

Section 4.4

1.) $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + \frac{1}{3}$, Domain: all x-values

$$y' = x^2 - x - 2 = (x-2)(x+1) = 0 \rightarrow x=-1, x=2$$

$$\begin{array}{r} + \quad 0 \quad - \quad 0 \quad + \\ \hline \end{array} \quad y^1$$

$$\text{rel. max. } \left\{ \begin{array}{ll} x = -1 & x = 2 \\ y = \frac{3}{2} & y = -3 \end{array} \right\} \text{ rel. min.}$$

$$y'' = 2x - 1 = 0 \rightarrow x = \frac{1}{2}$$

$$\begin{array}{c} - \quad 0 \quad + \\ \hline \end{array} \quad Y^{11}$$

$$\text{inf. } \left. \begin{array}{l} x = \frac{1}{2} \\ y = -\frac{3}{4} \end{array} \right\} \text{pt.}$$

y is \uparrow for $x < -1, x > 2$;

y is \downarrow for $-1 < x < 2$

$y \in U$ for $x > \frac{1}{2}$;

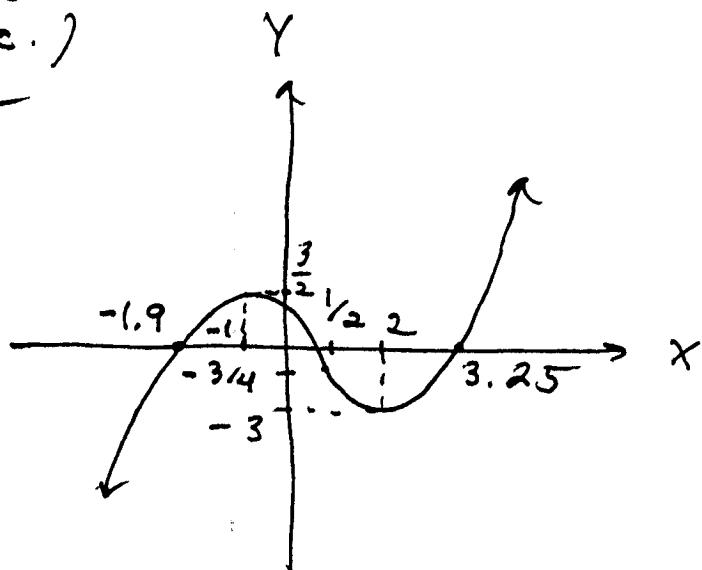
y is \wedge for $x < y_2$;

$$x=0 : y = \frac{1}{3}$$

$$Y=0 : \frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{1}{3} = 0$$

(use graphing calc.)

$$x \approx -1.9, 0.15, 3.25$$



2.) $y = \frac{x^4}{4} - 2x^2 + 4$, Domain: all x -values,
 $y' = x^3 - 4x = x(x-2)(x+2) = 0 \rightarrow$
 $x=0, x=2, x=-2$

$-$	0	$+$	0	$-$	0	$+$	y'
	$x = -2$	$x = 0$		$x = 2$			
	$\underbrace{y = 0}$ abs. min.	$\underbrace{y = 4}$ rel. max.		$\underbrace{y = 0}$ abs. min.			

$$y'' = 3x^2 - 4 = 0 \rightarrow x = \pm \frac{2}{\sqrt{3}}$$

$+$	0	$-$	0	$+$	y''
infl.	$\left\{ \begin{array}{l} x = -\frac{2}{\sqrt{3}} \\ y = \frac{16}{9} \end{array} \right.$	$\left\{ \begin{array}{l} x = \frac{2}{\sqrt{3}} \\ y = \frac{16}{9} \end{array} \right.$			infl.
pt.	$\left\{ \begin{array}{l} y = \frac{16}{9} \end{array} \right.$	$\left\{ \begin{array}{l} y = \frac{16}{9} \end{array} \right.$			pt.

y is \uparrow for $-2 < x < 0, x > 2$;
 y is \downarrow for $x < -2, 0 < x < 2$;
 y is \cup for $x < -\frac{2}{\sqrt{3}}, x > \frac{2}{\sqrt{3}}$;
 y is \cap for $-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$.

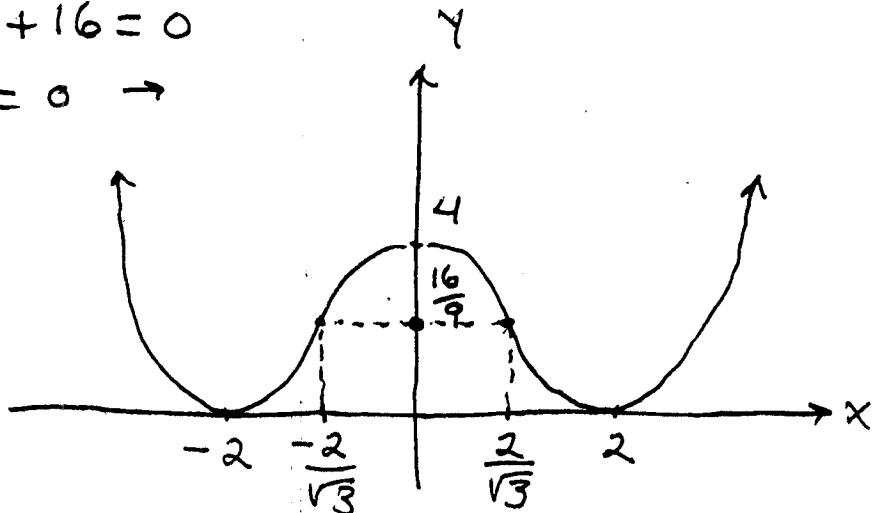
$$x=0: y=4$$

$$y=0: \frac{1}{4}(x^2)^2 - 2(x^2) + 4 = 0$$

$$\rightarrow (x^2)^2 - 8(x^2) + 16 = 0$$

$$(x^2 - 4)(x^2 + 4) = 0 \rightarrow$$

$$x=2, x=-2$$



$$10.) \quad Y = 6 - 2x - x^2, \quad \text{Domain: all } x\text{-values},$$

$$Y' = -2 - 2x = -2(1+x) = 0 \rightarrow x = -1$$

$$\begin{array}{c} + \\ \hline \text{abs.} \end{array} \quad \begin{array}{c} 0 \\ \text{max.} \end{array} \quad \begin{array}{c} - \\ \hline Y' \end{array}$$

$$\begin{array}{c} - \\ \hline Y'' \end{array}$$

$$Y'' = -2 \quad \begin{array}{c} - \\ \hline - \end{array} \quad \begin{array}{c} - \\ \hline Y'' \end{array}$$

Y is \uparrow for $x < -1$;

Y is \downarrow for $x > -1$;

Y is \wedge for all x -values

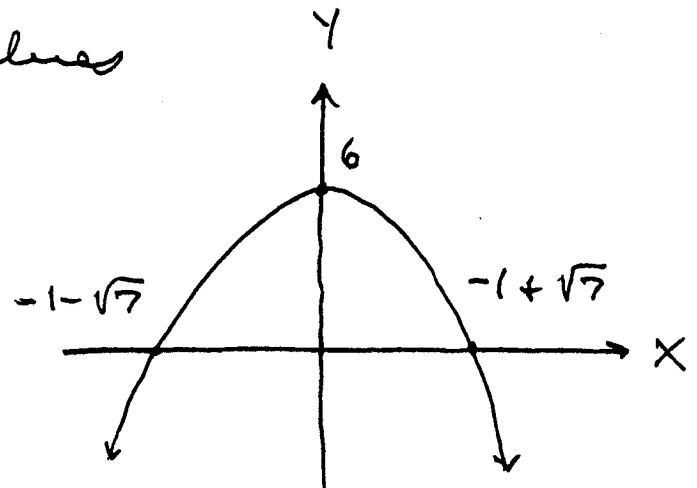
$$x=0: Y=6$$

$$Y=0: x^2+2x-6=0$$

$$\rightarrow x = \frac{-2 \pm \sqrt{4-(-24)}}{2}$$

$$= \frac{-2 \pm 2\sqrt{7}}{2}$$

$$= -1 \pm \sqrt{7}$$



$$17.) \quad Y = x^4 - 2x^2, \quad \text{Domain: all } x\text{-values}$$

$$Y' = 4x^3 - 4x = 4x(x-1)(x+1) = 0 \rightarrow$$

$$x=0, x=1, x=-1$$

$$\begin{array}{c} - \\ \hline \end{array} \quad \begin{array}{c} 0 \\ \text{abs. min.} \end{array} \quad \begin{array}{c} + \\ \text{rel. max.} \end{array} \quad \begin{array}{c} - \\ \hline \end{array} \quad \begin{array}{c} 0 \\ \text{abs. min.} \end{array} \quad \begin{array}{c} + \\ \hline Y' \end{array}$$

$$\begin{array}{c} x=-1 \\ Y=-1 \end{array} \quad \begin{array}{c} x=0 \\ Y=0 \end{array} \quad \begin{array}{c} x=1 \\ Y=-1 \end{array}$$

$$\begin{array}{c} \text{abs. min.} \\ \text{max.} \end{array} \quad \begin{array}{c} \text{rel.} \\ \text{min.} \end{array} \quad \begin{array}{c} \text{abs. min.} \\ \text{max.} \end{array}$$

$$Y'' = 12x^2 - 4 = 4(3x^2 - 1) = 0 \rightarrow x = \pm \frac{1}{\sqrt{3}}$$

$$\begin{array}{c} + \\ \hline + & - & + \end{array} \quad Y''$$

infl. pt. $\left\{ \begin{array}{l} x = \frac{-1}{\sqrt{3}} \\ y = -\frac{5}{9} \end{array} \right. \quad \left\{ \begin{array}{l} x = \frac{1}{\sqrt{3}} \\ y = \frac{5}{9} \end{array} \right. \right\}$ infl. pt.

y is \uparrow for $-1 < x < 0, x > 1;$

y is \downarrow for $x < -1, 0 < x < 1;$

y is \cup for $x < -\frac{1}{\sqrt{3}}, x > \frac{1}{\sqrt{3}};$

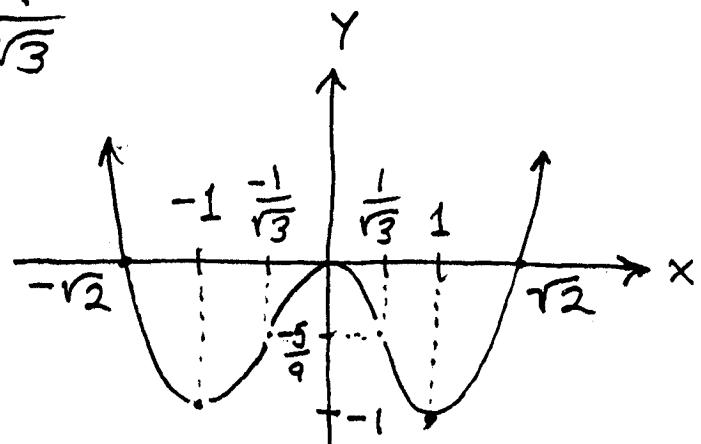
y is \cap for $\frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

$$x=0: y=0$$

$$y=0: x^4 - 2x^2 = 0$$

$$\rightarrow x^2(x^2 - 2) = 0$$

$$\rightarrow x=0, x=\pm\sqrt{2}$$



21.) $y = x^5 - 5x^4$, Domain: all x -values,

$$y' = 5x^4 - 20x^3 = 5x^3(x-4) = 0$$

$$\begin{array}{c} + \\ \hline + & - & + \end{array} \quad y'$$

rel. $\left\{ \begin{array}{l} x=0 \\ y=0 \end{array} \right. \quad \left\{ \begin{array}{l} x=4 \\ y=-256 \end{array} \right. \right\}$ rel.

max. $\left\{ \begin{array}{l} y=0 \\ y=-256 \end{array} \right. \right\}$ min.

$$y'' = 20x^3 - 60x^2 = 20x^2(x-3) = 0$$

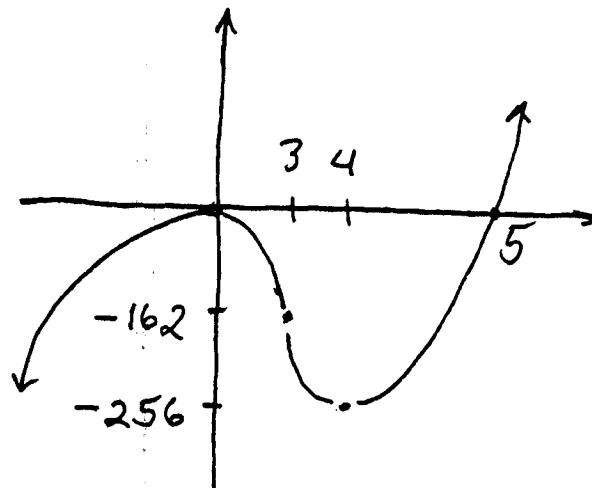
$$\begin{array}{c} - \\ \hline - & + & - & + \end{array} \quad y''$$

$$x=0$$

$$x=3$$

$\left\{ \begin{array}{l} y=-162 \\ y=-162 \end{array} \right. \right\}$ infl. pt.

$$\begin{aligned}
 x=0 : y=0 \\
 y=0 : x^5 - 5x^4 = 0 \\
 \rightarrow x^4(x-5) = 0 \\
 \rightarrow x=0, x=5
 \end{aligned}$$



23.) $y = x + \sin x \quad 0 \leq x \leq 2\pi$

$$y' = 1 + \cos x = 0 \rightarrow \cos x = -1 \rightarrow x = \pi$$

 abs. min.	0 $x=0$ $y=0$	0 $x=\pi$ $y=0$
		 y' abs. max.
		$x=2\pi$ $y=2\pi$

$$y'' = -\sin x = 0 \rightarrow x = 0, \pi, 2\pi$$

 $x=0$ inf. pt.	0 $x=\pi$ $y=\pi$	0 $x=2\pi$ $y=2\pi$
		 y''

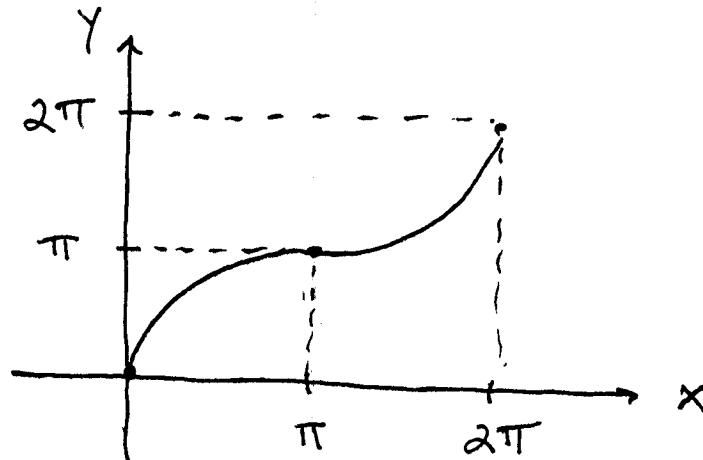
y is ↑ for $0 < x < \pi, \pi < x < 2\pi$;

y is U for $\pi < x < 2\pi$;

y is Λ for $0 < x < \pi$

$$x=0 : y=0$$

$$y=0 : x=0$$



$$29) Y = x^{1/5}$$

$$Y' = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}}$$

Domain: all x -values,

$$\begin{array}{c|cc} & + & + \\ \hline & | & | \\ & no & \end{array} \quad Y'$$

$\left. \begin{array}{l} x=0 \\ y=0 \end{array} \right\}$ vert.
 $\left. \begin{array}{l} y=0 \end{array} \right\}$ tan.

$$Y'' = -\frac{4}{25}x^{-9/5} = \frac{-4}{25x^{9/5}}$$

$$\begin{array}{c|cc} & + & - \\ \hline & | & | \\ & no & \end{array} \quad Y''$$

$\left. \begin{array}{l} x=0 \\ y=0 \end{array} \right\}$ infl.
 $\left. \begin{array}{l} y=0 \end{array} \right\}$ pt.

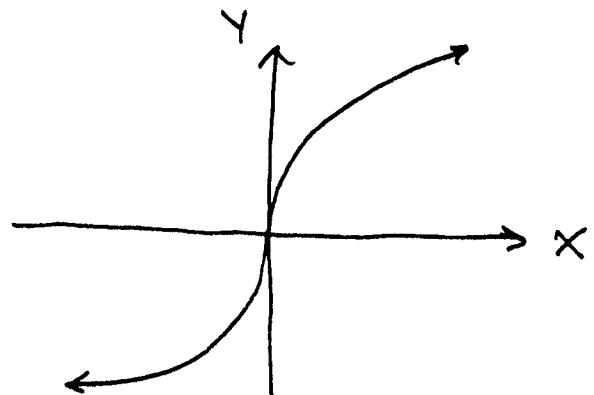
y is \uparrow for $x < 0, x > 0$;

y is \cup for $x < 0$;

y is \wedge for $x > 0$;

$$x=0 : y=0$$

$$y=0 : x=0$$



$$30) Y = x^{3/5}$$

$$Y' = \frac{3}{5}x^{-2/5} = \frac{3}{5x^{2/5}}$$

Domain: all x -values,

$$\begin{array}{c|cc} & - & + \\ \hline & | & | \\ & no & \end{array} \quad Y'$$

$\left. \begin{array}{l} x=0 \\ y=0 \end{array} \right\}$ abs. min.

$$Y'' = -\frac{6}{25}x^{-8/5} = -\frac{6}{25x^{8/5}}$$

$$\begin{array}{c|cc} & - & - \\ \hline & | & | \\ & no & \end{array} \quad Y''$$

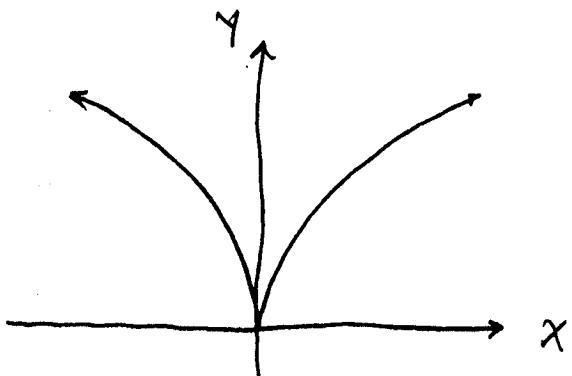
y is \uparrow for $x > 0$,

y is \downarrow for $x < 0$,

y is \wedge for $x < 0, x > 0$,

$$x=0 : y=0$$

$$y=0 : x=0$$



$$33) Y = 2x - 3x^{\frac{2}{3}}$$

Domain: all x -values,

$$Y' = 2 - \cancel{3} \cdot \frac{2}{\cancel{3}} x^{-\frac{1}{3}} = 2 - \frac{2}{x^{\frac{1}{3}}} = 2 \left(\frac{x^{\frac{1}{3}} - 1}{x^{\frac{1}{3}}} \right) = 0$$

$$\rightarrow x^{\frac{1}{3}} - 1 = 0 \rightarrow x = 1$$

$+$	NO	$-$
0	$+$	$+$

y'

corner $\begin{cases} x=0 \\ Y=0 \end{cases}$ rel. max.

$\begin{cases} x=1 \\ Y=-1 \end{cases}$ rel. min.

$$Y'' = -2 \cdot \frac{-1}{3} x^{-\frac{4}{3}} = \frac{2}{3x^{\frac{4}{3}}} = 0$$

$+$	NO	$+$
$x=0$	1	$+$

y''

Y is \uparrow for $x < 0, x > 1$,

Y is \downarrow for $0 < x < 1$,

Y is \cup for $x < 0, x > 1$;

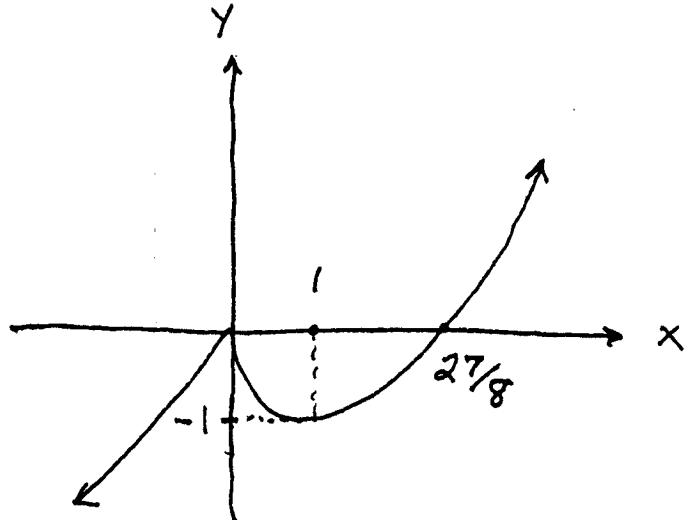
$$X=0: Y=0$$

$$Y=0: 2x - 3x^{\frac{2}{3}} = 0$$

$$\rightarrow x^{\frac{2}{3}}(2x^{\frac{1}{3}} - 3) = 0$$

$$\rightarrow x=0, x^{\frac{1}{3}} = \frac{3}{2}$$

$$\rightarrow x = \frac{27}{8}$$



$$41) Y = \frac{x^2 - 3}{x-2}, \quad \text{Domain: all } x \neq 2$$

$$Y' = \frac{(x-2) \cdot 2x - (x^2 - 3)(1)}{(x-2)^2} = \frac{2x^2 - 4x - x^2 + 3}{(x-2)^2}$$

$$= \frac{x^2 - 4x + 3}{(x-2)^2} = \frac{(x-3)(x-1)}{(x-2)^2} = 0$$

$+$	NO	$-$
0	$x=2$	0
$+$		$+$

y'

rel. $\begin{cases} x=1 \\ Y=2 \end{cases}$ max.

$\begin{cases} x=3 \\ Y=6 \end{cases}$ rel. min.

$$\begin{aligned}
 Y'' &= \frac{(x-2)^2 \cdot (2x-4) - (x^2-4x+3) \cdot 2(x-2)}{(x-2)^2} \\
 &= \frac{2(x-2) [x^2-4x+4 - x^2+4x-3]}{(x-2)^2} = \frac{2}{x-2} = 0
 \end{aligned}$$

—
 $\begin{array}{c} \text{---} \\ | \\ \text{No} \\ | \\ \text{---} \end{array}$ $x=2$ $\begin{array}{c} + \\ \text{---} \end{array}$ y'

$$x=0 : Y = \frac{3}{2}$$

$$Y=0 : x^2-3=0 \rightarrow x = \pm\sqrt{3}$$

$$\lim_{x \rightarrow \pm\infty} \frac{x^2-3}{x-2} \cdot \frac{1/x}{1/x} = \lim_{x \rightarrow \pm\infty} \frac{x - \frac{3}{x}}{1 - \frac{3}{x}} = \pm\infty,$$

$$\lim_{x \rightarrow 2^+} \frac{x^2-3}{x-2} = \frac{\text{"1"}}{0^+} = +\infty,$$

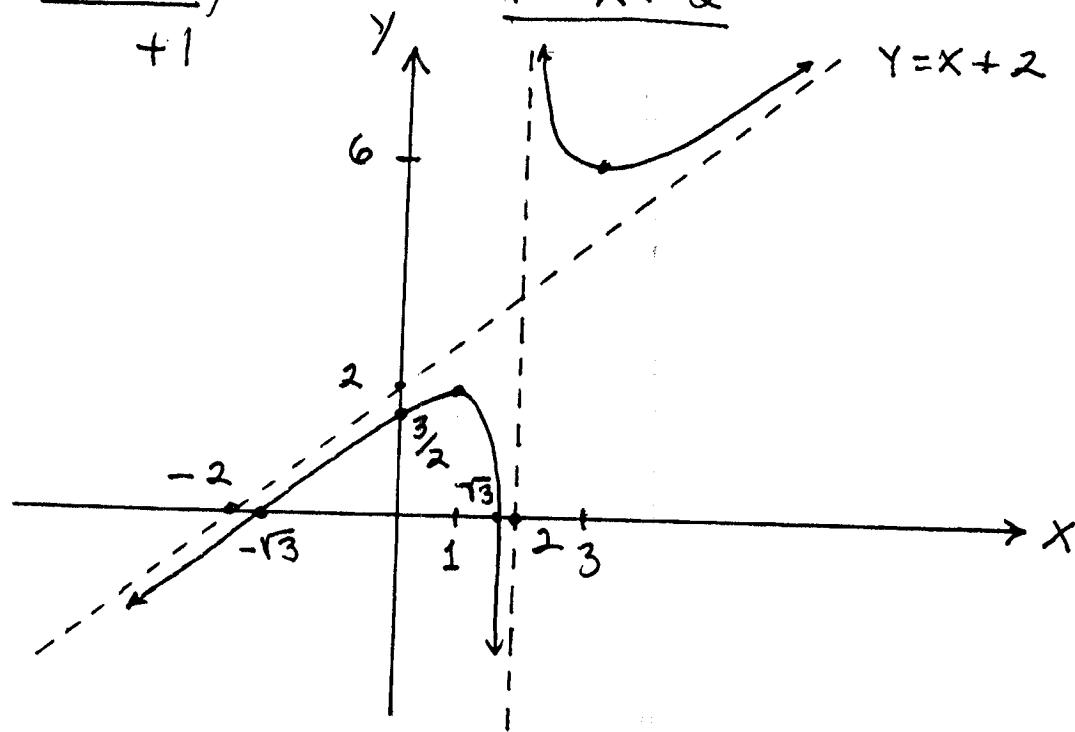
$$\lim_{x \rightarrow 2^-} \frac{x^2-3}{x-2} = \frac{\text{"1"}}{0^-} = -\infty, \quad \underline{\text{V.A. : } x=2};$$

$$\begin{array}{r}
 x+2 \\
 x-2 \overline{)x^2 - 3} \\
 - (x^2 - 2x) \\
 \hline
 2x - 3 \\
 - (2x - 4) \\
 \hline
 +1
 \end{array}$$

$$\text{so } \frac{x^2-3}{x-2} = x+2 + \frac{1}{x-2}$$

so tilted asymptote is

$$Y = x+2$$



$$52.) Y = x(\ln x)^2, \text{ Domain: } x > 0$$

$$Y' = x \cdot 2\ln x \cdot \frac{1}{x} + (\ln x)^2 = \ln x(2 + \ln x) = 0$$

$$\begin{array}{ccccccc|c} & & & & & & & Y' \\ \hline & & + & 0 & - & 0 & + & \\ x=0 & & | & | & | & | & | & \\ \end{array}$$

$$\text{rel. max. } \left\{ \begin{array}{l} x=e^{-2} \\ Y=4e^{-2} \end{array} \right. \quad \left. \begin{array}{l} x=1 \\ Y=0 \end{array} \right\} \begin{array}{l} \text{abs.} \\ \text{min.} \end{array}$$

$$Y'' = 2 \cdot \frac{1}{x} + 2\ln x \cdot \frac{1}{x} = 2 \cdot \frac{1}{x}(1 + \ln x) = 0$$

$$\begin{array}{ccccc|c} & & - & 0 & + & Y'' \\ \hline & & | & | & | & \\ x=0 & & & & & \\ x=e^{-1} & & & & \left. \begin{array}{l} \\ Y=e^{-1} \end{array} \right\} & \text{infl. pt.} \\ Y=e^{-1} & & & & & \end{array}$$

Y is \uparrow for $0 < x < \frac{1}{e^2}, x > 1,$

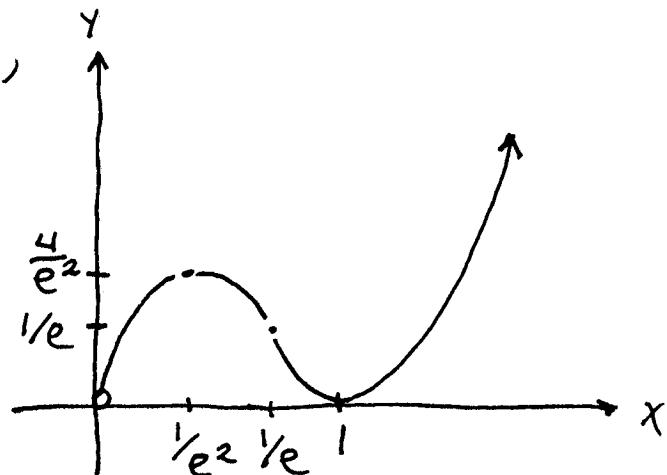
Y is \downarrow for $\frac{1}{e^2} < x < 1,$

Y is \cup for $x > \frac{1}{e},$

Y is \wedge for $0 < x < \frac{1}{e};$

$$x=0: (\text{No})$$

$$Y=0: x=1$$



$$58.) Y = \frac{e^x}{1+e^x}, \text{ Domain: all } x\text{-values,}$$

$$Y' = \frac{(1+e^x) \cdot e^x - e^x \cdot e^x}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2} = 0$$

$$\begin{array}{ccccc|c} & + & + & + & & Y' \\ \hline & & & & & \end{array}$$

$$Y'' = \frac{(1+e^x)^2 \cdot e^x - e^x \cdot 2(1+e^x) \cdot e^x}{(1+e^x)^4}$$

$$= \frac{e^x(1+e^x) \cdot [1+e^x - 2e^x]}{(1+e^x)^4} = \frac{e^x(1-e^x)}{(1+e^x)^3} = 0$$

$$\rightarrow 1-e^x = 0 \rightarrow e^x = 1 \rightarrow x = 0$$

+	0	-	
<hr/>			Y''
$x=0$			} infl.
$y=\frac{1}{2}$			pt.

y is \uparrow for all x -values,

y is U for $x < 0$,

y is \wedge for $x > 0$;

$$x=0 : y = \frac{1}{2}$$

$$y=0 : (no)$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{1+e^x} \cdot \frac{\frac{1}{e^x}}{\frac{1}{e^x}} = \lim_{x \rightarrow \infty} \frac{1}{\frac{1}{e^x} + 1} = \frac{1}{0+1} = 1$$

$$\text{so H.A. : } y = 1$$

$$\lim_{x \rightarrow -\infty} \frac{e^x}{1+e^x} = \frac{e^{-\infty}}{1+e^{-\infty}} = \frac{0}{1+0} = 0$$

$$\text{so H.A. : } y = 0$$

