1.) Show that the function $y = c_1 x + c_2 x^3$ solves the differential equation $x^2 y'' - 3xy' + 3y = 0$.

2.) Use any method to solve the following differential equations.

a.) $y' - y^2 x e^x = 0$

b.) $\frac{dy}{dx} + \frac{2y}{x} = 3x + 1$

3.) Consider the function $f(x, y) = 3 \sin \sqrt{y - e^x}$.

a.) Determine the domain of f. Sketch the domain in two-dimensional space.

b.) Determine the range of f. Briefly explain.

4.) Use Lagrange Multipliers to minimize the function $w = x^2 + y^2 + z^2$ subject to the constraint $x + 2y - z = 6$.

5.) Find and classify the critical points as determining relative maximum, relative minimum, or saddle points.

$$z = 2y^3 - 3x^2 - 3xy + 9x$$

6.) Find the interval of convergence and radius of convergence for the following power series :

$$\sum_{n=1}^{\infty} \frac{2^{n+1}(x-1)^n}{n^3}$$

7.) Find the 300th number in the following sequence :
$6, 9, 13, 18, 24, 31, \cdots$

8.) Build an open (no top) rectangular box with a volume of 32 ft.^{3} What dimensions will minimize the surface area of the box ? (Find the critical point, but you need not verify that it determines a minimum value.)

9.) What should n be in order that the Taylor Polynomial $P_n(x)$ of degree n centered at $c = 0$ estimate the value of $\frac{1}{1-x}$ with absolute error at most 0.0001 at the point $x = 1/3$.

10.) Use any method to find the first three (3) nonzero terms of the Taylor Series centered at the given value of c for each function.

a.) $f(x) = e^{-x} \cdot \cos\sqrt{x}$, $c = 0$

b.) $f(x) = \sqrt{x}$, $c = 4$

11.) Use the 6th-degree Taylor Polynomial $P_6(x)$ centered at $c = 0$ for $\dfrac{x^5}{1-x}$ to estimate the value of $\displaystyle\int_0^{1/2} \frac{x^5}{1-x}\,dx$

12.) Evaluate the following double intergrals.

a.) $\displaystyle\int_0^{\sqrt{\pi/2}} \int_0^{y^2} 2y\cos(x)\;dxdy$

b.) $\displaystyle\int_0^{6} \int_{y/2}^{3} e^{x^2}\,dxdy$

13.) Use Netwon's Method to estimate the solution of $x^4 - 3x = 8$. Start with $x_1 = 1$ and compute the next two Newton approximations, x_2 and x_3.

14.) A tank is holding 50 gallons of pure water. A solution containg 1/2 pound of salt per gallon begins flowing into the tank at the rate of 3 gallons per minute. At the same time the well-stirred mixture begins flowing out of the tank at the rate of 4 gallons per minute. How much salt is in the tank after $t = 10$ minutes ?

The following EXTRA CREDIT PROBLEM is worth   This problem is OPTIONAL.

1.) Determine the maximum amount of salt in the tank in problem 14.