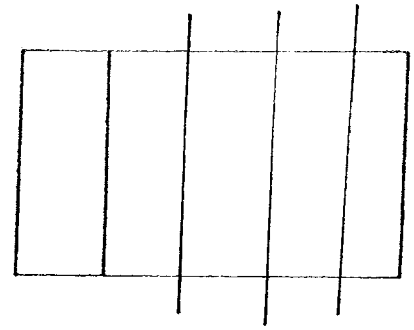
7.) What is the maximum number of rectangles which can be formed within the boundary of the given figure using 100 vertical lines ? Count all rectangles including overlapping ones.



lines	□'s
1	$6 = 1+2+3$
2	$10 = 1+2+3+4$
3	$15 = 1+2+3+4+5$
4	$21 = 1+2+3+4+5+6$
$\vdots$	$\vdots$
n	$1+2+3+\cdots+(n+2) = \dfrac{(n+2)((n+2)+1)}{2}$

If $n=100$ then $\dfrac{(102)(103)}{2} = 5253$ rectangles

2.) Evaluate the double integral $\int_0^1 \int_0^x (2x - 3y^2)\, dy\, dx$.

$$= \int_0^1 (2xy - y^3) \Big|_{y=0}^{y=x} dx$$

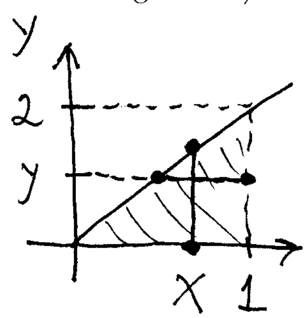
$$= \int_0^1 (2x^2 - x^3)\, dx$$

$$= \left(\frac{2}{3}x^3 - \frac{1}{4}x^4\right)\Big|_0^1$$

$$= \frac{2}{3} - \frac{1}{4}$$

$$= \frac{5}{12}$$

3.) Evaluate the double integral $\int_0^2 \int_{y/2}^1 e^{-x^2}\, dx\, dy$. (HINT : Switch the order of integration.)



$x = \frac{y}{2}$ or $y = 2x$

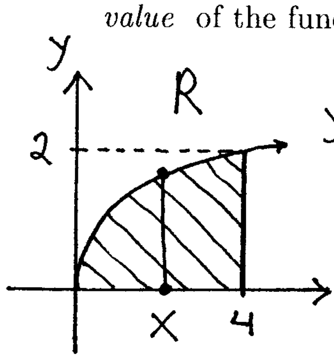
$$= \int_0^1 \int_0^{2x} e^{-x^2}\, dy\, dx$$

$$= \int_0^1 \left(e^{-x^2} \cdot y\right)\Big|_{y=0}^{y=2x} dx$$

$$= \int_0^1 2xe^{-x^2} dx = -e^{-x^2}\Big|_0^1$$

$$= -e^{-1} - -e^{0} = 1 - \frac{1}{e}$$

4.) SET BUT DO NOT EVALUATE the double integral(s) for the *average value* of the function $f(x, y) = xy$ over the region R given in the diagram.



$y = \sqrt{x}$

$$\text{Area } R = \int_0^4 \int_0^{\sqrt{x}} 1\, dy\, dx$$

and

$$\text{AVE} = \frac{1}{\text{Area } R} \cdot \int_0^4 \int_0^{\sqrt{x}} xy\, dy\, dx$$

5.) Find and classify each critical point as that which determines a relative maximum value, a relative minimum value, or a saddle point for the function

$f(x, y) = x^3 - y^3 + 3xy$:

$f_x = 3x^2 + 3y = 0 \rightarrow y = -x^2$ (SUB)

$f_y = -3y^2 + 3x = 0 \rightarrow x = y^2$

$\rightarrow x = (-x^2)^2 = x^4 \rightarrow x^4 - x = 0 \rightarrow$

$x(x^3 - 1) = 0 \rightarrow x = 0, y = 0$ or

$x = 1, y = -1$;

$z_{xx} = 6x, \; z_{yy} = -6y, \; z_{xy} = 3$

Test $(0,0)$: $D = z_{xx} \cdot z_{yy} - (z_{xy})^2 = (0)(0) - (3)^2 = -9 < 0$

so $(0,0)$ determines a saddle point with $z = 0$;

Test $(1,-1)$: $D = z_{xx} \cdot z_{yy} - (z_{xy})^2 = (6)(6) - (3)^2 = 27 > 0$

and $z_{xx} = 6 > 0$ so $(1,-1)$ determines a minimum value of $z = -1$.

6.) Use the Method of Lagrange Multipliers to minimize $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $x - 2y + 6z = 82$

$$F(x,y,z,\lambda) = x^2 + y^2 + z^2 - \lambda(x - 2y + 6z - 82)$$
$$= x^2 + y^2 + z^2 - \lambda x + 2\lambda y - 6\lambda z + 82\lambda \rightarrow$$

$$\begin{cases} F_x = 2x - \lambda = 0 \rightarrow \lambda = 2x \\ F_y = 2y + 2\lambda = 0 \rightarrow y = -\lambda \longrightarrow y = -2x \\ F_z = 2z - 6\lambda = 0 \rightarrow z = 3\lambda \rightarrow z = 3(2x) = 6x \\ F_\lambda = -x + 2y - 6z + 82 = 0 \end{cases}$$

then $-x + 2(-2x) - 6(6x) + 82 = 0 \rightarrow$

$-x - 4x - 36x = -82 \rightarrow$

$-41x = -82 \rightarrow$ $x = 2$,

$y = -4$, $z = 12$, $\lambda = 4$, and

minimum value

$f(2,-4,12) = 2^2 + (-4)^2 + (12)^2 = 164$

1.) Determine the nth term (starting with $n = 1$) of each of the following sequences.

a.) $2, 4, 6, 8, 10, \cdots$ $\boxed{2n}$

1 2 3 4 5 ... n

b.) $1, 5, 9, 13, 17, \cdots$ $\boxed{4n-3}$

1 2 3 4 5 ... n

c.) $\frac{2}{1}, \frac{-4}{4}, \frac{8}{9}, \frac{-16}{16}, \frac{32}{25}, \cdots$ $\boxed{\frac{(-1)^{n+1} 2^n}{n^2}}$

1 2 3 4 5 ... n

2.) Determine whether each series converges or diverges. Briefly explain and name the test that you are using.

a.) $\sum_{n=1}^{\infty} n^{-4} = \sum_{n=1}^{\infty} \frac{1}{n^4}$ converges by p-series since $p = 4 > 1$

b.) $\sum_{n=0}^{\infty} \frac{3^n}{(n+1)!}$ ratio test: converges since

$\lim_{n\to\infty} \frac{3^{n+1}}{(n+2)!} \cdot \frac{(n+1)!}{3^n} = \lim_{n\to\infty} \frac{3}{n+2} = 0 < 1$

c.) $\sum_{n=1}^{\infty} \cos\left(\pi + \frac{1}{n^2}\right)$ $\lim_{n\to\infty} \cos\left(\pi + \frac{1}{n^2}\right) = \cos \pi = -1 \neq 0$

so series diverges by nth term test

d.) $\sum_{n=2}^{\infty} \left(\frac{4}{3}\right)^n$ diverges by geometric series test since $r = \frac{4}{3}$ and $r \geq 1$.