Section 7.5

33.) Let x, y, z be positive #'s :
$x+y+z=30 \rightarrow z=30-x-y$;
maximize product
$P=xyz = xy(30-x-y) \rightarrow$
$\boxed{P=30xy-x^2y-xy^2}$; then
$P_x = 30y-2xy-y^2$
$= y(30-2x-y)=0 \rightarrow$
(No) $\boxed{\cancel{y=0}}$ or $30-2x-y=0 \rightarrow \boxed{y=30-2x}$;
and $P_y = 30x-x^2-2xy$
$= x(30-x-2y)=0 \rightarrow$
(No) $\boxed{\cancel{x=0}}$ or $\boxed{30-x-2y=0}$; then
(sub.) $30-x-2(30-2x)=0 \rightarrow$
$30-x-60+4x=0 \rightarrow 3x-30=0 \rightarrow$
$x=10$, $y=10$, $z=10$ and $P=1000$

34.) Let x, y, z be positive #'s :
$x+y+z=32 \rightarrow z=32-x-y$;
maximize $P=xy^2z = xy^2(32-x-y)$
$\rightarrow \boxed{P=32xy^2-x^2y^2-xy^3}$; then
$P_x = 32y^2-2xy^2-y^3$
$= y^2(32-2x-y)=0 \rightarrow \boxed{\cancel{y=0}}$ (No)
or $32-2x-y=0 \rightarrow \boxed{y=32-2x}$; and
$P_y = 64xy-2x^2y-3xy^2$
$= xy(64-2x-3y)=0 \rightarrow \boxed{\cancel{x=0}}$ (No),
or $\boxed{\cancel{y=0}}$ (No) or $\boxed{64-2x-3y=0}$; then

(sub.) $64-2X-3(32-2X)=0 \rightarrow$

$64-2X-96+6X=0 \rightarrow 4X-32=0 \rightarrow$

$X=8$, $Y=16$, and $Z=8$ and $P=16{,}384$.

35.) Let X, Y, Z be positive #'s :

$X+Y+Z=30 \rightarrow Z=30-X-Y$;

<u>minimize</u> $S=X^2+Y^2+Z^2 \rightarrow$

$\boxed{S=X^2+Y^2+(30-X-Y)^2}$; then

$$S_X = 2X+2(30-X-Y)(-1)$$
$$= 2X-60+2X+2Y$$
$$= 4X+2Y-60 = 2(2X+Y-30)=0 \rightarrow$$

$2X+Y-30=0 \rightarrow \boxed{Y=30-2X}$; and

$$S_Y = 2Y+2(30-X-Y)(-1)$$
$$= 2Y-60+2X+2Y$$
$$= 4Y+2X-60 = 2(2Y+X-30)=0 \rightarrow$$

$\boxed{2Y+X-30=0}$; then (sub.)

$2(30-2X)+X-30=0 \rightarrow 60-4X+X-30=0$

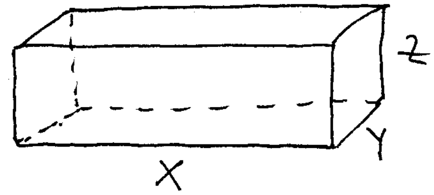
$\rightarrow 30=3X \rightarrow$ $X=10$, $Y=10$, $Z=10$.

37.) $R=-5x_1^2-8x_2^2-2x_1x_2+42x_1+102x_2 \rightarrow$

$$\left.\begin{aligned} R_{x_1} &= -10x_1-2x_2+42=0 \\ R_{x_2} &= -16x_2-2x_1+102=0 \end{aligned}\right\} \quad \left.\begin{aligned} x_2 &= 21-5x_1 \\ x_1 &= 51-8x_2 \end{aligned}\right\}$$

$x_2 = 21-5(51-8x_2) \rightarrow x_2 = 21-255+40x_2 \rightarrow$

$234 = 39x_2 \rightarrow$ $x_2=6$ and $x_1=3$ and $R=\$369$

43.) 

$X + 2Y + 2Z = 144 \rightarrow$
$\underline{X = 144 - 2Y - 2Z}$;

maximize volume

$V = XYZ = (144 - 2Y - 2Z)YZ \rightarrow$

$\boxed{V = 144YZ - 2Y^2Z - 2YZ^2}$; then

$V_y = 144Z - 4YZ - 2Z^2 = 2Z(72 - 2Y - Z) = 0$

$\rightarrow Z = 0$ (NO) or $72 - 2Y - Z = 0 \rightarrow \boxed{Z = 72 - 2Y}$;

$V_Z = 144Y - 2Y^2 - 4YZ = 2Y(72 - Y - 2Z) = 0$

$\rightarrow Y = 0$ (NO) or $\boxed{72 - Y - 2Z = 0}$; then (sub)

$72 - Y - 2(72 - 2Y) = 0 \longrightarrow$

$72 - Y - 144 + 4Y = 0 \rightarrow 3Y = 72 \rightarrow Y = 24 \text{ in.}$,

$Z = 24 \text{ in.}$, $X = 48 \text{ in.}$ and volume

$V = 27{,}648 \text{ in.}^3$.

46.) Total weight $T = T_{small} + T_{large}$

$= X(3 - 0.002X - 0.005Y)$

$\quad + Y(4.5 - 0.003X - 0.004Y) \rightarrow$

$T = 3X - 0.002X^2 - 0.005XY$

$\quad + 4.5Y - 0.003XY - 0.004Y^2 \rightarrow$

$\boxed{T = 3X + 4.5Y - 0.002X^2 - 0.004Y^2 - 0.008XY}$;

$T_x = 3 - 0.004x - 0.008y = 0 \rightarrow$

$3 - 0.004x = 0.008y \rightarrow \boxed{y = 375 - \tfrac{1}{2}x}$;

$T_y = 4.5 - 0.008y - 0.008x = 0 \rightarrow$

$\boxed{4.5 - 0.008y - 0.008x = 0}$; then (sub.)

$4.5 - 0.008(375 - \tfrac{1}{2}x) - 0.008x = 0 \rightarrow$

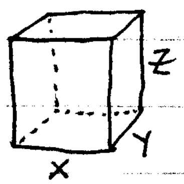
$4.5 - 3 - 0.004x - 0.008x = 0 \rightarrow$

$1.5 - 0.012x = 0 \rightarrow 1.5 = 0.012x \rightarrow$

$x = 125$ small , $y \approx 312$ large , and

$T \approx 1046.37$ lbs. (?)

Worksheet 5

1.) 

Volume 2

$8 = XYZ \rightarrow Z = \frac{8}{XY}$;

minimize surface area

$S = 2XY + 2YZ + 2XZ \rightarrow$

$$S = 2XY + 2Y\left(\frac{8}{XY}\right) + 2X\left(\frac{8}{XY}\right) = 2XY + \frac{16}{X} + \frac{16}{Y} \rightarrow$$

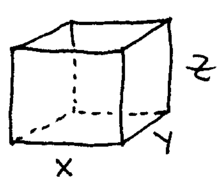
$S_X = 2Y - \frac{16}{X^2} = 0 \rightarrow Y = 8/X^2$ and

$S_Y = 2X - \frac{16}{Y^2} = 0 \rightarrow X = 8/Y^2 \rightarrow$

$$Y = \frac{8}{\left(\frac{8}{Y^2}\right)^2} = \frac{1}{8}Y^4 \rightarrow 8Y - Y^4 = Y(8 - Y^3) = 0 \rightarrow$$

$\cancel{Y=0}$ or $\boxed{Y=2}$, $\boxed{X=2}$, and $\boxed{Z=2}$ and

$S = 24\ ft^2$ is minimum surface area.

2.) 

Volume $\rightarrow 1 = XYZ \rightarrow Z = \frac{1}{XY}$;

minimize cost

$$C = \frac{3}{4}(XY) + 3(2XZ + 2YZ)$$
$$= \frac{3}{4}(XY) + 6XZ + 6YZ$$
$$= \frac{3}{4}XY + 6X\left(\frac{1}{XY}\right) + 6Y\left(\frac{1}{XY}\right)$$
$$= \frac{3}{4}XY + \frac{6}{Y} + \frac{6}{X} \quad \text{then}$$

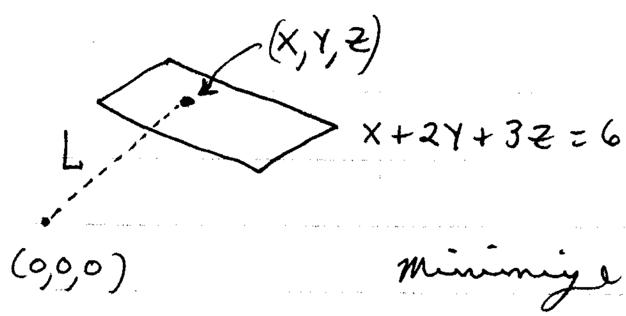
$$\left.\begin{array}{l} C_X = \frac{3}{4}Y - \frac{6}{X^2} = 0 \\ C_Y = \frac{3}{4}X - \frac{6}{Y^2} = 0 \end{array}\right\} \quad \begin{array}{l} Y = \frac{8}{X^2} \\ X = \frac{8}{Y^2} \end{array} \rightarrow X = \frac{8}{\left(\frac{8}{X^2}\right)^2} \rightarrow$$

$$X = \frac{1}{8}X^4 \rightarrow X^4 - 8X = 0 \rightarrow X(X^3 - 8) = 0 \rightarrow$$

$X \neq 0$ or $X = 2$ ft., $Y = 2$ ft., $Z = \frac{1}{4}$ ft. and

the minimum cost is $C = 9¢$.

3.)



minimize distance

$$L = \sqrt{(X-0)^2 + (Y-0)^2 + (Z-0)^2}$$
$$= \sqrt{(6-2Y-3Z)^2 + Y^2 + Z^2} \longrightarrow$$

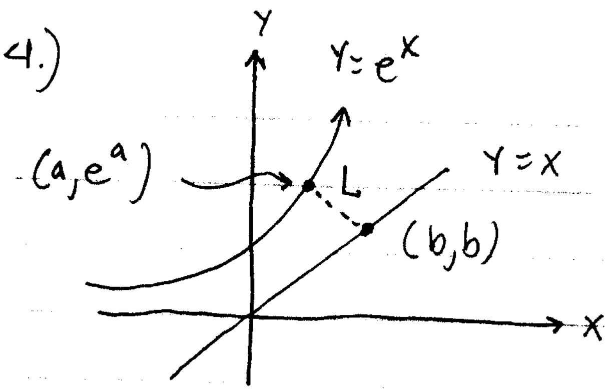
$$\left.\begin{array}{l} L_Y = \frac{1}{2}(\sim\sim)^{-1/2} \cdot [2(6-2Y-3Z)(-2) + 2Y] = 0 \\ L_Z = \frac{1}{2}(\sim\sim)^{-\frac{1}{2}} [2(6-2Y-3Z)(-3) + 2Z] = 0 \end{array}\right\} \rightarrow$$

$$\left.\begin{aligned}(6-2Y-3Z)(-2)+Y&=0\\(6-2Y-3Z)(-3)+Z&=0\end{aligned}\right\}\quad\left.\begin{aligned}-12+5Y+6Z&=0\\-18+6Y+10Z&=0\end{aligned}\right\}$$

$$\left.\begin{aligned}Y&=\tfrac{1}{5}(12-6Z)\\Y&=\tfrac{1}{6}(18-10Z)\end{aligned}\right\}\quad \tfrac{1}{5}(12-6Z)=\tfrac{1}{6}(18-10Z)\rightarrow$$

$72-36Z=90-50Z \rightarrow 14Z=18 \rightarrow Z=9/7$, $Y=\frac{6}{7}$, and $X=3/7$ determine a minimum distance of $\boxed{L=1.60}$.

4.)



Minimize distance L given by

$L=\sqrt{(a-b)^2+(e^a-b)^2} \rightarrow$

$L_a=\frac{1}{2}(\sim)^{-\frac{1}{2}}\left[2(a-b)+2(e^a-b)\cdot e^a\right]=0$ and

$L_b=\frac{1}{2}(\sim)^{-\frac{1}{2}}\cdot\left[2(a-b)(-1)+2(e^a-b)(-1)\right]=0 \rightarrow$

(*) $\left.\begin{aligned}a-b+(e^a-b)e^a&=0\\b-a+(e^a-b)(-1)&=0\end{aligned}\right\}$ (add equations) $\rightarrow$

$(e^a - b)(e^a - 1) = 0 \rightarrow \underline{\underline{\underline{b = e^a}}}$ or $\underline{\underline{\underline{e^a = 1}}}$;

if $\boxed{b = e^a}$ then equation

(*) $a - b + (e^a - b)e^a = 0$

becomes

$$a - e^a + (e^a - e^a)e^a = 0 \rightarrow$$

$$a - e^a = 0 \rightarrow a = e^a$$

(This is impossible! See graphs of $Y = X$ and $Y = e^X$.) ;

if $\boxed{e^a = 1}$ then $\boxed{a = 0}$ and equation

(*) $a - b + (e^a - b)e^a = 0$

becomes

$$0 - b + (1 - b)(1) = 0 \rightarrow$$

$1 - 2b = 0$ $\boxed{b = \frac{1}{2}}$; thus

$a = 0$ determines the point $(0, 1)$ on $Y = e^X$ and
$b = \frac{1}{2}$ determines the point $(\frac{1}{2}, \frac{1}{2})$ on $Y = X$

and the minimum distance is

$$\boxed{L = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{-1}{2}\right)^2} = \frac{1}{\sqrt{2}}}$$

.