Math 16C
Vogler

p- Series

Consider the following series:

1) $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{> \frac{2}{4} = \frac{1}{2}} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}}_{> \frac{4}{8} = \frac{1}{2}} + \ldots$

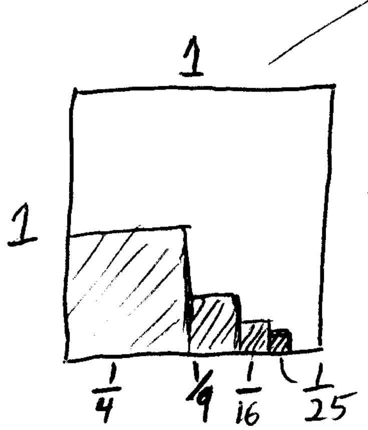
$\Rightarrow \underbrace{1 + \sum_{n=1}^{\infty} \frac{1}{2}}_{\infty \text{ or diverges}} < \underbrace{\sum_{n=1}^{\infty} \frac{1}{n}}_{} = \infty$ or diverges

∞ or diverges $\Rightarrow$

Hence, $\sum_{n=1}^{\infty} \frac{1}{n}$ (Called Harmonic Series) diverges

2) $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \underbrace{\frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \ldots}_{} < 2$

Hence, $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges



< 1

In general,

Defn Let p be constant. $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots$

is called the p-series

p-series test

- if $p > 1 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^p}$ converges
- if $p \leq 1 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges