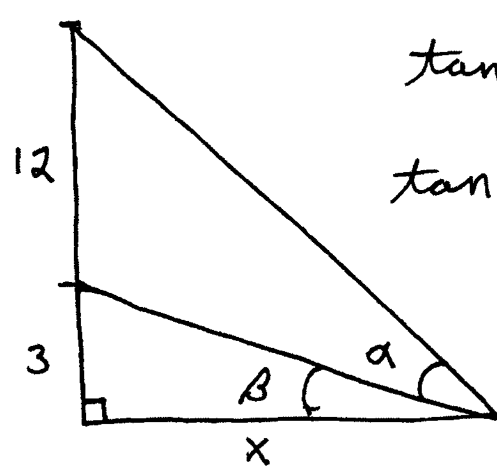
$$Y' = \frac{2x}{x^2+4} - \left\{ x \cdot \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} + (1) \cdot \arctan\left(\frac{x}{2}\right) \right\}$$

$$= \frac{2x}{x^2+4} - \frac{x}{2+\frac{x^2}{2}} \cdot \frac{2}{2} - \arctan\left(\frac{x}{2}\right)$$

$$= \cancel{\frac{2x}{x^2+4}} - \cancel{\frac{2x}{4+x^2}} - \arctan\left(\frac{x}{2}\right)$$

$$= -\arctan\left(x/2\right)$$

43.)



$\tan\beta = \frac{3}{x} \rightarrow \beta = \arctan\left(\frac{3}{x}\right);$

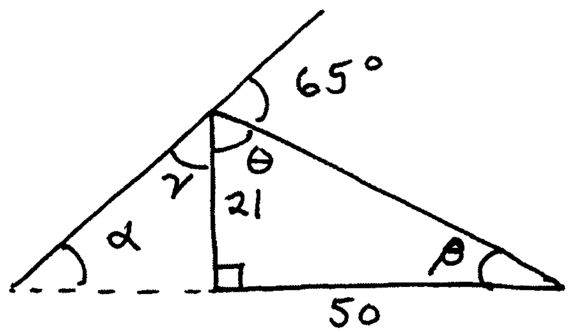
$\tan(\alpha+\beta) = 15/x \longrightarrow$

$\alpha+\beta = \arctan\left(15/x\right) \longrightarrow$

$$\alpha + \arctan\left(\frac{3}{x}\right) = \arctan\left(\frac{15}{x}\right) \longrightarrow$$

$$\alpha = \arctan\left(\frac{15}{x}\right) - \arctan\left(\frac{3}{x}\right) \quad \text{OR}$$
$$\alpha = \operatorname{arccot}\left(\frac{x}{15}\right) - \operatorname{arccot}\left(\frac{x}{3}\right)$$

44.)



$$\tan\beta = \frac{21}{50} \rightarrow \beta = \arctan\left(\frac{21}{50}\right)$$

so $\underline{\theta = 90° - \beta = 90° - \arctan\left(\frac{21}{50}\right)}$;

$$\gamma = 180° - 65° - \theta$$
$$= 115° - \left(90° - \arctan\left(21/50\right)\right) \rightarrow$$

$\underline{\gamma = 25° + \arctan\left(21/50\right)}$; then

$$\alpha = 90° - \gamma$$
$$= 90° - \left(25° + \arctan\left(21/50\right)\right) \rightarrow$$

$$\boxed{\alpha = 65° - \arctan\left(21/50\right)}$$