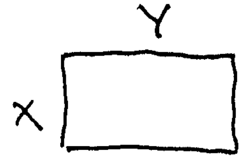
Section 4.6

1.) 

area $XY = 16$ in.^{2} $\rightarrow$

$\boxed{Y = 16/X}$; minimize (sub.)

perimeter $P = 2X + 2Y \rightarrow$

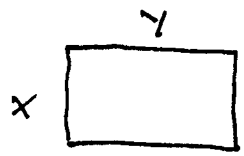
$P = 2X + 2\left(\frac{16}{X}\right) \rightarrow \boxed{P = 2X + \frac{32}{X}}$; then

$$P' = 2 - \frac{32}{X^2} = \frac{2X^2 - 32}{X^2} = \frac{2(X^2 - 16)}{X^2} = 0$$

$\rightarrow X^2 - 16 = 0 \rightarrow X = 4$

$-$ 0 $+$ P'

$X = 4$ in.
$Y = 4$ in.
min. $P = 16$ in.^{2}

2.) 

perimeter $2X + 2Y = 8$ in. $\rightarrow$

$X + Y = 4 \rightarrow \boxed{Y = 4 - X}$;

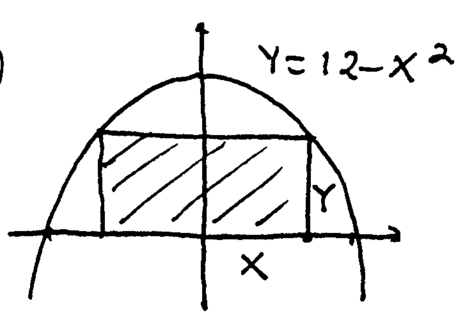
maximize area $A = XY \rightarrow$ (sub.)

$A = X \cdot (4 - X) \rightarrow \boxed{A = 4X - X^2}$; then

$A' = 4 - 2X = 0 \rightarrow X = 2$

0 A'

$X = 2$ in.
$Y = 2$ in.
max. $A = 4$ in.^{2}

4.) 

Maximize area

$A = 2X \cdot Y = 2X \cdot (12 - X^2) \rightarrow$

$\boxed{A = 24X - 2X^3}$; then

$A' = 24 - 6x^2 = 6(4-x^2) = 6(2-x)(2+x) = 0$

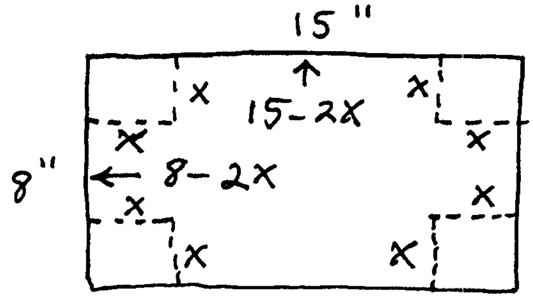
$\rightarrow x = 2$ or $x = -2$ (No)

$$\begin{array}{ccc} + & 0 & - \\ \hline & x=2 & \end{array} \quad A'$$

$\left.\begin{array}{l} x = 2 \\ Y = 8 \end{array}\right\}$ 4 by 8 rect.

max. $A = 32$

5.)



maximize volume

$V = (15-2x)(8-2x)\cdot x \xrightarrow{D}$

(triple product)

$V' = (-2)(8-2x)(x) + (15-2x)(-2)(x) + (15-2x)(8-2x)(1)$

$= (-16x + 4x^2) + (-30x + 4x^2) + (4x^2 - 46x + 120)$

$= 12x^2 - 92x + 120 = 4(3x^2 - 23x + 30) = 0 \rightarrow$

$x = \dfrac{23 \pm \sqrt{529 - 360}}{6} = \dfrac{23 \pm 13}{6} = 6$ or $\dfrac{5}{3} \rightarrow$

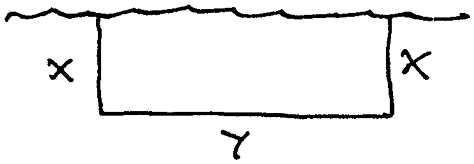
$x = 5/3''$

$$\begin{array}{ccc} + & 0 & - \\ \hline & x = 5/3'' & \end{array} \quad V'$$

so dim. are $\dfrac{35}{3}''$ by $\dfrac{14}{3}''$ by $\dfrac{5}{3}''$ and

max. $V = \dfrac{2450}{27}$ in.^{3} ≈ 90.74 in.^{3}

7.)



length $2x + Y = 800$ m. $\rightarrow$

$\boxed{Y = 800 - 2x}$;

maximize area $A = xY = x(800-2x) \rightarrow$

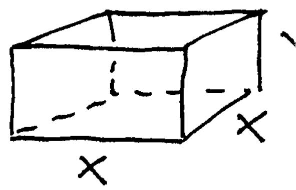
$\boxed{A = 800x - 2x^2} \xrightarrow{D} A' = 800 - 4x = 0 \rightarrow$

$x = 200$ m.

$$\begin{array}{ccc} + & 0 & - \end{array} \quad A'$$

$x = 200$ m.
$y = 400$ m.
max. $A = 80{,}000$ m.^{2}

9.) a.)



volume $x^2 y = 500$ ft.^{3} $\rightarrow$ $\boxed{y = \frac{500}{x^2}}$; minimize surface area (weight)

$S = x^2 + 4xy = x^2 + 4x \cdot \left(\frac{500}{x^2}\right) \rightarrow$

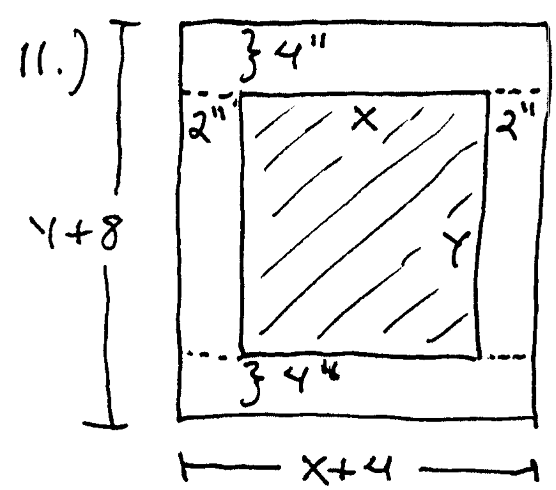
$\boxed{S = x^2 + \frac{2000}{x}} \xrightarrow{D} S' = 2x - \frac{2000}{x^2}$

$= \frac{2x^3 - 2000}{x^2} = \frac{2(x^3 - 1000)}{x^2} = 0 \rightarrow x = 10$ ft.

$$\begin{array}{ccc} - & 0 & + \end{array} \quad S'$$

$x = 10$ ft.
$y = 5$ ft.
min. $S = 300$ ft.^{2}

11.)



print area $xy = 50$ in.^{2}

$\rightarrow \boxed{y = \frac{50}{x}}$;

minimize paper area

$A = (x+4)(y+8)$

$= (x+4)\left(\frac{50}{x} + 8\right)$

$= 50 + \frac{200}{x} + 8x + 32 \rightarrow \boxed{A = \frac{200}{x} + 8x + 82} \xrightarrow{D}$

$$A' = -\frac{200}{x^2} + 8 = \frac{8x^2 - 200}{x^2} = \frac{8(x^2 - 25)}{x^2} = 0$$

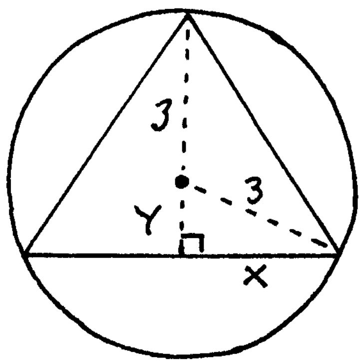
$\rightarrow x = 5$ in.

$$\begin{array}{ccc} - & 0 & + \end{array} \quad A'$$

page size: 18 in. by 9 in. $\begin{cases} x = 5 \text{ in.} \\ Y = 10 \text{ in.} \end{cases}$

min. $A = 162 \text{ in.}^2$

12.)



$x^2 + Y^2 = 3^2 \rightarrow$

$\boxed{x^2 = 9 - Y^2}$;

maximize volume

$$V = \frac{1}{3}\pi x^2 (Y + 3)$$

$$= \frac{1}{3}\pi (9 - Y^2)(Y + 3) = \frac{1}{3}\pi (9Y + 27 - Y^3 - 3Y^2) \rightarrow$$

$$\boxed{V = \frac{1}{3}\pi (9Y + 27 - Y^3 - 3Y^2)} \xrightarrow{D}$$

$$V' = \frac{1}{3}\pi (9 - 3Y^2 - 6Y) = \frac{1}{3}\pi \cdot (-3)(Y^2 + 2Y - 3)$$

$$= -\pi (Y - 1)(Y + 2) = 0 \rightarrow Y = 1 \text{ or } Y = -2 \text{ (no)}$$

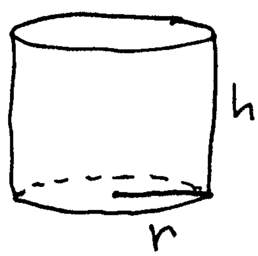
$$\begin{array}{ccc} + & 0 & - \end{array} \quad V'$$

cone dimensions: $\begin{cases} Y = 1 \\ x = \sqrt{8} \end{cases}$

$h = 4$, $r = \sqrt{8}$

max. $V = \frac{\pi}{3}(\sqrt{8})^2(4) = \frac{32}{3}\pi$

14.)



volume $\pi r^2 h = 1000 \text{ cm.}^3 \rightarrow$

$\boxed{h = \frac{1000}{\pi r^2}}$; minimize

surface area (weight)

$$S = \pi r^2 + 2\pi r h = \pi r^2 + 2\cancel{\pi} r \cdot \frac{1000}{\cancel{\pi} r^2} \rightarrow$$

$$\boxed{S = \pi r^2 + \frac{2000}{r}} \xrightarrow{D} S' = 2\pi r - \frac{2000}{r^2}$$

$$= \frac{2\pi r^3 - 2000}{r^2} = \frac{2(\pi r^3 - 1000)}{r^2} = 0 \rightarrow$$

$$\pi r^3 - 1000 = 0 \rightarrow r = \left(\frac{1000}{\pi}\right)^{1/3} \approx 6.83 \text{ in.}$$

$$\begin{array}{ccc} - & 0 & + \end{array} \quad S'$$

$$r = \left(\frac{1000}{\pi}\right)^{1/3} \text{in.} \approx 6.83 \text{ in.}$$

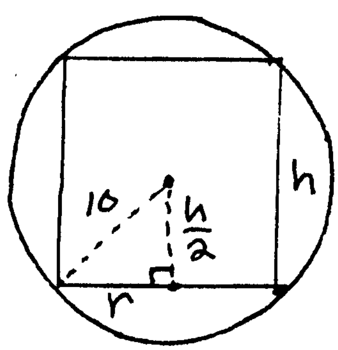
$$h = \frac{1000}{\pi\left(\frac{1000}{\pi}\right)^{2/3}} = \left(\frac{1000}{\pi}\right)^{1/3} \text{in.} \approx 6.83 \text{ in.}$$

$$\text{min. } S = \pi\left(\frac{1000}{\pi}\right)^{2/3} + \frac{2000}{\left(\frac{1000}{\pi}\right)^{1/3}} \rightarrow$$

$$S = \frac{1000^{2/3}}{\pi^{1/3}} + 2 \cdot 1000^{2/3} \cdot \pi^{1/3}$$

$$= \frac{1000^{2/3}(1 + 2\pi^{2/3})}{\pi^{1/3}} \approx 439.4 \text{ in.}^2$$

19.)



$$r^2 + \left(\frac{h}{2}\right)^2 = 10^2 \rightarrow$$

$$\boxed{r^2 = 100 - \frac{h^2}{4}} \; ;$$

maximize volume

$$V = \pi r^2 h = \pi\left(100 - \frac{h^2}{4}\right)h \rightarrow$$

$$\boxed{V = \pi\left(100h - \frac{1}{4}h^3\right)} \xrightarrow{D} V' = \pi\left(100 - \frac{3}{4}h^2\right) = 0$$

$$\rightarrow 100 = \frac{3}{4}h^2 \rightarrow h^2 = \frac{400}{3} \rightarrow h = \frac{20}{\sqrt{3}} \text{ cm.}$$

$$\begin{array}{ccc} + & 0 & - \end{array} \quad V'$$

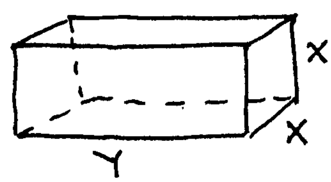
$$h = \frac{20}{\sqrt{3}} \approx 11.55 \text{ cm.}$$

$$r = \sqrt{100 - \frac{1}{4}\cdot\frac{400}{3}} = 10\sqrt{\frac{2}{3}} \approx 8.16 \text{ cm.}$$

max. $V = \pi\left(10\sqrt{\frac{2}{3}}\right)^2\left(\frac{20}{\sqrt{3}}\right)$

$$= \frac{4000}{3\sqrt{3}}\pi \approx 2418 \text{ cm}^3$$

20.)



$Y + 4X = 108'' \rightarrow \boxed{Y = 108 - 4X}$;

maximize volume $V = X^2 Y = X^2(108 - 4X) \rightarrow$

$\boxed{V = 108X^2 - 4X^3} \xrightarrow{D} V' = 216X - 12X^2$

$= 12X(18 - X) \rightarrow X = 18$ or $X = 0$ (NO)

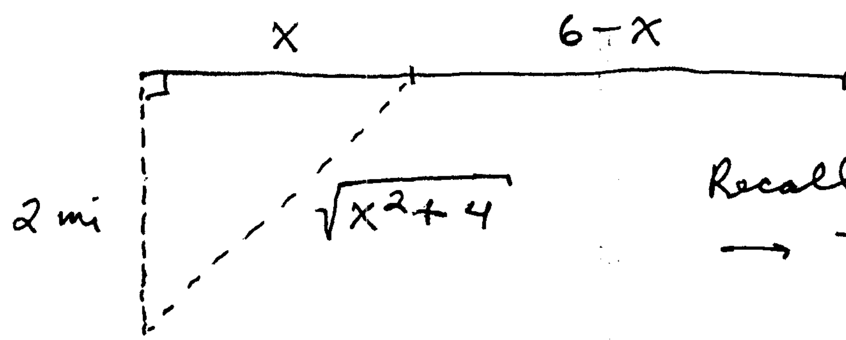
$$\begin{array}{ccc} + & 0 & - \end{array} \quad V'$$

$X = 18$ in.

$Y = 36$ in.

max. $V = 11{,}664 \text{ in}^3$

38.)



Recall : $D = RT$

$\rightarrow T = \frac{D}{R}$:

minimize time $T = T_{row} + T_{walk} \rightarrow$

$$\boxed{T = \frac{\sqrt{x^2+4}}{2} + \frac{6-x}{5}} \xrightarrow{D}$$

$T' = \frac{1}{2} \cdot \frac{1}{\cancel{2}}(x^2+4)^{-1/2} \cdot \cancel{2}x - \frac{1}{5} = 0 \longrightarrow$

$\frac{x}{2\sqrt{x^2+4}} = \frac{1}{5} \longrightarrow 5x = 2\sqrt{x^2+4} \longrightarrow$

$25x^2 = 4(x^2+4) = 4x^2+16 \longrightarrow$

$21x^2 = 16 \longrightarrow x^2 = \frac{16}{21} \longrightarrow x = \frac{4}{\sqrt{21}} \approx 0.873$ mi.

$-\quad 0 \quad +$ $\quad T'$

$x = \frac{4}{\sqrt{21}} \approx 0.873$ mi

min $T \approx 2.12$ hr.

52.) Let x: # of people added to 50 ;

Revenue = (\$/person)(# people)

$= (200-2x)(50+x)$;

Cost $= 6000 + 32(50+x)$;

maximize profit

$P =$ Revenue $-$ Cost

$= (200-2x)(50+x) - 6000 - 32(50+x) \xrightarrow{D}$

$P' = (200-2x)(1) + (-2)(50+x) - 32$

$= 200 - 2x - 100 - 2x - 32$

$= 68 - 4x = 0 \longrightarrow x = 17$ people

$+ \quad 0 \quad -$ $\quad P'$

$x = 17$

~~total~~ people : 67

\$/person : \$166

Revenue : \$11,122

Cost : \$8144

max. profit $P = \$2978$