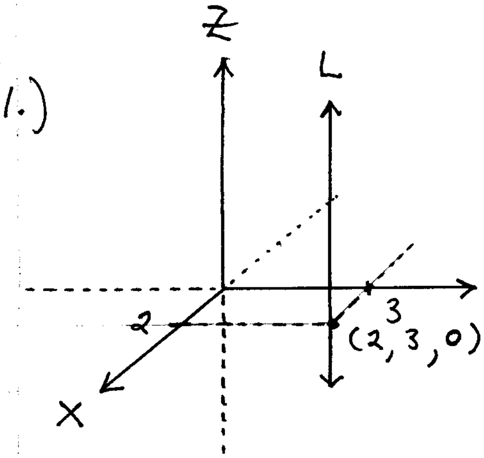
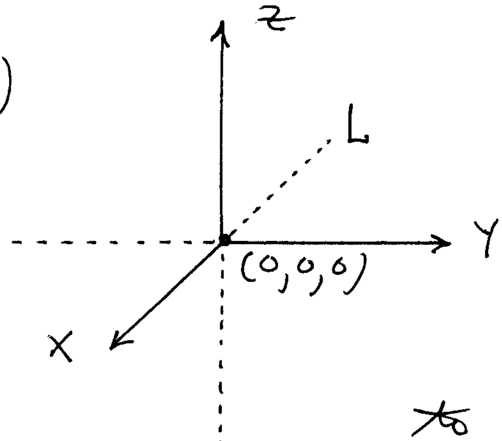
Section 12.1

1.)



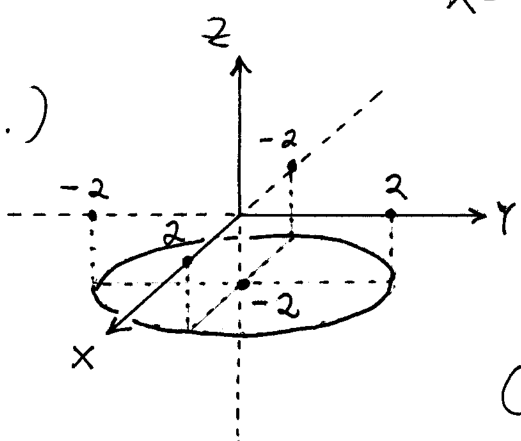
The set of points with $X=2$ and $Y=3$ is the line L passing through the point $(2,3,0)$ and parallel to the z-axis

3.)



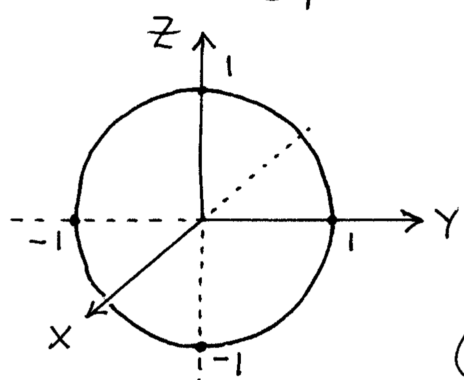
The set of points with $Y=0$ and $Z=0$ is the line L passing through the point $(0,0,0)$ and parallel to the x-axis (L is the X-axis)

6.)



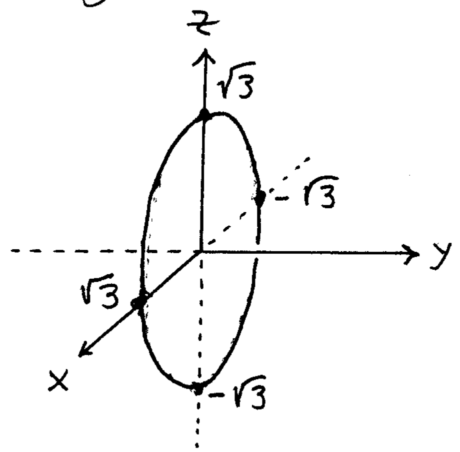
The set of points with $X^2+Y^2=4$ and $Z=-2$ is the set of points on the circle $X^2+Y^2=4$ (center $(0,0)$, radius 2) lying in the plane $Z=-2$ (parallel to the XY-plane)

8.)



The set of points with $Y^2+Z^2=1$ and $X=0$ is the set of points on the circle $Y^2+Z^2=1$ (center $(0,0)$, radius 1) lying in the plane $x=0$ (yz-plane)

12.)



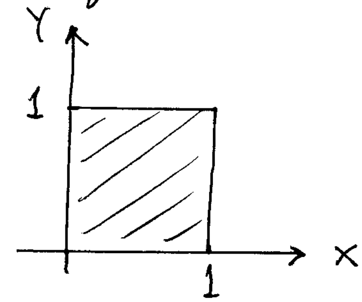
The set of points with $x^2+(y-1)^2+z^2=4$ and $y=0$ $\to$ $x^2+(-1)^2+z^2=4$ $\to$ $x^2+z^2=3=(\sqrt{3})^2$ is the set of points lying on the circle $x^2+z^2=3$ (center $(0,0)$, radius $\sqrt{3}$) and in the plane $y=0$ (xz-plane)

13.) a.) $x \geq 0$, $y \geq 0$, $z=0$: The set of points in the 1st quadrant of the xy-plane

b.) $x \geq 0$, $y \leq 0$, $z=0$: The set of points in the 4th quadrant of the xy-plane

14.) a.) $0 \leq x \leq 1$: The set of points lying on and between the parallel planes $x=0$ (yz-plane) and $x=1$.

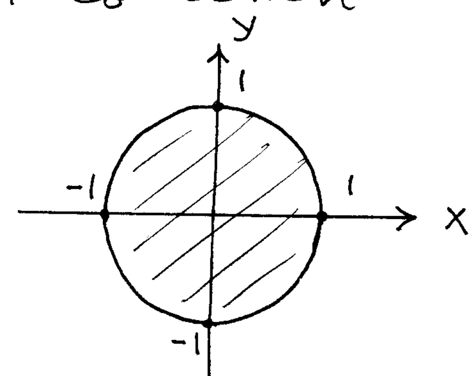
b.) $0 \leq x \leq 1$, $0 \leq y \leq 1$: The set of points on and inside the vertical (parallel to z-axis) square column passing through the given 1 by 1 square in the xy-plane.



c.) $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1$: The set of points on and inside the 1 by 1 by 1 cube in the 1st octant.

16.) a.) $x^2 + y^2 \le 1,\ z = 0$: The set of points lying on and inside the circle $x^2 + y^2 = 1$ (center $(0,0)$, radius 1) in the plane $z = 0$ (xy-plane)

c.) $x^2 + y^2 \le 1$: The set of points on and inside the vertical (parallel to z-axis) circular column passing through the given circle of radius 1



17.) b.) $x^2 + y^2 + z^2 = 1,\ z \ge 0$: The set of points lying on or inside the top half of the sphere $x^2 + y^2 + z^2 = 1$ (center $(0,0,0)$, radius 1)

18.) a.) $x = y,\ z = 0$: The set of points lying on the line $x = y$ in the plane $z = 0$ (xy-axis)

b.) $x = y$: The set of points on the plane passing through the line $x = y$ (in the xy-plane) and parallel to the z-axis.

20.) a.) $X = 3$ b.) $Y = -1$ c.) $Z = 2$

21.) a.) $Z = 1$ b.) $X = 3$ c.) $Y = -1$

22.) a.) $X^2 + Y^2 = 2^2$, $Z = 0$
b.) $Y^2 + Z^2 = 2^2$, $X = 0$
c.) $X^2 + Z^2 = 2^2$, $Y = 0$

24.) a.) $(X+3)^2 + (Y-4)^2 = 1^2$, $Z = 1$
b.) $(Y-4)^2 + (Z-1)^2 = 1^2$, $X = -3$
c.) $(X+3)^2 + (Z-1)^2 = 1^2$, $Y = 4$

25.) a.) $Y = 3$, $Z = -1$
b.) $X = 1$, $Z = -1$
c.) $X = 1$, $Y = 3$

26.) all points (X, Y, Z) equidistant from $(0,0,0)$ and $(0,2,0)$:

$$\sqrt{(X-0)^2 + (Y-0)^2 + (Z-0)^2} = \sqrt{(X-0)^2 + (Y-2)^2 + (Z-0)^2}$$

$$\rightarrow \cancel{X^2} + Y^2 + \cancel{Z^2} = \cancel{X^2} + (Y-2)^2 + \cancel{Z^2}$$

$$\rightarrow \cancel{Y^2} = \cancel{Y^2} - 4Y + 4 \rightarrow 4Y = 4 \rightarrow$$

$\boxed{Y = 1}$ (a plane parallel ~~to~~ the XZ-plane)

28.) all points (X, Y, Z) 2 units from $(0,0,1)$ and 2 units $(0,0,-1)$:

$$\sqrt{(x-0)^2+(y-0)^2+(z-1)^2} = 2 \quad \text{and}$$

$$\sqrt{(x-0)^2+(y-0)^2+(z+1)^2} = 2 \rightarrow$$

$$x^2+y^2+(z-1)^2 = 4 \quad \text{and}$$

$$x^2+y^2+(z+1)^2 = 4 \rightarrow$$

$$\cancel{x^2}+\cancel{y^2}+(z-1)^2 = \cancel{x^2}+\cancel{y^2}+(z+1)^2 \rightarrow$$

$$\cancel{z^2}-2z+\cancel{1} = \cancel{z^2}+2z+\cancel{1} \rightarrow 4z=0$$

$$\rightarrow z=0 \text{ ; then}$$

$$x^2+y^2+(0-1)^2=4 \rightarrow$$

$$\boxed{x^2+y^2=3 \text{ and } z=0}$$

30.) $0 \le x \le 2,\ 0 \le y \le 2,\ 0 \le z \le 2$

31.) $z \le 0$

32) $x^2+y^2+z^2=1$ and $z \ge 0$

33.) a.) $(x-1)^2+(y-1)^2+(z-1)^2 < 1^2$

b.) $(x-1)^2+(y-1)^2+(z-1)^2 > 1^2$

36.) $D = \sqrt{(2-{-1})^2+(5-1)^2+(0-5)^2}$

$= \sqrt{9+16+25} = \sqrt{50} = 5\sqrt{2}$

37.) $D = \sqrt{(4-1)^2+(-2-4)^2+(7-5)^2}$

$= \sqrt{9+36+4} = \sqrt{49} = 7$

41.) $(x-(-2))^2+(y-0)^2+(z-2)^2=(2\sqrt{2})^2$
$\rightarrow$ center $(-2,0,2)$, radius $2\sqrt{2}$

46.) $(x-0)^2+(y-(-1))^2+(z-5)^2=2^2 \rightarrow$
$x^2+(y+1)^2+(z-5)^2=4$

49.) $x^2+y^2+z^2+4x-4z=0 \rightarrow$
$(x^2+4x+\underline{\underline{4}})+y^2+(z^2-4z+\underline{\underline{4}})=4+4 \rightarrow$
$(x+2)^2+y^2+(z-2)^2=8=(2\sqrt{2})^2 \rightarrow$
center $(-2,0,2)$, radius $2\sqrt{2}$

52.) $3x^2+3y^2+3z^2+2y-2z=9 \rightarrow$
$3x^2+(3y^2+2y \quad)+(3z^2-2z \quad)=9 \rightarrow$
$3x^2+3(y^2+\frac{2}{3}y \quad)+3(z^2-\frac{2}{3}z \quad)=9 \rightarrow$
$x^2+(y^2+\frac{2}{3}y \quad)+(z^2-\frac{2}{3}z \quad)=3 \rightarrow$
$x^2+(y^2+\frac{2}{3}y+\frac{1}{9})+(z^2-\frac{2}{3}z+\frac{1}{9})=3+\frac{1}{9}+\frac{1}{9} \rightarrow$
$(x-0)^2+(y+\frac{1}{3})^2+(z-\frac{1}{3})^2=\frac{29}{9}=\left(\frac{\sqrt{29}}{3}\right)^2 \rightarrow$
center $(0,-1/3,1/3)$, radius $\frac{\sqrt{29}}{3}$

53.) a.) Distance from (x,y,z) and point $(x,0,0)$ (on the x-axis):
$$D=\sqrt{(x-x)^2+(y-0)^2+(z-0)^2}$$
$$=\sqrt{y^2+z^2}.$$

b.) Distance from (X, Y, Z) and point $(0, Y, 0)$ (on the Y-axis):

$$D = \sqrt{(X-0)^2 + (Y-Y)^2 + (Z-0)^2}$$

$$= \sqrt{X^2 + Z^2}$$

c.) Distance from (X, Y, Z) and point $(0, 0, Z)$ (on the Z-axis):

$$D = \sqrt{(X-0)^2 + (Y-0)^2 + (Z-Z)^2}$$

$$= \sqrt{X^2 + Y^2}$$

54.) a.) Distance from (X, Y, Z) and point $(X, Y, 0)$ (on XY-plane):

$$D = \sqrt{(X-X)^2 + (Y-Y)^2 + (Z-0)^2}$$

$$= \sqrt{Z^2} = |Z|$$

b.) Distance from (X, Y, Z) and point $(0, Y, Z)$ (on YZ-plane):

$$D = \sqrt{(X-0)^2 + (Y-Y)^2 + (Z-Z)^2} = \sqrt{X^2} = |X|$$

c.) Distance from (X, Y, Z) and point $(X, 0, Z)$ (on XZ-plane):

$$D = \sqrt{(X-X)^2 + (Y-0)^2 + (Z-Z)^2} = \sqrt{Y^2} = |Y|$$

Section 12.2

1.) a.) $3\vec{u} = 3\,\overrightarrow{(3,-2)} = \overrightarrow{(9,-6)}$

b.) $|3\vec{u}| = \sqrt{9^2+(-6)^2} = \sqrt{117} = 3\sqrt{13}$

4.) a.) $\vec{u}-\vec{v} = \overrightarrow{(3,-2)} - \overrightarrow{(-2,5)} = \overrightarrow{(5,-7)}$

b.) $|\vec{u}+\vec{v}| = \sqrt{5^2+(-7)^2} = \sqrt{74}$

6.) a.) $-2\vec{u}+5\vec{v} = -2\,\overrightarrow{(3,-2)} + 5\,\overrightarrow{(-2,5)}$

$= \overrightarrow{(-6,4)} + \overrightarrow{(-10,25)} = \overrightarrow{(-16,29)}$

b.) $|-2\vec{u}+5\vec{v}| = \sqrt{(-16)^2+(29)^2} = \sqrt{1097}$

7.) a.) $\frac{3}{5}\vec{u} + \frac{4}{5}\vec{v} = \frac{3}{5}\,\overrightarrow{(3,-2)} + \frac{4}{5}\,\overrightarrow{(-2,5)}$

$= \overrightarrow{\left(\frac{9}{5}, \frac{-6}{5}\right)} + \overrightarrow{\left(\frac{-8}{5}, 4\right)} = \overrightarrow{\left(\frac{1}{5}, \frac{14}{5}\right)}$

b.) $\left|\frac{3}{5}\vec{u} + \frac{4}{5}\vec{v}\right| = \sqrt{\left(\frac{1}{5}\right)^2+\left(\frac{14}{5}\right)^2} = \sqrt{\frac{197}{25}} = \frac{\sqrt{197}}{5}$

9.) $\overrightarrow{PQ} = \overrightarrow{(2-1,-1-3)} = \overrightarrow{(1,-4)}$

10.) $O = (0,0)$, $P = \left(\frac{2+-4}{2}, \frac{-1+3}{2}\right) = (-1,1)$, so

$\overrightarrow{OP} = \overrightarrow{(-1-0, 1-0)} = \overrightarrow{(-1,1)}$

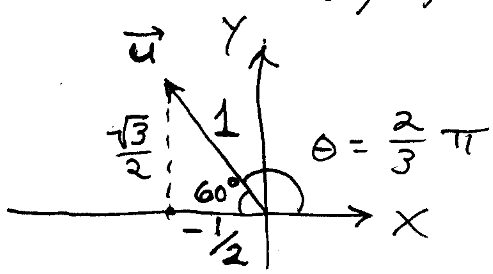
11.) $A = (2,3)$, $O = (0,0)$, so

$\overrightarrow{AO} = \overrightarrow{(0-2, 0-3)} = \overrightarrow{(-2,-3)}$

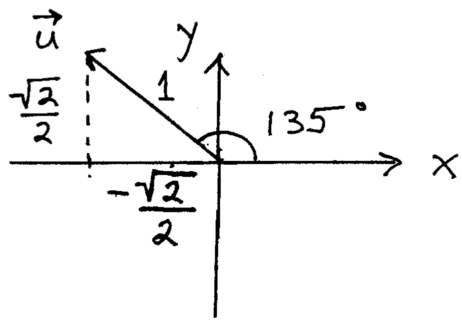
12.) $\overrightarrow{AB} = \overrightarrow{(2-1,\ 0-(-1))} = \overrightarrow{(1,1)}$,

$\overrightarrow{CD} = \overrightarrow{(-2-(-1),\ 2-3)} = \overrightarrow{(-1,-1)}$, so

$\overrightarrow{AB} + \overrightarrow{CD} = \overrightarrow{(0,0)}$

13.) 

$\vec{u} = \overrightarrow{\left(-\tfrac{1}{2},\ \tfrac{\sqrt{3}}{2}\right)}$

16.) 

$\vec{u} = \overrightarrow{\left(-\tfrac{\sqrt{2}}{2},\ \tfrac{\sqrt{2}}{2}\right)}$

18.) $\overrightarrow{P_1P_2} = \overrightarrow{(-3-1,\ 0-2,\ 5-0)} = \overrightarrow{(-4,-2,5)}$

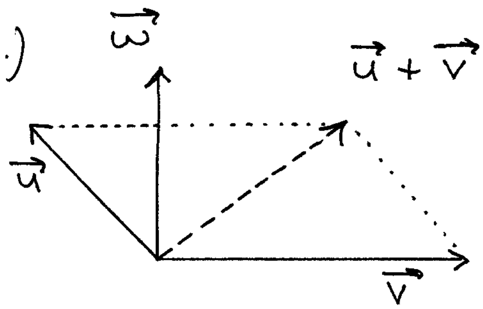
$= -4\vec{i} - 2\vec{j} + 5\vec{k}$

21.) $5\vec{u} - \vec{v} = 5\,\overrightarrow{(1,1,-1)} - \overrightarrow{(2,0,3)}$

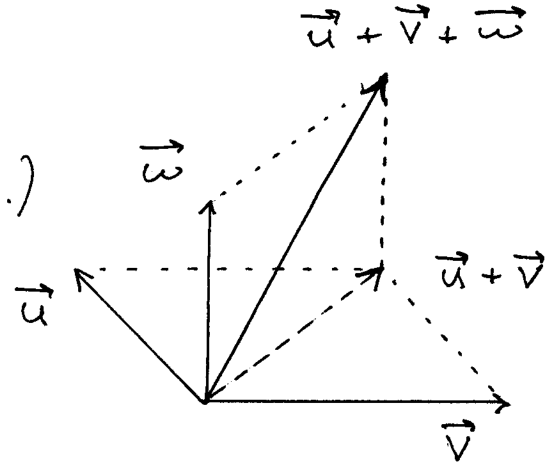
$= \overrightarrow{(5,5,-5)} - \overrightarrow{(2,0,3)} = \overrightarrow{(3,5,-8)}$

$= 3\vec{i} + 5\vec{j} - 8\vec{k}$

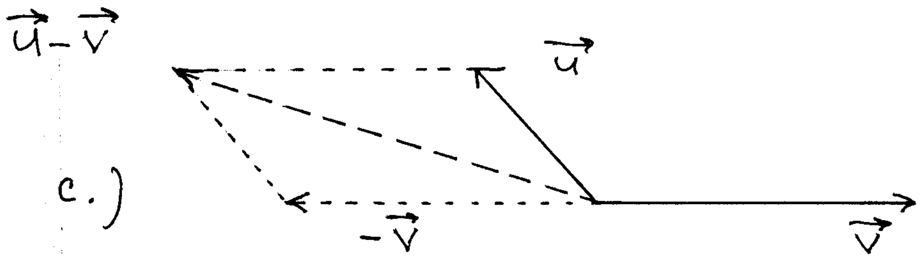
23.) a.)



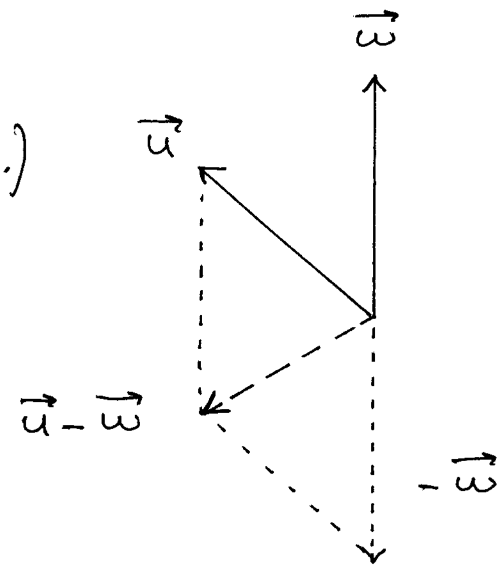
b.)



c.)



d.)



25.) $\left|2\vec{i}+\vec{j}-2\vec{k}\right| = \sqrt{2^2+1^2+(-2)^2}$

$= \sqrt{9} = 3$, so

$2\vec{i}+\vec{j}-2\vec{k} = 3\cdot\frac{1}{3}\left(2\vec{i}+\vec{j}-2\vec{k}\right)$

$= 3\cdot\left(\frac{2}{3}\vec{i}+\frac{1}{3}\vec{j}-\frac{2}{3}\vec{k}\right)$

28.) $\left|\frac{3}{5}\vec{i}+\frac{4}{5}\vec{k}\right| = \sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2} = \sqrt{\frac{25}{25}} = 1$, so

$$\frac{3}{5}\vec{i} + \frac{4}{5}\vec{k} = 1 \cdot \left(\frac{3}{5}\vec{i} + \frac{4}{5}\vec{k}\right)$$

29.) $\left|\frac{1}{\sqrt{6}}\vec{i} - \frac{1}{\sqrt{6}}\vec{j} - \frac{1}{\sqrt{6}}\vec{k}\right| = \sqrt{\left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{-1}{\sqrt{6}}\right)^2 + \left(\frac{-1}{\sqrt{6}}\right)^2}$

$= \sqrt{\frac{1}{6} + \frac{1}{6} + \frac{1}{6}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$, so

$\frac{1}{\sqrt{6}}\vec{i} - \frac{1}{\sqrt{6}}\vec{j} - \frac{1}{\sqrt{6}}\vec{k} = \frac{1}{\sqrt{2}} \cdot \frac{1}{1/\sqrt{2}}\left(\frac{1}{\sqrt{6}}\vec{i} - \frac{1}{\sqrt{6}}\vec{j} - \frac{1}{\sqrt{6}}\vec{k}\right)$

$= \frac{1}{\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{\sqrt{6}}\vec{i} - \frac{\sqrt{2}}{\sqrt{6}}\vec{j} - \frac{\sqrt{2}}{\sqrt{6}}\vec{k}\right)$

$= \frac{1}{\sqrt{2}} \cdot \left(\frac{1}{\sqrt{3}}\vec{i} - \frac{1}{\sqrt{3}}\vec{j} - \frac{1}{\sqrt{3}}\vec{k}\right)$

31.) a.) $\vec{w} = 2 \cdot \vec{i}$

b.) $\vec{w} = \sqrt{3} \cdot -\vec{k} = -\sqrt{3} \cdot \vec{k}$

c.) $\vec{w} = \frac{1}{2} \cdot \left(\frac{3}{5}\vec{j} + \frac{4}{5}\vec{k}\right) = \frac{3}{10}\vec{j} + \frac{4}{10}\vec{k}$

d.) $\vec{w} = 7 \cdot \left(\frac{6}{7}\vec{i} - \frac{2}{7}\vec{j} + \frac{3}{7}\vec{k}\right) = 6\vec{i} - 2\vec{j} + 3\vec{k}$

33.) $\vec{v} = 12\vec{i} - 5\vec{k} \rightarrow |\vec{v}| = \sqrt{12^2 + (-5)^2} = 13$

so direction is $\vec{u} = \frac{1}{13}\vec{v} = \frac{12}{13}\vec{i} - \frac{5}{13}\vec{k}$,
so vector in same direction
of magnitude 7 is

$\vec{w} = 7\vec{u} = \frac{84}{13}\vec{i} - \frac{35}{13}\vec{k}$

35.) a.) $\overrightarrow{P_1P_2} = \overrightarrow{(2-(-1), 5-1, 0-5)}$

$= \overrightarrow{(3, 4, -5)} \rightarrow |\overrightarrow{(3,4,-5)}| = \sqrt{3^2 + 4^2 + (-5)^2}$

$= \sqrt{50} = 5\sqrt{2}$, so direction is

$\vec{u} = \frac{1}{5\sqrt{2}} \overrightarrow{(3,4,-5)} = \left(\frac{3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

b.) midpoint: $\left(\frac{2-1}{2}, \frac{5+1}{2}, \frac{0+5}{2}\right) = \left(\frac{1}{2}, 3, \frac{5}{2}\right)$

38.) a.) $\overrightarrow{P_1P_2} = \overrightarrow{(2,-2,-2)} \longrightarrow$

$\left|\overrightarrow{(2,-2,-2)}\right| = \sqrt{2^2+(-2)^2+(-2)^2} = \sqrt{12} = 2\sqrt{3}$,

so direction is

$\vec{u} = \frac{1}{2\sqrt{3}} \overrightarrow{(2,-2,-2)} = \overrightarrow{\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)}$

b.) midpoint: $\left(\frac{0+2}{2}, \frac{0-2}{2}, \frac{0-2}{2}\right) = (1,-1,-1)$

40.) $\overrightarrow{AB} = -7\vec{i} + 3\vec{j} + 8\vec{k}$, $A = (-2,-3,6)$,

$B = (a,b,c)$, so

$(a-(-2), b-(-3), c-6) = (a+2, b+3, c-6) = (-7,3,8)$

$$\left.\begin{aligned} &\rightarrow\ a+2 = -7 \ \rightarrow\ a = -9 \\ &\rightarrow\ b+3 = 3 \ \rightarrow\ b = 0 \\ &\rightarrow\ c-6 = 8 \ \rightarrow\ c = 14 \end{aligned}\right\} \text{ so}$$

$B = -9\vec{i} + 0\vec{j} + 14\vec{k} = -9\vec{i} + 14\vec{k}$

41.) $\vec{u} = 2\vec{i} + \vec{j}$, $\vec{v} = \vec{i} + \vec{j}$, and

$\vec{w} = \vec{i} - \vec{j}$; we want

$\vec{u} = a\vec{v} + b\vec{w} \longrightarrow$

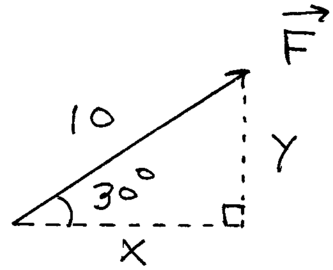
$\overrightarrow{(2,1)} = a\overrightarrow{(1,1)} + b\overrightarrow{(1,-1)} \longrightarrow$

$$\overrightarrow{(2,1)} = \overrightarrow{(a+b,\ a-b)} \longrightarrow$$

$$\left.\begin{array}{l} a+b=2 \\ a-b=1 \end{array}\right\} \quad 2a = 3 \rightarrow a = 3/2$$

$$\frac{3}{2} + b = 2 \rightarrow b = 2 - \frac{3}{2} \rightarrow b = 1/2$$

43.)



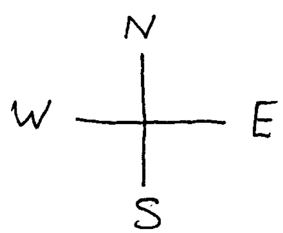
$$\cos 30° = \frac{X}{10} \rightarrow$$

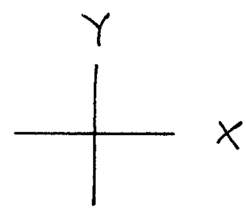
$$\frac{\sqrt{3}}{2} = \frac{X}{10} \rightarrow X = 5\sqrt{3}\ ;$$

$$\sin 30° = \frac{Y}{10} \rightarrow Y = 10 \sin 30° = 10\left(\frac{1}{2}\right) = 5\ ;$$

$$\vec{F} = 5\sqrt{3}\ \vec{i} + 5\ \vec{j}$$

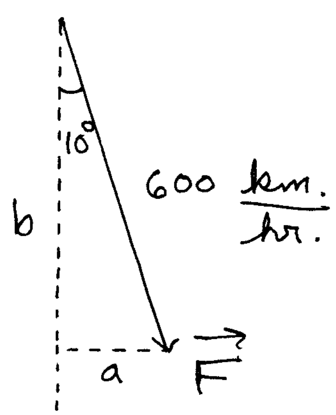
46.)





$$\cos 10° = \frac{b}{600} \rightarrow b = 600 \cos 10°\ ;$$

$$\sin 10° = \frac{a}{600} \rightarrow a = 600 \sin 10°\ ;$$



$$\vec{F} = 600 \sin 10° \cdot \vec{i} - 600 \cos 10° \cdot \vec{j}$$