Math 21D
Vogler
Discussion Sheet 10

1.) Find the area of the following surfaces \( S \), which are directly above the rectangular region \( R \) with vertices \((0,0), (2,0), (2,4), \) and \((0,4)\) in the \( xy \)-plane.
   a.) plane \( z = 5 \)
   b.) plane \( z = 2y \)
   c.) plane \( x + 2y + 3z = 12 \)

2.) Find the area of the following surfaces \( S \), which are directly above the disc \( x^2 + y^2 \leq 9 \) in the \( xy \)-plane.
   a.) top half of sphere \( x^2 + y^2 + z^2 = 64 \)
   b.) paraboloid \( z = x^2 + y^2 + 1 \)
   c.) cone \( z = \sqrt{x^2 + y^2} \)

3.) Let surface \( S \) be the top half of the sphere \( x^2 + y^2 + z^2 = 4 \). Define the following function \( g \) on \( S \): \( g(P) = g(x, y, z) \) is the square of the distance from \( P \) to the \( xy \)-plane. Compute the surface integral of \( g \) over \( S \).

4.) Let surface \( S \) be that portion of the paraboloid \( z = x^2 + y^2 + 4 \) directly above the disc \( x^2 + y^2 \leq 1 \) in the \( xy \)-plane. Let function \( g(x, y, z) = \sqrt{x^2 + y^2} \). Compute the surface integral of \( g \) over \( S \).

5.) Let surface \( S \) be that portion of the paraboloid \( z = 4 - x^2 - y^2 \) cut by the plane \( z = 0 \). Find the Flux of the vector field \( \vec{F}(x, y, z) = (y)\hat{i} + (x)\hat{j} + (z)\hat{k} \) outward through the surface \( S \).

6.) Find the Flux of the vector field \( \vec{F}(x, y, z) = (2x)\hat{i} + (-3y)\hat{j} + (z)\hat{k} \) in the direction away from the origin and across the region \( S \) in the plane \( x + 2y + 3z = 12 \), which is directly above the triangle with vertices \((0,0), (0,2), \) and \((2,6)\) in the \( xy \)-plane.

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

7.) Two bicyclists are twelve miles apart. They begin riding toward each other, one pedaling at 4 mph and the other at 2 mph. At the same time a bumblebee begins flying back and forth between the riders at a constant speed of 10 mph. What is the total distance the bumblebee travels by the time the riders meet?
1.) Compute the divergence of $\vec{F}$ and the curl of $\vec{F}$ for each of the following vector fields.
\[ \text{a.) } \vec{F}(x, y, z) = (x^4)\vec{i} + (-x^3z^2)\vec{j} + (4xy^2z)\vec{k} \]
\[ \text{b.) } \vec{F}(x, y, z) = (xy\sin z)\vec{i} + (\cos(xz))\vec{j} + (y\cos z)\vec{k} \]

2.) Verify Stoke's Theorem for $\vec{F}(x, y, z) = (y^2)\vec{i} + (x)\vec{j} + (z^2)\vec{k}$, where surface $S$ is that portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$.

3.) Use Stoke's Theorem to evaluate $\int \int_S \nabla \times \vec{F} \cdot \vec{n} \ dS$, where $\vec{F}(x, y, z) = (yz)\vec{i} + (xz)\vec{j} + (xy)\vec{k}$ and surface $S$ is that portion of the paraboloid $z = 9 - x^2 - y^2$ above the plane $z = 5$.

4.) Use Stoke's Theorem to evaluate $\oint_C \vec{F} \cdot \vec{T} \ ds$, where $\vec{F}(x, y, z) = (e^{-x})\vec{i} + (e^x)\vec{j} + (e^z)\vec{k}$ and surface $S$ is that portion of the plane $2x + y + 2z = 2$ in the first octant.

5.) Verify the Divergence Theorem for $\vec{F}(x, y, z) = (xy)\vec{i} + (yz)\vec{j} + (xz)\vec{k}$, where the solid $D$ is the cylinder $x^2 + y^2 = 1$ for $0 \leq z \leq 1$.

6.) Use the Divergence Theorem to evaluate $\int \int_S \vec{F} \cdot \vec{n} \ dS$, where $\vec{F}(x, y, z) = (e^x \sin y)\vec{i} + (e^x \cos y)\vec{j} + (yz^2)\vec{k}$ and surface $S$ is the box bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 2$.

7.) Use the Divergence Theorem to evaluate $\int \int \int_D \text{div} \vec{F} \ dV$, where $\vec{F}(x, y, z) = (xe^y)\vec{i} + (xz)\vec{j} + (x \sin z)\vec{k}$ and the solid $D$ is the cube with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

"An individual has not started living until he can rise above the narrow confines of his individualistic concerns to the broader concerns of all humanity." – Martin Luther King, Jr.