Section 13.1

5) $\vec{r}(t) = (\sin t,\ \cos t)$ $\quad \vec{v}(t) = \vec{r}\,'(t) = (\cos t, -\sin t)$

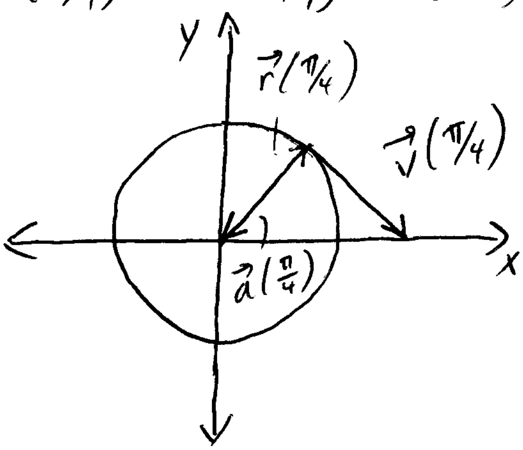
$\vec{a}(t) = \vec{v}\,'(t) = (-\sin t, -\cos t) = -\vec{r}(t)$

$\underline{t = \frac{\pi}{4}}$

$\vec{r}(\frac{\pi}{4}) = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

$\vec{v}(\frac{\pi}{4}) = (\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$

$\vec{a}(\pi/4) = -\vec{r}(\pi/4) = -(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

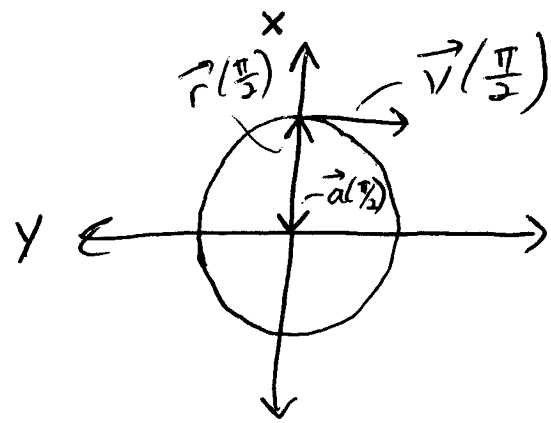


$\underline{t = \frac{\pi}{2}}$

$\vec{r}(\frac{\pi}{2}) = (1, 0)$

$\vec{v}(\pi/2) = (0, -1)$

$\vec{a}(\pi/2) = -(1, 0)$



14) $\vec{r}(t) = (e^{-t}, 2\cos 3t, 2\sin 3t)$ $\quad t = 0$

$\vec{v}(t) = (-e^{-t}, -6 \overset{\sin}{\cancel{\cos}} 3t, 6\cos 3t)$

$\vec{a}(t) = (e^{-t}, -18\cos 3t, -18\sin 3t)$

$\vec{v}(0) = (-1, 0, 6)$, $\vec{a}(0) = (1, -18, 0)$

speed $= |\vec{v}(0)| = \sqrt{1^2 + 0^2 + 6^2} = \sqrt{37}$

direction $= \frac{\vec{v}(0)}{|\vec{v}(0)|} = (-\frac{1}{\sqrt{37}}, 0, \frac{6}{\sqrt{37}})$

30) DE: $\vec{v} = \frac{d\vec{r}}{dt} = (t^3 + 4t,\ t,\ 2t^2)$

IC: $\vec{r}(0) = (1, 1, 0)$

$\vec{r}(t) = \int \vec{v}\, dt = \left(\frac{t^4}{4} + 2t^2,\ \frac{t^2}{2},\ \frac{2}{3}t^3\right) + \vec{C}$

$(1,1,0) = \vec{r}(0) = (0, 0, 0) + \vec{C}$

$\Rightarrow \vec{C} = (1, 1, 0)$

$\Rightarrow \vec{r}(t) = \left(\frac{t^4}{4} + 2t^2 + 1,\ \frac{t^2}{2} + 1,\ \frac{2}{3}t^3\right)$

33) $\vec{r}(t) = (\sin t,\ t^2 - \cos t,\ e^t)$

$\vec{v}(t) = (\cos t,\ 2t + \sin t,\ e^t)$

Want tangent line @ $\underset{\vee}{t=0}$ so build line that goes through $\vec{r}(0)$ parallel to $\vec{v}(0)$.

$\vec{r}(0) = (0, -1, 1)$ $\quad \vec{v}(0) = (1, 0, 1)$

$\Rightarrow \vec{\ell}(t) = \vec{r}(0) + \vec{v}(0)t$ tangent line of curve @ $t = 0$

$= (t, -1, 1 + t)$

all $-\infty < t < \infty$

Section 13.2

1) With $V_0 = 840\,m/sec$ & $\alpha = 60^\circ = \pi/3$

$\Rightarrow \vec{r}(t) = ((840 \cdot \cos \pi/3)t,\ \cancel{60}(840 \cdot \sin \pi/3)t - \frac{9.8}{2}t^2)\ m/s$

$= (420t,\ 420\sqrt{3}\,t - 4.9t^2)\ m/s$

Want to find t^* s.t. $x(t^*) = 21{,}000\ m$

$x(t^*) = 420\frac{m}{s}t^* = 21{,}000\ m \Rightarrow t^* = 50\ sec$

4) $y(t) = y_0 + (V_0 \sin\alpha)t - \frac{1}{2}g t^2$

$\Rightarrow y(t) = \cancel{30}\ 32ft + (32\ ft/sec \cdot \sin 30^\circ)t - \frac{1}{2}(32ft/sec^2)t^2$

$= (32 + 16t - 16t^2)\ ft$

Ball hits the ground when $y = 0$

$\Rightarrow 0 = 32 + 16t - 16t^2 \Leftrightarrow t^2 - t - 2 = 0$

$\Leftrightarrow (t-2)(t+1) = 0 \Rightarrow t = -1$ or $t = 2$

time has to be positive $\Rightarrow t^* = 2\ sec$

$x(t^*) = (32ft/sec \cdot \cos 30^\circ)t^* = 32 \cdot \frac{\sqrt{3}}{\cancel{2}} \cdot \cancel{2} \approx 55.4ft$

7) a) Range $R = \frac{V_0^2}{g}\sin 2\alpha \Rightarrow 10m = \frac{V_0^2}{9.8m/s^2} \cdot \sin 90^\circ$

$\Rightarrow V_0^2 = 98\ m^2/s^2 \Rightarrow V_0 = 9.9\ m/s$

b) $\cancel{6m \approx \frac{98\,m^2s^2}{9.8}}$

$6\,m = \frac{98\,m^2/s^2}{9.8\,m/s}\sin 2\alpha \Rightarrow \sin 2\alpha \approx 0.59999$

$\Rightarrow 2\alpha \approx 36.87^\circ$ or 143.12°

$\Rightarrow \alpha \approx 18.4^\circ$ or 71.6°

Section 13.3

2) $\vec{r}(t) = (6\sin 2t,\ 6\cos 2t,\ 5t) \qquad 0 \le t \le \pi$

$\vec{v}(t) = (12\cos 2t,\ -12\sin 2t,\ 5)$

$\Rightarrow |\vec{v}| = \sqrt{144\cos^2 2t + 144\sin^2 2t + 25} = \sqrt{169} = 13$

$\vec{T} = \dfrac{\vec{v}(t)}{|\vec{v}(t)|} = \left(\frac{12}{13}\cos 2t,\ -\frac{12}{13}\sin 2t,\ \frac{5}{13}\right)$

$\text{Length} = \int_0^\pi |v|\,dt = \int_0^\pi 13\,dt = 13t\Big|_0^\pi = 13\pi$

7) $\vec{r}(t) = \left(t\cos t,\ t\sin t,\ \left(2\cdot\frac{\sqrt{2}}{3}\right)t^{3/2}\right) \qquad 0 \le t \le \pi$

$\vec{v}(t) = (\cos t - t\sin t,\ \sin t + t\cos t,\ \sqrt{2}\,t^{1/2})$

$\Rightarrow |\vec{v}| = \sqrt{(\cos t - t\sin t)^2 + (\sin t + t\cos t)^2 + (\sqrt{2t})^2}$

$= [\cos^2 t - \cancel{2t\cos t\sin t} + t^2\sin^2 t + \sin^2 t + \cancel{2t\sin t\cos t} + t^2\cos^2 t + 2t]^{1/2}$

$= \sqrt{1 + 2t + t^2} = \sqrt{(1+t)^2} = |t+1| = t+1$ since $t \ge 0$

$\Rightarrow \vec{T} = \dfrac{\vec{v}(t)}{|\vec{v}|} = \left(\dfrac{\cos t - t\sin t}{t+1},\ \dfrac{\sin t + t\cos t}{t+1},\ \dfrac{\sqrt{2}\,t^{1/2}}{t+1}\right)$

$\text{Length} = \int_0^\pi |\vec{v}(t)|\,dt = \int_0^\pi t+1\,dt = \frac{t^2}{2} + t\Big|_0^\pi = \frac{\pi^2}{2} + \pi$

9) $\vec{r}(t) = (5\sin t,\ 5\cos t,\ 12t)$

$\vec{v}(t) = (5\cos t,\ -5\sin t,\ 12)$

Let t^* be when arc length $= 26\pi$

$|\vec{v}| = \sqrt{25\cos^2 t + 25\sin^2 t + 144} = \sqrt{169} = 13$

$\Rightarrow 26\pi = \int_0^{t^*} |\vec{v}|\,dt = \int_0^{t^*} 13\,dt = 13t\Big|_0^{t^*} = 13t^*$

$\Rightarrow t^* = 2\pi \qquad \Rightarrow \vec{r}(t^*) = \vec{r}(2\pi) = (0,\ 5,\ 24\pi)$

12) $\vec{r}(t) = (\cos t + t\sin t,\ \sin t - t\cos t)$ $\quad \frac{\pi}{2} \le t \le \pi$

$\vec{v}(t) = (-\cancel{\sin t} + \cancel{\sin t} + t\cos t,\ \cancel{\cos t} - \cancel{\cos t} + t\sin t)$

$= (t\cos t,\ t\sin t)$

$|\vec{v}(t)| = \sqrt{(t\cos t)^2 + (t\sin t)^2} = \sqrt{t^2} = |t| = t$ since $t \ge 0$

$\Rightarrow s(t) = \int_0^t \tau \, d\tau = \frac{\tau^2}{2}\Big|_0^t = \frac{t^2}{2}$

Length $= s(\pi) - s(\frac{\pi}{2}) = \frac{\pi^2}{2} - \frac{\pi^2}{8} = \frac{3\pi^2}{8}$