Section 4.7

2.) $f(x) = x^3 + 3x + 1 \Rightarrow f'(x) = 3x^2 + 3$

$$\Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 + 3x_n + 1}{3x_n^2 + 3}$$

$$= \frac{3x_n^3 + \cancel{3x_n} - x_n^3 - \cancel{3x_n} - 1}{3x_n^2 + 3} = \frac{2x_n^3 - 1}{3x_n^2 + 3}$$

Hence, $x_{n+1} = \dfrac{2x_n^3 - 1}{3x_n^2 + 3}$

$x_0 = 0 \Rightarrow x_1 = -\frac{1}{3} \Rightarrow x_2 \approx -0.32222$

4.) $f(x) = 2x - x^2 + 1 \Rightarrow f'(x) = 2 - 2x$

$$\Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{2x_n - x_n^2 + 1}{2 - 2x_n}$$

$$= \frac{\cancel{2x_n} - 2x_n^2 - \cancel{2x_n} + x_n^2 - 1}{2 - 2x_n} = \frac{-x_n^2 - 1}{2 - 2x_n}$$

Hence, $x_{n+1} = \dfrac{-x_n^2 - 1}{2 - 2x_n}$

1st Root: $x_0 = 0 \Rightarrow x_1 = -\frac{1}{2} \Rightarrow x_2 \approx -.41667$

2nd Root: $x_0 = 2 \Rightarrow x_1 = \frac{5}{2} \Rightarrow x_2 \approx 2.41667$

6.) $f(x) = x^4 - 2 \Rightarrow f'(x) = 4x^3$

$$\Rightarrow X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)} = X_n - \frac{X_n^4 - 2}{4X_n^3}$$

$$= \frac{4X_n^4 - X_n^4 + 2}{4X_n^3} = \frac{3X_n^4 + 2}{4X_n^3}$$

Hence, $X_{n+1} = \dfrac{3X_n^4 + 2}{4X_n^3}$

$X_0 = -1 \Rightarrow X_1 = -\frac{5}{4} \Rightarrow X_2 \approx -1.1935$

22.) First, verify $\sqrt{x} = 3 - x^2$ is solvable by IMVT.

Let $f(x) = \sqrt{x} - 3 + x^2$ & $m = 0$

f is cont. (sqrt + poly is cont.)

$f(0) = -3 < 0$ & $f(4) = 15 > 0$

Choose interval $[0,4]$ so $f(0) < m < f(4)$

By IMVT there exists @ least one $c \in [0,4]$ satisfying $f(c) = m \Leftrightarrow \sqrt{c} - 3 + c^2 = 0$

$\Leftrightarrow \sqrt{c} = 3 - c^2$

Second, find c using Newton's method

$f(x) = \sqrt{x} - 3 + x^2 \Rightarrow f'(x) = \frac{1}{2\sqrt{x}} + 2x = \dfrac{1 + \cancel{4}x^{3/2}}{2\sqrt{x}}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\sqrt{x_n} - 3 + x_n^2}{\frac{1 + 4x_n^{3/2}}{2\sqrt{x_n}}}$$

$$= x_n - \frac{2x_n - 6x_n^{1/2} + 2x_n^{5/2}}{1 + 4x_n^{3/2}} = \frac{x_n + 4x_n^{5/2} - 2x_n + 6x_n^{1/2} - 2x_n^{5/2}}{1 + 4x_n^{3/2}}$$

Hence, $$x_{n+1} = \frac{2x_n^{5/2} - x_n + 6x_n^{1/2}}{1 + 4x_n^{3/2}}$$

Choose $x_0 = 1 \Rightarrow x_1 = 7/5 \Rightarrow x_2 \approx 1.35555$

$\Rightarrow x_3 \approx 1.35497 \Rightarrow x_4 \approx 1.35497$

$\Rightarrow c \approx 1.3550$

25.) First, verity $x^3 + 2x - 4 = 0$ is solvable by IMVT

Let $f(x) = x^3 + 2x - 4$ & $m = 0$

f is cont. (poly.)

$f(1) = -1 < 0$ & $f(2) = 8 > 0$

Choose interval $[1, 2]$ so $f(1) < m < f(2)$

By IMVT, there exists @ least one c b/w 1 & 2 satisfying $f(c) = m \Leftrightarrow c^3 + 2c - 4 = 0$

Second, find c using Newton's Method

$f(x) = x^3 + 2x - 4 \Rightarrow f'(x) = 3x^2 + 2$

$$\Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 + 2x_n - 4}{3x_n^2 + 2}$$

$$= \frac{3x_n^3 + \cancel{2x_n} - x_n^3 - \cancel{2x_n} + 4}{3x_n^2 + 2} = \frac{2x_n^3 + 4}{3x_n^2 + 2}$$

Hence, $x_{n+1} = \dfrac{2x_n^3 + 4}{3x_n^2 + 2}$

Choose $x_0 = 1 \Rightarrow x_1 = 1.2 = \frac{6}{5} \Rightarrow x_2 \approx 1.17975$

$\Rightarrow x_3 \approx 1.179509 \Rightarrow x_4 \approx 1.179509$

$\Rightarrow$ root $= c \approx 1.17951$