Vogler
Math 21D

Moments & Center of Mass in 3D

Let $\delta(P) = \delta(x,y,z)$ be density $\left(\frac{\text{mass}}{\text{vol.}} \text{ units}\right)$

1st Moments about Arbitrary Planes

About plane $x=\bar{x}$: $M_{x=\bar{x}} = \iiint_R (x-\bar{x})\,\delta(P)\,dV$

About plane $y=\bar{y}$: $M_{y=\bar{y}} = \iiint_R (y-\bar{y})\,\delta(P)\,dV$

About plane $z=\bar{z}$: $M_{z=\bar{z}} = \iiint_R (z-\bar{z})\,\delta(P)\,dV$

1st Moments about Coordinate Planes

$M_{yz} = \iiint_R x\,\delta(P)\,dV \qquad M_{xz} = \iiint_R y\,\delta(P)\,dV \qquad M_{xy} = \iiint_R z\,\delta(P)\,dV$

Note: $M_{yz} = M_{x=0} \qquad M_{xz} = M_{y=0} \qquad M_{xy} = M_{z=0}$

Mass

$m = \iiint_R \delta(P)\,dV$

Center of Mass

$\bar{x} = \frac{M_{yz}}{m} \qquad \bar{y} = \frac{M_{xz}}{m} \qquad \bar{z} = \frac{M_{xy}}{m}$

Volume

$V = \iiint_R 1\,dV$

Centroid

$\bar{x} = \frac{\iiint_R x\,dV}{V} \qquad \bar{y} = \frac{\iiint y\,dV}{V} \qquad \bar{z} = \frac{\iiint z\,dV}{V}$

Moments of Inertia (2nd Moments)

About line L or pt. P_0: $I = \iiint_R r^2\,\delta(P)\,dV$

w/ $r := r(x,y,z)$ is distance from line L or pt. P_0

About x-axis: $I_x = \iiint_R (y^2+z^2)\,\delta(P)\,dV$

About y-axis: $I_y = \iiint_R (x^2+z^2)\,\delta(P)\,dV$

About z-axis: $I_z = \iiint_R (x^2+y^2)\,\delta(P)\,dV$