Procedure for method of Lagrange Multipliers

For 1 constraint:

1) Identify fn f to be minimized or/and maximized & constraint $g(x,y,z)=0$
2) Build Lagrangian by
$$L(x,y,z,\lambda) = f(x,y,z) - \lambda g(x,y,z)$$
3) Find critical points of Lagrangian
$$L_x = f_x - \lambda g_x = 0$$
$$L_y = f_y - \lambda g_y = 0 \iff \vec{\nabla f} - \lambda \vec{\nabla g} = 0 \iff \vec{\nabla f} = \lambda \vec{\nabla g}$$
$$L_z = f_z - \lambda g_z = 0 \qquad g(x,y,z)=0$$
$$L_\lambda = -g = 0 \iff g(x,y,z)=0$$
4) Now that you have 4 equations & 4 unknowns solve for x,y,z,λ (It is usually best to solve for λ first then x,y,z). This will give you a list of ~~canid~~ candidates for maxs & mins
5) Look through lists to find maximums and/or minimums depending on what problem asks for.

For 2 constraints: $g(x,y,z)=0$ & $h(x,y,z)=0$
Procedure is same except for following
2) Lagragian $L(x,y,z,\lambda,\mu) = f(x,y,z) - \lambda g(x,y,z) - \mu h(x,y,z)$

3) $L_x = f_x - \lambda g_x - \mu h_x = 0$

$L_y = f_y - \lambda g_y - \mu h_y = 0$

$L_z = f_z - \lambda g_z - \mu h_z = 0 \iff \vec{\nabla f} = \lambda \vec{\nabla g} + \mu \vec{\nabla h}$

$L_\lambda = -g = 0 \qquad g(x,y,z) = 0$

$L_\mu = -h = 0 \qquad h(x,y,z) = 0$

4) You now have 5 equations with 5 unknowns & you need to solve for x, y, z, λ, μ (Again, it usually is best to solve for λ & μ then x, y, z).