1.) Consider the mapping $F$ given by $F(u, v) = (3u - 2v, u + v) = (x, y)$. Let $R$ be the rectangle and its interior in the $uv$-plane with vertices $(0, 0), (2, 0), (2, 3)$, and $(0, 3)$.
   a.) Find the image $S$ of $R$ under $F$ and the area of $S$.
   b.) Find a mapping $G$ which maps $S$ to $R$.

2.) Redo problem 1.) where $R$ is the triangle and its interior with vertices $(0, 0), (-2, 3)$, and $(2, 0)$.

3.) Consider the mapping $F$ given by $F(u, v, w) = (u - v + 2w, 2u + v - w, 3u + 2v + w) = (x, y, z)$. Let $R$ be the rectangle box and its interior in the $uvw$-space with vertices $(0, 0, 0), (2, 0, 0), (2, 3, 0), (0, 3, 0), (0, 0, 4), (2, 0, 4), (2, 3, 4), (0, 3, 4)$.
   a.) Find the image $S$ of $R$ under $F$ and the volume of $S$.
   b.) Find a mapping $G$ which maps $S$ to $R$.

4.) Plot the curve $C$ determined by each vector function.
   a.) $\mathbf{r}(t) = e^{t} \mathbf{i} + e^{2t} \mathbf{j}$ for $-1 \leq t \leq 1$
   b.) $\mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j}$ for $0 \leq t \leq 2\pi$
   c.) $\mathbf{r}(t) = \sqrt{t} \cos t \mathbf{i} + \sqrt{t} \sin t \mathbf{j}$ for $0 \leq t \leq 4\pi$
   d.) $\mathbf{r}(t) = 2t \mathbf{i} + 3t \mathbf{j} + 4t \mathbf{k}$ for $0 \leq t \leq 2$
   e.) $\mathbf{r}(t) = \sin t \mathbf{i} + \cos t \mathbf{j} + t \mathbf{k}$ for $0 \leq t \leq 4\pi$

5.) Assume that the motion of a particle along path $C$ is determined by the position function $\mathbf{r}(t) = f(t) \mathbf{i} + g(t) \mathbf{j} + h(t) \mathbf{k}$. We know that the speed of motion at time $t$ is $\left| \mathbf{v}(t) \right| = \frac{ds}{dt} = \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}$. Show that the acceleration of motion at time $t$ is given by $a(t) = \frac{\mathbf{v}(t) \cdot \mathbf{a}(t)}{\left| \mathbf{v}(t) \right|}$.

6.) Assume that the path $C$ of a bird in flight is determined by the vector function $\mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j} + 2t \mathbf{k}$ for $t \geq 0$. Find the bird's position vector, velocity vector, speed, acceleration vector, and acceleration at time
   a.) $t = 0$.
   a.) $t = 1$.
   a.) $t = 2$.

7.) The position of a bicyclist is determined by the vector function $\mathbf{r}(t) = (3t) \mathbf{i} + (3\sin t) \mathbf{j}$ for $0 \leq t \leq 2\pi$. Determine the bicyclist's maximum speed.
8.) Find vector function \( \mathbf{r}(t) \) if \( \mathbf{r}'(t) = \mathbf{i} + t \mathbf{j} + \cos 2t \mathbf{k} \), \( \mathbf{r}'(0) = \mathbf{i} + \mathbf{j} + \mathbf{k} \), and \( \mathbf{r}(0) = 2\mathbf{i} - \mathbf{j} - \mathbf{k} \).

9.) A super ball is projected at an angle of 75° with initial speed 100 m/sec.
   a.) How high does the ball go?
   b.) How long is the ball in the air?
   c.) How far downrange does the ball travel?

10.) A ball bearing is projected at an angle of 60° and lands 500 feet downrange. What was the ball bearing’s initial speed?

11.) A kiwi is projected at an angle of \( \alpha \) degrees with an initial speed of 100 m/sec. If it lands 200 meters downrange, what is \( \alpha \)?

12.) Assume that \( \mathbf{u}(t) = a(t) \mathbf{i} + b(t) \mathbf{j} + c(t) \mathbf{k} \), \( \mathbf{v}(t) = f(t) \mathbf{i} + g(t) \mathbf{j} + h(t) \mathbf{k} \), and \( y = k(t) \).
   a.) (Dot Product Rule) Prove that \( D\{\mathbf{u}(t) \cdot \mathbf{v}(t)\} = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t) \).
   b.) (Chain Rule) Prove that \( D\{\mathbf{u}(k(t))\} = \mathbf{u}''(g(t))k'(t) \).

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

13.) Find the limit of the following sequence of numbers:

\[
2, 2 - \frac{1}{2}, 2 - \frac{1}{2 - \frac{1}{2}}, 2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2}}}, 2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2}}}} \ldots
\]