1.) Show that the function \( y = c_1 x + c_2 x^3 \) solves the differential equation \( x^2 y'' - 3xy' + 3y = 0 \).

2.) Use any method to solve the following differential equations.

a.) \( y' - y^2 xe^x = 0 \)

b.) \( \frac{dy}{dx} + \frac{2y}{x} = 3x + 1 \)
3.) Consider the function \( f(x, y) = 3 \sin \sqrt{y - e^x} \).

a.) Determine the domain of \( f \). Sketch the domain in two-dimensional space.

b.) Determine the range of \( f \). Briefly explain.

4.) Use Lagrange Multipliers to minimize the function \( w = x^2 + y^2 + z^2 \) subject to the constraint \( x + 2y - z = 6 \).
5.) Find and classify the critical points as determining relative maximum, relative minimum, or saddle points.

\[ z = 2y^3 - 3x^2 - 3xy + 9x \]

6.) Find the interval of convergence and radius of convergence for the following power series:

\[ \sum_{n=1}^{\infty} \frac{2^{n+1}(x - 1)^n}{n^3} \]
7.) Find the 300th number in the following sequence:
   6, 9, 13, 18, 24, 31, \ldots

8.) Build an open (no top) rectangular box with a volume of 32 ft.\(^3\). What dimensions will minimize the surface area of the box? (Find the critical point, but you need not verify that it determines a minimum value.)

9.) What should \( n \) be in order that the Taylor Polynomial \( P_n(x) \) of degree \( n \) centered at \( c = 0 \) estimate the value of \( \frac{1}{1 - x} \) with absolute error at most 0.0001 at the point \( x = \frac{1}{3} \).
10.) Use any method to find the first three nonzero terms of the Taylor Series centered at the given value of \( c \) for each function.

a.) \( f(x) = e^{-x} \cdot \cos \sqrt{x}, \ c = 0 \)

b.) \( f(x) = \sqrt{x}, \ c = 4 \)
11.) Use the 6th-degree Taylor Polynomial $P_6(x)$ centered at $c = 0$ for $\frac{x^5}{1-x}$ to estimate the value of $\int_0^{1/2} \frac{x^5}{1-x} \, dx$.

12.) Evaluate the following double integrals.
   
   a.) $\int_0^{\pi/2} \int_0^{y^2} 2y \cos(x) \, dxdy$
   
   b.) $\int_0^6 \int_{y/2}^3 e^{x^2} \, dxdy$
13.) Use Newton's Method to estimate the solution of \( x^4 - 3x = 8 \). Start with \( x_1 = 1 \) and compute the next two Newton approximations, \( x_2 \) and \( x_3 \).

14.) A tank is holding 50 gallons of pure water. A solution containing 1/2 pound of salt per gallon begins flowing into the tank at the rate of 3 gallons per minute. At the same time, the well-stirred mixture begins flowing out of the tank at the rate of 4 gallons per minute. How much salt is in the tank after \( t = 10 \) minutes?
The following EXTRA CREDIT PROBLEM is worth

This problem is OPTIONAL.

1.) Determine the maximum amount of salt in the tank in problem 14.