

Math 21C Final Study List

11.1 Sequences

- Definition: Sequences
- Properties of sequences
- Sandwich Theorem for Sequences

11.2 Series

- Definition: Series, Sequence of Partial Sums, Nth Partial Sum
- Converting sequence of partial sums to/from sequence of terms
- Definition: Geometric Series
- Know and use Geometric Series Test
- Using geometric series to convert a repeating decimal into a fraction
- Using telescoping series to find sum of series
- Know and use Nth Term Test

11.3 Integral Test

- Know and use Integral Test
- Definition: p-series
- Using p-series Test

11.4 Comparison Tests

- Know and use Comparison Test
- Know and use Limit Comparison Test

11.5 Ratio and Root Test

- Know and use Ratio Test
- Know and use Root Test

11.6 Alternating Series, Absolute and Conditional Convergence

- Definition: Alternating Series
- Know and use Alternating Series Test
- Estimating error for a convergent alternating series
- Definition: Absolute Convergence
- Definition: Conditional Convergence
- Know and use Absolute Convergence Test

11.7 Power Series

- Definition: Power Series about $x=a$
- Corollary to the Convergence Theorem for Power Series
- Definition: Center, Radius of Convergence, and Interval of Convergence
- Finding center, radius of convergence, and interval of convergence for given power series
- Term-by-Term Differentiation Theorem
- Term-by-Term Integration Theorem

11.8 Taylor and Maclaurin Series

- Definition: Taylor Series of $f(x)$ at $x=a$, and Maclaurin Series
- Definition: Taylor Polynomial of Order n for $f(x)$ at $x=a$
- Computing a Taylor Polynomial of Order n

11.9 Convergence of Taylor Series; Error Estimates

- Definition: Taylor's Formula, and Remainder of Order n (Error Term)
- Taylor's Formula Theorem
- Remainder Estimation Theorem
- Showing a Taylor Series converges for $\sin(x)$
- Finding truncation error for a Taylor polynomial

12.1 3D Coordinate Systems

- With equations describe regions in space
- Distance between points
- Equation for a sphere

12.2 Vectors

- Definition: Vector
- Compute a vector from 2 points
- Compute length of vector and direction of vector (unit vector)
- Vector addition, subtraction, & multiplication by scalar (Know geometric pictures)
- Definition: Midpoint between points

12.3 Dot Product

- Definition: Dot Product
- Compute angle between vectors using dot product (You need to know $\cos\theta$ for angles: 0, 30, 45, 60, & 90 degrees)
- Definition: Vectors being perpendicular
- Properties of the dot product
- Definition: Vector projection and orthogonal vector
- Computing projection and orthogonal decomposition
- Computing work done

12.4 Cross Product

- Definition: Cross Product
- Definition: Parallel vectors
- Properties of cross product
- Computing area of parallelogram using cross product
- Definition: Triple scalar product
- Computing volume of parallelepiped using triple scalar product

12.5 Lines and Planes in Space

- Definition: Parametric, symmetric, and vector equation for a line
- Find equation of a line, point on a line, or vector parallel to the line
- Compute distance from point to a line
- Definition: Equation for a plane
- Find equation of plane, point on plane, or normal to the plane
- Compute distance from point to plane

14.1 Functions of Several Variables

- Definition: Domain and Range
- Find the domain and range of a function

14.2 Limits and Continuity in Higher Dimensions

- Definition: Limit
- Properties of limits
- Taking limits of 2D functions including limits of quotients
- Definition: Continuity
- Using two path test to show limit does not exist

14.3 Partial Derivatives

- Definition: Partial Derivative with respect to x or y
- Finding 1st order partial derivatives
- Finding 2nd order partial derivatives
- Mixed Derivative Theorem

14.4 Chain Rule

- Chain rule tree and formula for 2 intermediate & 1 independent variables, 3 intermediate & 1 independent variables, and 3 intermediate & 2 independent variables
- Using chain rule in above cases
- Using formula for implicit differentiation

14.5 Directional Derivatives and Gradient Vectors

- Definition: Directional derivative of function in a direction of a unit vector
- Definition: Gradient vector
- Compute gradient vector
- Compute directional derivative using formula with the gradient vector
- Properties of directional derivative

14.6 Tangent Planes and Differentials

- Definition: Tangent Plane and Normal Line
- Finding equations for the tangent plane and normal line of a level surface of the form $f(x,y,z) = c$ or $z = f(x,y)$
- Finding equation for the tangent line of a curve that intersects two surfaces
- Estimating the change in $f(x,y)$ using the directional derivative

14.7 Extreme Values and Saddle Points

- Definition: Local maximum and local minimum
- Definition: Critical Point
- First Derivative Test Theorem
- Definition: Saddle Point
- Second Derivative Test Theorem (Make sure you know what the Hessian of $f(x)$ is)
- Finding all the local max/min and saddle points of $f(x,y)$
- Finding the absolute max/min of $f(x,y)$ on a given domain

14.8 Lagrange Multipliers

- Using the method of Lagrange Multipliers to find min/max of a function with 1 or 2 constraints.

13.1 Vector Functions

- Definition: Limit
- Definition: Continuity
- Definition: Derivative, Smooth
- Definition: Velocity Vector, Speed, Direction of Velocity, and Acceleration Vector
- Finding velocity vector, speed, direction of velocity, and acceleration vector for a given position vector function
- Definition: Indefinite and definite integral
- Solving differential equation with given initial conditions
- Finding tangent line for a curve in space

13.2 Modeling Projectile Motion

- Formula for Ideal Projectile Motion at the origin
- Using above formula to find things like maximum height, flight time, or range (Note: any use of the formula on the HW problems could be asked)

13.3 Arc Length and Unit Tangent Vector

- Definition: Length of a curve in space
- Computing the length of a given curve
- Finding the arc length from a base point equation for a given curve
- Definition: Unit Tangent Vector
- Computing the tangent vector for a given curve

Notes:

- Any definition listed can be asked on the test.
- Any formula used in any computation above can be asked on the test.
- I might ask you to prove one of the following formulas: area of parallelogram, volume of parallelepiped, distance from point to line, or distance from point to plane.
- The test will have 10 questions: 5 on the new material (14.6~8, 13.1~3) and 5 on the old material (previous sections). I will take one question (exact copy) from each of the two midterms.