Section 14.6

1.) $\underbrace{x^2+y^2+z^2}_{f(x,y,z)} = 3$ and $P=(1,1,1)$;

find gradient vector :

$f_x = 2x$, $f_y = 2y$, $f_z = 2z$ so

$\vec{\nabla} f(1,1,1) = f_x(1,1,1)\,\vec{i} + f_y(1,1,1)\,\vec{j} + f_z(1,1,1)\,\vec{k} \Rightarrow$

$\vec{\nabla} f(1,1,1) = 2\vec{i} + 2\vec{j} + 2\vec{k}$; then

a.) tangent plane :

$2(x-1) + 2(y-1) + 2(z-1) = 0 \Rightarrow x+y+z=3$.

b.) normal line :

$$L: \begin{cases} x = 1+2t \\ y = 1+2t \\ z = 1+2t \end{cases}$$

4.) $\underbrace{x^2+2xy-y^2+z^2}_{f(x,y,z)} = 7$ and $P=(1,-1,3)$;

find gradient vector :

$f_x = 2x+2y$, $f_y = 2x-2y$, $f_z = 2z$ so

$\vec{\nabla} f(1,-1,3) = f_x(1,-1,3)\,\vec{i} + f_y(1,-1,3)\,\vec{j} + f_z(1,-1,3)\,\vec{k} \Rightarrow$

$\vec{\nabla} f(1,-1,3) = (0)\vec{i} + (4)\vec{j} + (6)\vec{k} = 4\vec{j} + 6\vec{k}$;

then

a.) tangent plane :

$(0)(x-1) + 4(y+1) + 6(z-3) = 0 \Rightarrow$

$4y + 4 + 6z - 18 = 0 \Rightarrow 4y + 6z = 14 \Rightarrow$

$2y + 3z = 7$.

b.) normal line :

$$L: \begin{cases} x = 1 + (0)t \\ y = -1 + (4)t \\ z = 3 + (6)t \end{cases} \Rightarrow L: \begin{cases} x = 1 \\ y = -1 + 4t \\ z = 3 + 6t \end{cases}$$

9.) $z = \ln(x^2+y^2) \Rightarrow \underbrace{z - \ln(x^2+y^2)}_{f(x,y,z)} = 0$

and $P = (1,0,0)$;

find gradient vector :

$f_x = \dfrac{-2x}{x^2+y^2}$, $f_y = \dfrac{-2y}{x^2+y^2}$, $f_z = 1$ so

$\vec{\nabla} f(1,0,0) = f_x(1,0,0)\,\vec{i} + f_y(1,0,0)\,\vec{j} + f_z(1,0,0)\,\vec{k} \Rightarrow$

$\vec{\nabla} f(1,0,0) = (-2)\vec{i} + (0)\,\vec{j} + (1)\,\vec{k} = -2\vec{i} + \vec{k}$;

then tangent plane is

$(-2)(x-1) + (0)(y-0) + (1)(z-0) = 0 \Rightarrow$

$-2x + 2 + z = 0 \Rightarrow z = 2x - 2$

12.) $z = 4x^2 + y^2 \Rightarrow \underbrace{z - 4x^2 - y^2}_{f(x,y,z)} = 0$

and $P = (1,1,5)$;

find gradient vector :

$f_x = -8x$, $f_y = -2y$, $f_z = 1$ so

$\vec{\nabla} f(1,1,5) = f_x(1,1,5)\,\vec{i} + f_y(1,1,5)\,\vec{j} + f_z(1,1,5)\,\vec{k} \Rightarrow$

$\vec{\nabla} f(1,1,5) = -8\vec{i} - 2\vec{j} + \vec{k}$; then tangent plane

is $-8(x-1) - 2(y-1) + (z-5) = 0 \Rightarrow$

$-8x + 8 - 2y + 2 + z - 5 = 0 \Rightarrow -8x - 2y + z = -5$

14.) $\underbrace{xyz}_{f(x,y,z)} = 1$ and $\underbrace{x^2 + 2y^2 + 3z^2}_{g(x,y,z)} = 6$

and $P = (1,1,1)$; find gradient vectors:

$f_x = yz$, $f_y = xz$, $f_z = xy$ $\Rightarrow$

$\vec{\nabla} f(1,1,1) = (1)\vec{i} + (1)\vec{j} + (1)\vec{k} = \underline{\underline{\vec{i} + \vec{j} + \vec{k}}}$;

$g_x = 2x$, $g_y = 4y$, $g_z = 6z$ $\Rightarrow$

$\vec{\nabla} g(1,1,1) = (2)\vec{i} + (4)\vec{j} + (6)\vec{k} = \underline{\underline{2\vec{i} + 4\vec{j} + 6\vec{k}}}$;

vector $\vec{\nabla} f(1,1,1)$ is $\perp$ to level surface $f(x,y,z) = 1$ at $(1,1,1)$, and vector $\vec{\nabla} g(1,1,1)$ is $\perp$ to level surface $g(x,y,z) = 6$, so vector

$$\boxed{\vec{T} = \vec{\nabla} f(1,1,1) \times \vec{\nabla} g(1,1,1)}$$

is tangent to both curves (curve of intersection) at $\underline{\underline{P = (1,1,1)}}$; then

$$\vec{T} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 4 & 6 \end{vmatrix} = (6-4)\vec{i} - (6-2)\vec{j} + (4-2)\vec{k}$$
$$= 2\vec{i} - 4\vec{j} + 2\vec{k},$$

so tangent line is

$$L: \begin{cases} x = 1 + (2)t \\ y = 1 + (-4)t \\ z = 1 + (2)t \end{cases} \Rightarrow L: \begin{cases} x = 1 + 2t \\ y = 1 - 4t \\ z = 1 + 2t \end{cases}$$

15.) $\underbrace{x^2+2y+2z}_{f(x,y,z)}=4$ and $\underbrace{y}_{g(x,y,z)}=1$

and $P=(1,1,\tfrac{1}{2})$; find gradient vectors:

$f_x=2x,\ f_y=2,\ f_z=2$ so

$\vec{\nabla}f\,(1,1,\tfrac{1}{2}) = 2\vec{i}+2\vec{j}+2\vec{k}$;

$g_x=0,\ g_y=1,\ g_z=0$ so

$\vec{\nabla}g\,(1,1,\tfrac{1}{2}) = (0)\vec{i}+(1)\vec{j}+(0)\vec{k} = \vec{j}$; then

$$\vec{T} = \vec{\nabla}f(1,1,\tfrac{1}{2}) \times \vec{\nabla}g(1,1,\tfrac{1}{2}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 2 \\ 0 & 1 & 0 \end{vmatrix}$$

$= (0-2)\vec{i}-(0-0)\vec{j}+(2-0)\vec{k} = -2\vec{i}+2\vec{k}$;

so tangent line is

$$L: \begin{cases} x = 1+(-2)t \\ y = 1+(0)t \\ z = \frac{1}{2}+(2)t \end{cases} \Rightarrow L: \begin{cases} x = 1-2t \\ y = 1 \\ z = \frac{1}{2}+2t. \end{cases}$$

20.) $f(x,y,z) = e^x \cos yz$, $P=(0,0,0)$,

$ds = 0.1$, and $\vec{A} = 2\vec{i}+2\vec{j}-2\vec{k}$; then

$\vec{u} = \dfrac{1}{|\vec{A}|}\vec{A} = \dfrac{1}{2\sqrt{3}}(2\vec{i}+2\vec{j}-2\vec{k}) \Rightarrow$

$\vec{u} = \dfrac{1}{\sqrt{3}}\vec{i}+\dfrac{1}{\sqrt{3}}\vec{j}-\dfrac{1}{\sqrt{3}}\vec{k}$; find

gradient vector :

$f_x = e^x \cos yz$, $f_y = -ze^x \sin yz$, $f_z = -ye^x \sin yz$

so $\vec{\nabla} f(0,0,0) = (1)\vec{i} + (0)\vec{j} + (0)\vec{k} = \vec{i}$; then the <u>differential</u> is

$$df = \left(D_{\vec{u}} f(0,0,0)\right) \cdot ds$$
$$= \left(\vec{\nabla} f(0,0,0) \bullet \vec{u}\right) \cdot ds$$
$$= \left(\vec{i} \bullet \left(\frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} - \frac{1}{\sqrt{3}}\vec{k}\right)\right) \bullet (0.1)$$
$$= \frac{1}{\sqrt{3}}(0.1) \approx 0.0577$$

21.) $g(x,y,z) = x + x\cos z - y\sin z + y$, $P = (2,-1,0)$, $ds = 0.2$, move toward $Q = (0,1,2)$ so $\overrightarrow{PQ} = \overrightarrow{(-2,2,2)}$; then $\vec{u} = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} \Rightarrow$

$\vec{u} = \frac{1}{2\sqrt{3}}\overrightarrow{(-2,2,2)} = \overrightarrow{\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$;

find gradient vector :

$g_x = 1 + \cos z$, $g_y = -\sin z + 1$,

$g_z = -x\sin z - y\cos z$ so

$\vec{\nabla} g(2,-1,0) = (2)\vec{i} + (1)\vec{j} + (1)\vec{k} = 2\vec{i} + \vec{j} + \vec{k}$; then the <u>differential</u> is

$$df = \left(D_{\vec{u}}\, g(2,-1,0)\right) \cdot ds$$
$$= \left(\vec{\nabla} g(2,-1,0) \bullet \vec{u}\right) \cdot ds$$

$$= \left((2\vec{i} + \vec{j} + \vec{k}) \cdot \left(-\frac{1}{\sqrt{3}} \vec{i} + \frac{1}{\sqrt{3}} \vec{j} + \frac{1}{\sqrt{3}} \vec{k} \right) \right) \cdot (0.2)$$

$$= \left(-\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \right) (0.2) = (0)(0.2) = 0.$$

Section 14.7

1.) $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4 \Rightarrow$

$f_x = 2x + y + 3 = 0 \Rightarrow \boxed{y = -2x - 3}$;

$f_y = x + 2y - 3 = 0 \Rightarrow \boxed{x = 3 - 2y}$;

substitute $\Rightarrow$

$$y = -2x - 3 = -2(3 - 2y) - 3 \Rightarrow$$

$$y = -6 + 4y - 3 \Rightarrow 9 = 3y \Rightarrow$$

$y = 3 \Rightarrow x = -3$ so $\boxed{(-3,3)}$ is critical point ; $f_{xx} = 2$, $f_{yy} = 2$, $f_{xy} = 1$; then

$$D = (f_{xx})(f_{yy}) - (f_{xy})^2$$

$$= (2)(2) - (1)^2 = 3 > 0,$$

and $f_{xx} = 2 > 0$, so $(-3,3)$ determines a <u>minimum</u> <u>value</u> of $f(3,-3) = -5$.

5.) $f(x,y) = x^2 + xy + 3x + 2y + 5 \Rightarrow$

$f_x = 2x + y + 3 = 0 \Rightarrow \boxed{y = -2x - 3}$;

$f_y = x + 2 = 0 \Rightarrow \boxed{x = -2}$;

substitute $\Rightarrow$

$y = -2x - 3 = -2(-2) - 3 = 1 \Rightarrow$

$\boxed{(-2,1)}$ is critical point ;

$f_{xx} = 2$, $f_{yy} = 0$, $f_{xy} = 1$; then

$$D = (f_{xx})(f_{yy}) - (f_{xy})^2$$

$$= (2)(0) - (1)^2 = -1 < 0,$$ so

$(-2,1)$ determines a <u>saddle</u> <u>point</u>.

19.) $f(x,y) = 6x^2 - 2x^3 + 3y^2 + 6xy \Rightarrow$

$f_x = 12x - 6x^2 + 6y = 6(2x - x^2 + y) = 0 \Rightarrow$

$2x - x^2 + y = 0 \Rightarrow \boxed{y = x^2 - 2x}$;

$f_y = 6y + 6x = 6(y+x) = 0 \Rightarrow y + x = 0 \Rightarrow$

$\boxed{y = -x}$; substitute $\Rightarrow$

$y = x^2 - 2x \Rightarrow -x = x^2 - 2x \Rightarrow$

$0 = x^2 - x = x(x-1) \Rightarrow \underline{x=0}$ or $\underline{x=1}$;

if $x=0$, then $y=0$ so $\boxed{(0,0)}$ is critical point ; if $x=1$, then $y=-1$, so $\boxed{(1,-1)}$ is critical point ;

$f_{xx} = 12 - 12x$, $f_{yy} = 6$, $f_{xy} = 6$;

<u>check</u> $(0,0)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$

$= (12)(6) - (6)^2 = 36 > 0$, and

$f_{xx} = 12 > 0$ so $(0,0)$ determines a <u>minimum</u> <u>value</u> of $f(0,0) = 0$.

<u>check</u> $(1,-1)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$

$= (0)(6) - (6)^2 = -36 < 0$, so

$(1,-1)$ determines a <u>saddle point</u>.

23.) $f(x,y) = x^3 + y^3 + 3x^2 - 3y^2 - 8 \Rightarrow$

$f_x = 3x^2 + 6x = 0 \Rightarrow 3x(x+2) = 0 \Rightarrow$

$\boxed{x=0}$ or $\boxed{x=-2}$;

$f_y = 3y^2 - 6y = 3y(y-2) \Rightarrow$

$\boxed{y=0}$ or $\boxed{y=2}$; then

critical points are

$(0,0)$, $(0,2)$, $(-2,0)$, $(-2,2)$;

$f_{xx} = 6x+6$, $f_{yy} = 6y-6$, $f_{xy} = 0$;

check $(0,0)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$
$= (6)(-6) - (0)^2 = -36 < 0$, so
$(0,0)$ determines a saddle point ;

check $(0,2)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$
$= (6)(6) - (0)^2 = 36 > 0$, and
$f_{xx} = 6 > 0$ so $(0,2)$ determines a minimum value of $f(0,2) = -12$.

check $(-2,0)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$
$= (-6)(-6) - (0)^2 = 36 > 0$, and
$f_{xx} = -6 < 0$ so $(-2,0)$ determines a maximum value of $f(-2,0) = -4$;

check $(-2,2)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$
$= (-6)(6) - (0)^2 = -36 < 0$, so
$(-2,2)$ determines a saddle point.

25.) $f(x,y) = 4xy - x^4 - y^4 \Rightarrow$

$f_x = 4y - 4x^3 = 4(y-x^3) = 0 \Rightarrow \boxed{y = x^3}$;

$f_y = 4x - 4y^3 = 4(x-y^3) = 0 \Rightarrow \boxed{x = y^3}$;

substitute $\Rightarrow$

$y = x^3 = (y^3)^3 = y^9 \Rightarrow$

$$0 = y^9 - y = y(y^8-1) = y(y^4-1)(y^4+1)$$
$$= y(y^2-1)(y^2+1)(y^4+1)$$
$$= y(y-1)(y+1)(y^2+1)(y^4+1)$$

$y=0$ or $y=1$ or $y=-1$;

if $y=0$, then $x=0$ so $\boxed{(0,0)}$ is critical point ; if $y=1$, then $x=1$ so $\boxed{(1,1)}$ is critical point ;
if $y=-1$, then $x=-1$ so $\boxed{(-1,-1)}$ is critical point ;

$f_{xx} = -12x^2$, $f_{yy} = -12y^2$, $f_{xy} = 4$;

check $(0,0)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$
$= (0)(0) - (4)^2 = -16 < 0$, so
$(0,0)$ determines a saddle point ;
check $(1,1)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$
$= (-12)(-12) - (4)^2 = 128 > 0$, and
$f_{xx} = -12 < 0$ so $(1,1)$ determines a maximum value of $f(1,1) = 2$;

check $(-1,-1)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$
$= (-12)(-12) - (4)^2 = 128 > 0$, and
$f_{xx} = -12 < 0$ so $(-1,-1)$ determines a maximum value of $f(-1,-1) = 2$.

28.) $f(x,y) = \frac{1}{x} + xy + \frac{1}{y} \Rightarrow$

$f_x = \frac{-1}{x^2} + y = 0 \Rightarrow \boxed{y = 1/x^2}$;

$f_y = x - \frac{1}{y^2} = 0 \Rightarrow \boxed{x = \frac{1}{y^2}}$;

substitute $\Rightarrow$

$y = \frac{1}{x^2} = \frac{1}{\left(\frac{1}{y^2}\right)^2} = y^4 \Rightarrow$

$y = y^4 \Rightarrow 0 = y^4 - y = y(y^3 - 1) \Rightarrow$

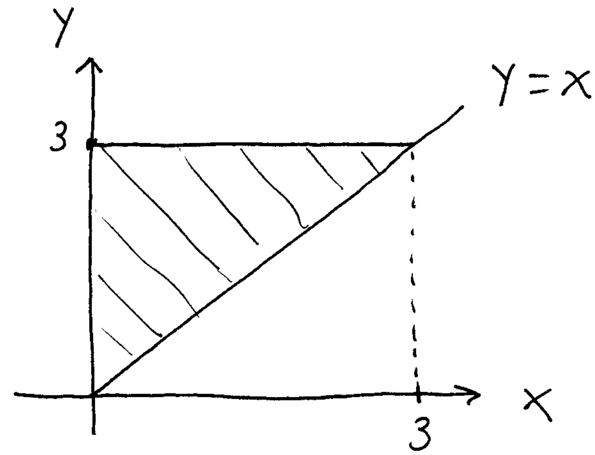
$y = 0$ or $y = 1$; if $y = 0$, ~~then~~ $x = \frac{1}{y^2}$ (impossible!) ; if $y = 1$, then $x = 1$ so $\boxed{(1,1)}$ is critical point ; $f_{xx} = \frac{2}{x^3}$, $f_{yy} = \frac{2}{y^3}$, $f_{xy} = 1$; then

$$D = (f_{xx})(f_{yy}) - (f_{xy})^2$$
$$= (2)(2) - (1)^2 = 3 > 0 \text{, and}$$

$f_{xx} = 2 > 0$, so $(1,1)$ determines a minimum value of

$f(1,1) = 3$.

31.)



$f(x,y) = 2x^2 - 4x + y^2 - 4y + 1 \Rightarrow$
$f_x = 4x - 4 = 4(x-1) = 0 \Rightarrow x = 1$;
$f_y = 2y - 4 = 2(y-2) = 0 \Rightarrow y = 2$,
so $\boxed{(1,2)}$ is critical point ;
corners are $\boxed{(0,0)}$, $\boxed{(3,3)}$, and $\boxed{(0,3)}$;

<u>along path $x=0$:</u> $z = y^2 - 4y + 1 \Rightarrow$
$z' = 2y - 4 = 2(y-2) = 0 \Rightarrow y = 2$ so
$\boxed{(0,2)}$ is critical point ;

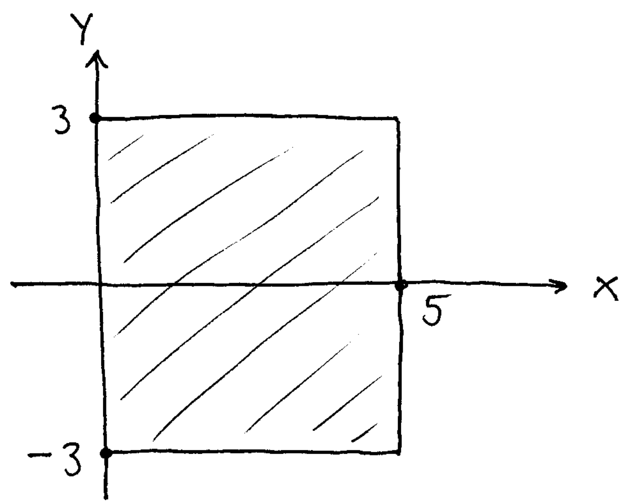
<u>along path $y=3$:</u> $z = 2x^2 - 4x - 2 \Rightarrow$
$z' = 4x - 4 = 4(x-1) = 0 \Rightarrow x = 1$ so
$\boxed{(1,3)}$ is critical point ;

<u>along path $y=x$:</u>
$z = 2x^2 - 4x + x^2 - 4x + 1 \Rightarrow$
$z = 3x^2 - 8x + 1 \Rightarrow$
$z' = 6x - 8 = 0 \Rightarrow x = 4/3$, so
$\boxed{(4/3, 4/3)}$ is critical point :

compare function values:

critical points and corners	function values	
$(1,2)$	$f(1,2)=-5$	
$(0,0)$	$f(0,0)=1$	
$(3,3)$	$f(3,3)=4$	MAX
$(0,3)$	$f(0,3)=-2$	
$(0,2)$	$f(0,2)=-3$	
$(1,3)$	$f(1,3)=-12$	MIN
$(4/3, 4/3)$	$f(4/3, 4/3)=-37/9$	

34.)



$T(x,y) = x^2 + xy + y^2 - 6x \Rightarrow$

$T_x = 2x + y - 6 = 0 \Rightarrow \underline{y = -2x + 6}$;

$T_y = x + 2y = 0 \Rightarrow \underline{x = -2y}$; substitute

$\Rightarrow y = -2x + 6 = -2(-2y) + 6 = 4y + 6 \Rightarrow$

$0 = 3y + 6 \Rightarrow y = -2 \Rightarrow x = 4$ so

$\boxed{(4,-2)}$ is critical point ;

corners are $(0,3)$, $(0,-3)$, $(5,3)$, $(5,-3)$;

along path $X=0$: $Z = Y^2 \Rightarrow Z' = 2Y = 0 \Rightarrow$
$Y = 0$ so $\boxed{(0,0)}$ is critical point ;

along path $X=5$: $Z = Y^2 + 5Y - 5 \Rightarrow$
$Z' = 2Y + 5 = 0 \Rightarrow Y = -5/2$, so
$\boxed{(5,-5/2)}$ is critical point ;

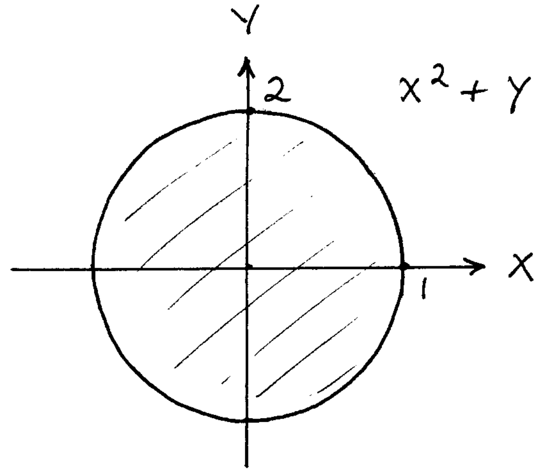
along path $Y=3$: $Z = X^2 - 3X + 9 \Rightarrow$
$Z' = 2X - 3 = 0 \Rightarrow X = 3/2$, so
$\boxed{(3/2, 3)}$ is critical point ;

along path $Y=-3$: $Z = X^2 - 9X + 9 \Rightarrow$
$Z' = 2X - 9 = 0 \Rightarrow X = 9/2$, so
$\boxed{(9/2, 3)}$ is critical point ;

compare function values :

critical points and corners	function values	
$(4,-2)$	$T(4,-2) = -12$	MIN
$(0,3)$	$T(0,3) = 9$	
$(0,-3)$	$T(0,-3) = 9$	
$(5,3)$	$T(5,3) = 19$	MAX
$(5,-3)$	$T(5,-3) = -11$	
$(0,0)$	$T(0,0) = 0$	
$(5,-5/2)$	$T(5,-5/2) = -45/4$	
$(3/2, 3)$	$T(3/2,3) = 27/4$	
$(9/2, 3)$	$T(9/2,3) = 63/4$	

41.)



$x^2 + y^2 = 1$

$T(x,y) = x^2 + 2y^2 - x \Rightarrow$

$T_x = 2x - 1 = 0 \Rightarrow x = 1/2$;

$T_y = 4y = 0 \Rightarrow y = 0$, so

$\boxed{(1/2, 0)}$ is <u>critical point</u> ;

<u>along path $x^2+y^2=1$</u> : $\begin{cases} x = \cos t \\ y = \sin t \end{cases}$

for $0 \le t \le 2\pi$:

$z = (\cos t)^2 + 2(\sin t)^2 - \cos t$

$= \underline{\cos^2 t + \sin^2 t} + \sin^2 t - \cos t$

$= 1 + \sin^2 t - \cos t \Rightarrow$

$z' = 2 \sin t \cos t + \sin t$

$= \sin t \, (2 \cos t + 1) = 0 \Rightarrow$

$\sin t = 0 \Rightarrow \underline{t = 0°}$ or $\underline{t = 180°}$ OR

$\cos t = -1/2 \Rightarrow \underline{t = 120°}$ or $\underline{t = 240°}$;

so <u>critical points</u> are :

$t = 0°$: $(1, 0)$

$t = 180°$: $(-1, 0)$

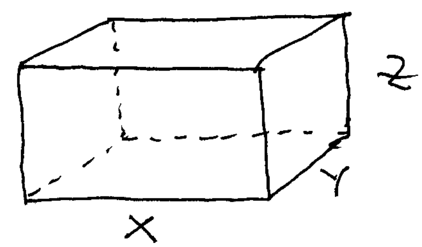
$t = 120°$: $(-1/2, \sqrt{3}/2)$

$t = 240°$: $(-1/2, -\sqrt{3}/2)$

compare function values :

critical points	function values	
$(1/2, 0)$	$T(1/2, 0) = \boxed{-1/4\ ^\circ F}$	MIN
$(1, 0)$	$T(1, 0) = 0^\circ F$	
$(-1, 0)$	$T(-1, 0) = 2^\circ F$	
$(-1/2, \sqrt{3}/2)$	$T(-1/2, \sqrt{3}/2) = \boxed{2\tfrac{1}{4}\,^\circ F}$	MAX
$(-1/2, -\sqrt{3}/2)$	$T(-1/2, -\sqrt{3}/2) = \boxed{2\tfrac{1}{4}\,^\circ F}$	MAX

I.)



Volume

$V = XYZ = 1$ ~~so~~

$\boxed{Z = \dfrac{1}{XY}}$;

minimize cost (¢)

$C = C_{top} + C_{bottom} + C_{sides}$

$= 3XY + 3XY + 2(2XZ + 2YZ)$

$= 6XY + 4(X+Y) \cdot Z$

$= 6XY + 4(X+Y) \cdot \dfrac{1}{XY} \Rightarrow$

$\boxed{C = 6XY + \dfrac{4}{Y} + \dfrac{4}{X}}$; ~~then~~

$C_X = 6Y - \dfrac{4}{X^2} = 0 \Rightarrow \underline{Y = \dfrac{2}{3X^2}}$;

$C_Y = 6X - \dfrac{4}{Y^2} = 0 \Rightarrow \underline{X = \dfrac{2}{3Y^2}}$;

substitute $\Rightarrow$

$Y = \dfrac{2}{3X^2} = \dfrac{2}{3\left(\frac{2}{3Y^2}\right)^2} = \dfrac{2}{\frac{4}{3Y^4}} = 2 \cdot \dfrac{3}{4} Y^4 \Rightarrow$

$$Y = \frac{3}{2}Y^4 \Rightarrow 0 = \frac{3}{2}Y^4 - Y = Y\left(\frac{3}{2}Y^3 - 1\right)$$

$$\Rightarrow Y = 0 \text{ (NO!) OR } Y = \left(\frac{2}{3}\right)^{1/3} \text{ ft.} \Rightarrow$$

$$X = \frac{2}{3\left(\frac{2}{3}\right)^{2/3}} = \frac{2}{3 \cdot \frac{2^{2/3}}{3^{2/3}}} = \left(\frac{2}{3}\right)^{1/3} \text{ ft.} \Rightarrow$$

$$Z = \frac{1}{XY} = \frac{1}{\left(\frac{2}{3}\right)^{1/3}\left(\frac{2}{3}\right)^{1/3}} = \frac{1}{\left(\frac{2}{3}\right)^{2/3}} = \left(\frac{3}{2}\right)^{2/3} \text{ ft.}$$

and minimum cost is

$$C = 6XY + \frac{4}{Y} + \frac{4}{X}$$

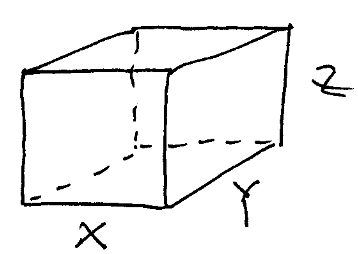
$$= 6\left(\frac{2}{3}\right)^{1/3}\left(\frac{2}{3}\right)^{1/3} + \frac{4}{\left(\frac{2}{3}\right)^{1/3}} + \frac{4}{\left(\frac{2}{3}\right)^{1/3}}$$

$$= 6\left(\frac{2}{3}\right)^{2/3} + 8 \cdot \left(\frac{3}{2}\right)^{1/3}$$

$$= 2^{5/3}\,3^{1/3} + 2^{5/3} \cdot 3^{1/3}$$

$$= 2^{8/3}\,3^{1/3} \approx 9.16 \text{ ¢}$$

II.)



Surface area

$$S = 2XY + 2XZ + 2YZ = 12$$

$$\Rightarrow XY + XZ + YZ = 6$$

$$\Rightarrow XY + (X+Y)Z = 6$$

$$\Rightarrow \boxed{Z = \frac{6 - XY}{X+Y}} \;;$$

maximize volume

$$V = XYZ = XY \cdot \frac{6-XY}{X+Y} = \frac{6XY - X^2Y^2}{X+Y} \Rightarrow$$

$$\boxed{V = \frac{6XY - X^2Y^2}{X+Y}} \quad ; \quad \text{then}$$

$$V_X = \frac{(X+Y)(6Y - 2XY^2) - (6XY - X^2Y^2)}{(X+Y)^2} = 0 \Rightarrow$$

$$Y\left[(X+Y)\cdot(6-2XY) - (6X - X^2Y)\right] = 0 \Rightarrow$$

$$Y=0 \text{ (NO!) OR } \cancel{6X} + 6Y - 2X^2Y - 2XY^2 - \cancel{6X} + X^2Y = 0 \Rightarrow$$

$$Y\cdot\left[6 - 2X^2 - 2XY + X^2\right] = 0 \Rightarrow Y = 0 \text{ (NO!)}$$

$$\text{OR} \quad 6 - X^2 - 2XY = 0 \Rightarrow$$

$$2XY = 6 - X^2 \Rightarrow \quad \boxed{Y = \frac{6-X^2}{2X}} \quad ; \quad \text{and}$$

$$V_Y = \frac{(X+Y)(6X - 2X^2Y) - (6XY - X^2Y^2)}{(X+Y)^2} = 0 \Rightarrow$$

$$X\left[(X+Y)(6-2XY) - (6Y - XY^2)\right] = 0 \Rightarrow$$

$$X=0 \text{ (NO!) OR } 6X + \cancel{6Y} - 2X^2Y - 2XY^2 - \cancel{6Y} + XY^2 = 0 \Rightarrow$$

$$X\cdot\left[6 - 2XY - 2Y^2 + Y^2\right] = 0 \Rightarrow X = 0 \text{ (NO!) OR}$$

$$6 - 2XY - Y^2 = 0 \Rightarrow 2XY = 6 - Y^2 \Rightarrow$$

$$\boxed{X = \frac{6-Y^2}{2Y}} \quad ; \quad \text{substitute} \Rightarrow$$

$$X = \frac{6-Y^2}{2Y} = \frac{6 - \left(\frac{6-X^2}{2X}\right)^2}{2\left(\frac{6-X^2}{2X}\right)} \cdot \frac{(2X)^2}{(2X)^2} \Rightarrow$$

$$X = \frac{6(2X)^2 - (6-X^2)^2}{2(2X)(6-X^2)} \Rightarrow$$

$4x^2(6-x^2) = 24x^2 - (36-12x^2+x^4) \Rightarrow$

$\cancel{24x^2} - 4x^4 = \cancel{24x^2} - 36 + 12x^2 - x^4 \Rightarrow$

$$\begin{aligned} 0 &= 3x^4 + 12x^2 - 36 \\ &= 3\left((x^2)^2 + 4(x^2) - 12\right) \\ &= 3(x^2-2)(x^2+6) \Rightarrow \end{aligned}$$

$x^2 - 2 = 0 \Rightarrow \boxed{x = \sqrt{2}\text{ m.}}$ or $x = -\sqrt{2}$ (NO!);

if $x = \sqrt{2}$, then

$$Y = \frac{6-(\sqrt{2})^2}{2(\sqrt{2})} = \frac{6-2}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} \Rightarrow$$

$\boxed{Y = \sqrt{2}\text{ m.}}$, then $z = \dfrac{6-(\sqrt{2})(\sqrt{2})}{\sqrt{2}+\sqrt{2}} = \dfrac{4}{2\sqrt{2}}$

$\Rightarrow \boxed{z = \sqrt{2}\text{ m.}}$; and max. volume is $V = (\sqrt{2})^3 \Rightarrow$

$\boxed{V = 2\sqrt{2}\text{ m.}^3}$.

Section 14.8

1.) Find extrema for $f(x,y) = xy$
s.t. $x^2 + 2y^2 = 1$:
Let $F(x,y,\lambda) = xy - \lambda(x^2 + 2y^2 - 1) \longrightarrow$

$$\left.\begin{aligned} F_x &= y - 2\lambda x = 0 \\ F_y &= x - 4\lambda y = 0 \end{aligned}\right\} \quad \left.\begin{aligned} \lambda &= y/2x \\ \lambda &= x/4y \end{aligned}\right\} \quad \frac{y}{2x} = \frac{x}{4y} \longrightarrow \boxed{2y^2 = x^2}$$

$F_\lambda = -x^2 - 2y^2 + 1 = 0$ (sub.)

$\longrightarrow -x^2 - x^2 + 1 = 0 \longrightarrow 2x^2 = 1 \longrightarrow$
$x^2 = 1/2 \Rightarrow x = \pm 1/\sqrt{2}$; if $x = \frac{1}{\sqrt{2}}$, then
$y = \pm 1/2$ so $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$ and $\left(\frac{1}{\sqrt{2}}, \frac{-1}{2}\right)$ are
critical points; if $x = -1/\sqrt{2}$ then
$y = \pm 1/2$ so $\left(\frac{-1}{\sqrt{2}}, \frac{1}{2}\right)$ and $\left(\frac{-1}{\sqrt{2}}, -1/2\right)$ are
critical points :

crit. pts.	values of $f(x,y)$	
$\left(\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$	$1/2\sqrt{2}$	MAX
$\left(\frac{1}{\sqrt{2}}, \frac{-1}{2}\right)$	$-1/2\sqrt{2}$	MIN
$\left(\frac{-1}{\sqrt{2}}, \frac{1}{2}\right)$	$-1/2\sqrt{2}$	MIN
$\left(\frac{-1}{\sqrt{2}}, \frac{-1}{2}\right)$	$1/2\sqrt{2}$	MAX

3.) Maximize $f(x,y) = 49 - x^2 - y^2$ s.t.
$x + 3y = 10$:
Let $F(x,y,\lambda) = (49 - x^2 - y^2) - \lambda(x + 3y - 10)$

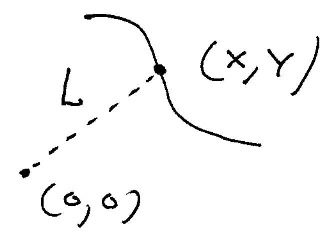
$$\longrightarrow \left.\begin{aligned} F_x &= -2x - \lambda = 0 \\ F_y &= -2y - 3\lambda = 0 \end{aligned}\right\} \quad \left.\begin{aligned} \lambda &= -2x \\ \lambda &= -\tfrac{2}{3}y \end{aligned}\right\} \quad -2x = -\tfrac{2}{3}y \longrightarrow \boxed{y = 3x}$$

$F_\lambda = -x - 3y + 10 = 0$ (sub.)

$\longrightarrow -x - 3(3x) + 10 = 0 \longrightarrow$

$-10x + 10 = 0 \rightarrow x = 1$ and $y = 3$ so $\boxed{(1,3)}$ is critical point with maximum value $f(1,3) = 39$.

8.) Distance



$L = \sqrt{(x-0)^2 + (y-0)^2} \rightarrow$

$L^2 = x^2 + y^2$;

Minimize and maximize

$f(x,y) = x^2 + y^2$ s.t. $x^2 + xy + y^2 = 1$:

Let $F(x,y,\lambda) = (x^2+y^2) - \lambda(x^2+xy+y^2-1) \rightarrow$

$$\left.\begin{array}{l} F_x = 2x - \lambda(2x+y) = 0 \rightarrow \lambda = \dfrac{2x}{2x+y} \\ F_y = 2y - \lambda(x+2y) = 0 \rightarrow \lambda = \dfrac{2y}{x+2y} \end{array}\right\} \rightarrow$$

$$\frac{2x}{2x+y} = \frac{2y}{x+2y} \rightarrow 2x^2 + \cancel{4xy} = \cancel{4xy} + 2y^2 \rightarrow$$

$\boxed{y^2 = x^2} \rightarrow \underline{y = x}$ or $\underline{\underline{y = -x}}$;

$F_\lambda = -x^2 - xy - y^2 + 1 = 0$ (sub.)

If $y = x$, then $-x^2 - x^2 - x^2 + 1 = 0 \rightarrow$

$3x^2 = 1 \rightarrow x = \pm 1/\sqrt{3} \rightarrow \underline{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$ and $\underline{\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)}$ are critical points ;

If $y = -x$, then $\cancel{-x^2} + \cancel{x^2} - x^2 + 1 = 0 \rightarrow$

$x^2 = 1 \rightarrow x = \pm 1 \rightarrow \underline{(1,-1)}$ and $\underline{(-1,1)}$ are critical points ; then

crit. pts.	values of $f(x,y)$	
$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$	$\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$	MIN
$\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$	$\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$	MIN
$(1,-1)$	$1+1=2$	MAX
$(-1,1)$	$1+1=2$	MAX

So min. distance is $\sqrt{\frac{2}{3}}$; max. distance is $\sqrt{2}$.

14.) maximize and minimize

$f(x,y) = 3x-y+6$ s.t. $x^2+y^2=4$:

Let $F(x,y,\lambda) = (3x-y+6)-\lambda(x^2+y^2-4) \rightarrow$

$$\left.\begin{aligned} F_x &= 3-\lambda(2x)=0 \rightarrow \lambda = \frac{3}{2x} \\ F_y &= -1-\lambda(2y)=0 \rightarrow \lambda=\frac{-1}{2y} \end{aligned}\right\} \frac{3}{\cancel{2}x} = \frac{-1}{\cancel{2}y} \rightarrow \boxed{y=\frac{-1}{3}x};$$

$F_\lambda = -x^2-y^2+4=0$ (sub.)

$\rightarrow -x^2-\left(\frac{-1}{3}x\right)^2+4=0 \rightarrow 4=\frac{10}{9}x^2 \rightarrow$

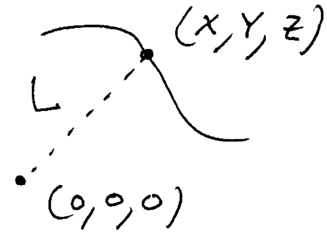
$x^2=\frac{18}{5} \rightarrow x=\pm\sqrt{\frac{18}{5}} = \pm 3\sqrt{\frac{2}{5}}$;

if $x=3\cdot\sqrt{\frac{2}{5}}$, then $y=-\sqrt{\frac{2}{5}}$ and $\left(3\sqrt{\frac{2}{5}}, -\sqrt{\frac{2}{5}}\right)$ is a critical point ;

if $x=-3\sqrt{\frac{2}{5}}$, then $y=\sqrt{\frac{2}{5}}$ and $\left(-3\sqrt{\frac{2}{5}}, \sqrt{\frac{2}{5}}\right)$ is a critical point ; then

crit. pts.	values of $f(x,y)$	
$\left(3\sqrt{\frac{2}{5}}, -\sqrt{\frac{2}{5}}\right)$	$9\sqrt{\frac{2}{5}}+\sqrt{\frac{2}{5}}+6 = 10\sqrt{\frac{2}{5}}+6$	MAX
$\left(-3\sqrt{\frac{2}{5}}, \sqrt{\frac{2}{5}}\right)$	$-9\sqrt{\frac{2}{5}}-\sqrt{\frac{2}{5}}+6 = -10\sqrt{\frac{2}{5}}+6$	MIN

21.) Distance



$L = \sqrt{(x-0)^2 + (Y-0)^2 + (z-0)^2}$

$\rightarrow \quad L^2 = x^2 + Y^2 + z^2$;

minimize $f(x,Y,z) = x^2 + Y^2 + z^2$

s.t. $\quad z^2 = xY + 4 \quad$:

Let $F(x,Y,z,\lambda) = (x^2 + Y^2 + z^2) - \lambda(z^2 - xY - 4) \rightarrow$

$F_x = 2x - \lambda(-Y) = 0 \rightarrow \lambda = \frac{-2x}{Y}$ $\quad$ $1 = \frac{-2x}{Y} \rightarrow \underline{Y = -2x}$,

$F_Y = 2Y - \lambda(-x) = 0 \rightarrow \lambda = \frac{-2Y}{x}$ $\quad$ $1 = \frac{-2Y}{x} \rightarrow \underline{Y = \frac{-1}{2}x}$

$F_z = 2z - \lambda(2z) = 0 \rightarrow \lambda = 1$ $\quad \rightarrow -2x = \frac{-1}{2}x$

$\rightarrow 4x = x \rightarrow 3x = 0 \rightarrow x = 0,\ Y = 0$;

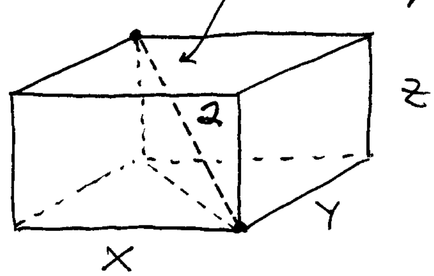
$F_\lambda = -z^2 + xY + 4 = 0 \rightarrow -z^2 + (0)(0) + 4 = 0$

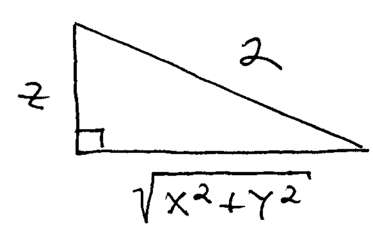
$\rightarrow z^2 = 4 \rightarrow z = \pm 2$ so $\boxed{(0,0,2)}$ and $\boxed{(0,0,-2)}$ are critical points ;

<u>crit. pts.</u>	<u>values of $f(x,Y)$</u>	
$(0,0,2)$	$0+0+4=4$	MIN
$(0,0,-2)$	$0+0+4=4$	MIN

So minimum distance is $\sqrt{4} = 2$.

27.) diagonal of box is 2





then $\quad x^2 + Y^2 + z^2 = 4$;

maximize volume $V = XYZ$

s.t. $X^2 + Y^2 + Z^2 = 4$: $(X>0, Y>0, Z>0)$

Let $F(X,Y,Z,\lambda) = XYZ - \lambda(X^2+Y^2+Z^2-4) \rightarrow$

$$\left.\begin{array}{l} F_X = YZ - \lambda(2X) = 0 \rightarrow \lambda = YZ/2X \\ F_Y = XZ - \lambda(2Y) = 0 \rightarrow \lambda = XZ/2Y \\ F_Z = XY - \lambda(2Z) = 0 \rightarrow \lambda = \frac{XY}{2Z} \end{array}\right\} \text{(pair them)} \rightarrow$$

$$\left.\begin{array}{l} \frac{YZ}{2X} = \frac{XZ}{2Y} \longrightarrow \boxed{X^2 = Y^2} \\ \frac{XZ}{2Y} = \frac{XY}{2Z} \longrightarrow \boxed{Z^2 = Y^2} \end{array}\right\} \text{(sub)}$$

$F_\lambda = -X^2 - Y^2 - Z^2 + 4 = 0$

$\rightarrow -(Y^2) - Y^2 - (Y^2) + 4 = 0$

$\rightarrow 3Y^2 = 4 \rightarrow Y = \pm 2/\sqrt{3} = \frac{2}{\sqrt{3}}$ (ONLY +!)

if $Y = \frac{2}{\sqrt{3}}$, then $X = \frac{2}{\sqrt{3}}$ and $Z = \frac{2}{\sqrt{3}}$

so critical point is $\boxed{\left(\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)}$ and maximum volume is

$$V = \left(\frac{2}{\sqrt{3}}\right)^3 = \frac{8}{3\sqrt{3}}$$

30.) maximize and minimize $T = 400XYZ^2$

s.t. $X^2 + Y^2 + Z^2 = 1$:

Let $F(X,Y,Z,\lambda) = 400XYZ^2 - \lambda(X^2+Y^2+Z^2-1) \rightarrow$

$F_X = 400YZ^2 - \lambda(2X) = 0 \rightarrow \lambda = \frac{400YZ^2}{2X}$

$$\rightarrow \boxed{\lambda = \frac{200YZ^2}{X}}\ ;$$

$$F_Y = 400XZ^2 - \lambda(2Y) = 0 \rightarrow \lambda = \frac{400XZ^2}{2Y}$$

$$\rightarrow \boxed{\lambda = \frac{200XZ^2}{Y}}\ ;$$

$$F_Z = 800XYZ - \lambda(2Z) = 0 \rightarrow \lambda = \frac{800XYZ}{2Z}$$

$$\rightarrow \boxed{\lambda = \frac{400XYZ}{Z}} = 400XY\ ;$$

(pair λ's) $\rightarrow$

$$\left.\begin{array}{l} \frac{200YZ^2}{X} = \frac{200XZ^2}{Y} \rightarrow Y^2Z^2 = X^2Z^2 \\ \frac{200XZ^2}{Y} = 400XY \rightarrow XZ^2 = 2XY^2 \end{array}\right\} \rightarrow$$

$$\left.\begin{array}{l} Y^2Z^2 - X^2Z^2 = 0 \rightarrow (Y^2 - X^2)Z^2 = 0 \\ XZ^2 - 2XY^2 = 0 \rightarrow X(Z^2 - Y^2) = 0 \end{array}\right\} \rightarrow$$

(*) $\left.\boxed{\begin{array}{l} Y^2 = X^2 \text{ or } Z = 0 \\ Z^2 = Y^2 \text{ or } X = 0 \end{array}}\right\}$ (sub) $\rightarrow$

$$F_\lambda = \boxed{-X^2 - Y^2 - Z^2 + 1 = 0}\ ; \text{ then}$$

<u>case</u> 1: If $Y^2 = X^2$ and $Z^2 = Y^2$, then

$-(Y^2) - Y^2 - (Y^2) + 1 = 0 \rightarrow 3Y^2 = 1 \rightarrow$

$Y = \pm 1/\sqrt{3}$; if $Y = \frac{1}{\sqrt{3}}$, then $X = \pm\frac{1}{\sqrt{3}}$ and $Z = \pm 1/\sqrt{3}$ and critical points are

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right), \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right);$$

if $Y = \frac{-1}{\sqrt{3}}$, then $X = \frac{\pm 1}{\sqrt{3}}$ and $Z = \frac{\pm 1}{\sqrt{3}}$
and critical points are
$\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$, $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$, $\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$, $\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$;

<u>case</u> 2: If $Y^2 = X^2$ and $X = 0$, then $Y = 0$ and
$0 - 0 - Z^2 + 1 = 0 \rightarrow Z^2 = 1 \rightarrow Z = \pm 1$
so critical points are $(0,0,1)$, $(0,0,-1)$;

<u>case</u> 3: If $Z = 0$ and $Z^2 = Y^2$, then $Y = 0$
and $-X^2 - 0 - 0 + 1 = 0 \rightarrow X^2 = 1 \rightarrow X = \pm 1$
~~so~~ critical points are $(1,0,0)$, $(-1,0,0)$;

<u>Case</u> 4: If $Z = 0$ and $X = 0$, then
$0 - Y^2 - 0 + 1 = 0 \rightarrow Y^2 = 1 \rightarrow Y = \pm 1$
so critical points are $(0,1,0)$, $(0,-1,0)$;
then

critical pts.	values of f(x,y)	
$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$	$\frac{400}{9}\,°C$	MAX
$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$	$\frac{400}{9}\,°C$	MAX
$\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$	$-\frac{400}{9}\,°C$	MIN
$\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$	$-\frac{400}{9}\,°C$	MIN
$\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$	$-\frac{400}{9}\,°C$	MIN
$\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$	$-\frac{400}{9}\,°C$	MIN
$\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$	$\frac{400}{9}\,°C$	MAX
$\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$	$\frac{400}{9}\,°C$	MAX
$(0,0,1)$	$0\,°C$	
$(0,0,-1)$	$0\,°C$	
$(1,0,0)$	$0\,°C$	
$(-1,0,0)$	$0\,°C$	
$(0,1,0)$	$0\,°C$	
$(0,-1,0)$	$0\,°C$	

33.) maximize $f(x,y,z) = x^2 + 2y - z^2$
s.t. $2x - y = 0$ and $y + z = 0$:

Let $F(x,y,z,\lambda,\mu) = (x^2 + 2y - z^2)$
$- \lambda(2x - y) - \mu(y + z) \rightarrow$

$$\left.\begin{array}{l} F_x = 2x - \lambda(2) - \mu(0) = 2x - 2\lambda = 0 \rightarrow \lambda = x \\ F_y = 2 - \lambda(-1) - \mu(1) = 2 + \lambda - \mu = 0 \quad \text{(sub.)} \\ F_z = -2z - \lambda(0) - \mu(1) = -2z - \mu = 0 \rightarrow \mu = -2z \end{array}\right\}$$

$\rightarrow 2 + x - (-2z) = 0 \rightarrow \boxed{x + 2z = -2}$;

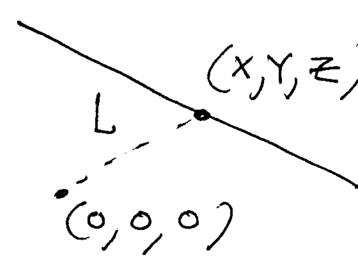
$F_\lambda = -2x + y = 0 \rightarrow \boxed{2x - y = 0}$;
$F_\mu = -y - z = 0 \rightarrow \boxed{y + z = 0}$; then solve

$$\left.\begin{array}{l} x + 2z = -2 \\ 2x - y = 0 \\ y + z = 0 \end{array}\right\} \quad \left.\begin{array}{l} x + 2z = -2 \\ 2x + z = 0 \end{array}\right\} \rightarrow$$

$-3z = 4 \rightarrow z = -4/3$, $y = 4/3$, $x = 2/3$ so
critical point is $\boxed{(2/3, 4/3, -4/3)}$ and

maximum value is
$f\left(\frac{2}{3}, \frac{4}{3}, \frac{-4}{3}\right) = \frac{4}{9} + \frac{24}{9} - \frac{16}{9} = \frac{12}{9} = \frac{4}{3}$.

35.) Distance



$L = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$

$\rightarrow L^2 = x^2 + y^2 + z^2$;

minimize $f(x,y,z) = x^2+y^2+z^2$

s.t. $y+2z=12$ and $x+y=6$:

Let $F(x,y,z,\lambda,\mu) = (x^2+y^2+z^2)$

$-\lambda(y+2z-12)-\mu(x+y-6) \rightarrow$

$F_x = 2x-\lambda(0)-\mu(1) = 2x-\mu=0 \rightarrow \mu=2x$

$F_y = 2y-\lambda(1)-\mu(1) = 2y-\lambda-\mu=0$ (sub.)

$F_z = 2z-\lambda(2)-\mu(0) = 2z-2\lambda=0 \rightarrow \lambda=z$

$\rightarrow 2y-z-2x=0 \rightarrow \boxed{2x-2y+z=0}$;

$F_\lambda = -y-2z+12=0 \rightarrow \boxed{y+2z=12}$;

$F_\mu = -x-y+6=0 \rightarrow \boxed{x+y=6}$; solve

$$\left.\begin{aligned} x+y \quad &= 6 \\ y+2z &= 12 \\ 2x-2y+z &= 0 \end{aligned}\right\} \left.\begin{aligned} x-2z &= -6 \\ 2x+5z &= 24 \end{aligned}\right\} 9z=36 \rightarrow$$

$z=4$, $x=2$, $y=4$ so critical point is $\boxed{(2,4,4)}$ and minimum

function value is $f(2,4,4)=4+16+16=36$

so minimum distance is $\sqrt{36} = 6$.