Section 7.8

1.) $\int_0^x (2x-y)\,dy = (2xy - \frac{1}{2}y^2)\Big|_{y=0}^{y=x}$
$= (2x^2 - \frac{1}{2}x^2) - (0-0) = \frac{3}{2}x^2$

2.) $\int_x^{x^2} \frac{y}{x}\,dy = \frac{1}{x}\cdot\frac{1}{2}y^2\Big|_{y=x}^{y=x^2} = \frac{1}{x}\left(\frac{1}{2}x^4\right) - \frac{1}{x}\left(\frac{1}{2}x^2\right)$
$= \frac{1}{2}x^3 - \frac{1}{2}x$

3.) $\int_1^{2y} \frac{y}{x}\,dx = \int_1^{2y} y\cdot\frac{1}{x}\,dx = y\ln|x|\Big|_{x=1}^{x=2y}$
$= y\ln|2y| - y\cancel{\ln 1}^{\,0} = y\ln|2y|$

7.) $\int_{e^y}^{y} \frac{y\ln x}{x}\,dx = \int_{e^y}^{y} y\cdot\frac{\ln x}{x}\,dx$ (Let $u = \ln x$
$\rightarrow du = \frac{1}{x}\,dx$) $= \int_{e^y}^{y} y\cdot u\,du = y\cdot\frac{u^2}{2}\Big|_{x=e^y}^{x=y}$
$= y\,\frac{(\ln x)^2}{2}\Big|_{x=e^y}^{x=y} = y\cdot\frac{1}{2}(\ln y)^2 - y\cdot\frac{1}{2}(\ln e^y)^2$
$= \frac{y}{2}(\ln y)^2 - \frac{y}{2}(y)^2 = \frac{y}{2}(\ln y)^2 - \frac{y^3}{2}$

9.) $\int_0^{x^3} y e^{-y/x}\,dy = \int_0^{x^3} y\cdot e^{\left(\frac{-1}{x}\right)y}\,dy$ (Int. by parts:
Let $u = y$, $dv = e^{(-1/x)y}\,dy \rightarrow$
$du = dy$, $v = -xe^{\left(\frac{-1}{x}\right)y}$)
$= -xye^{-y/x} - \int_0^{x^3} -x\cdot e^{(-1/x)y}\,dy = -xye^{-y/x}\Big|_{y=0}^{y=x^3} + \int_0^{x^3} e^{\left(\frac{-1}{x}\right)y}\,dy$
$= \left(-xye^{-y/x} + -xe^{((-1/x)y)}\right)\Big|_{y=0}^{y=x^3}$
$= \left(-x^4e^{-x^2} - xe^{-x^2}\right) - \left(0 - x\cancel{e^0}^{\,1}\right) = -x^4e^{-x^2} - xe^{-x^2} + x$

10.) $\int_y^3 \frac{xy}{\sqrt{x^2+1}}\,dx = \int_y^3 y\cdot x(x^2+1)^{-1/2}\,dx$
(Let $u = x^2+1 \rightarrow du = 2x\,dx \rightarrow \frac{1}{2}du = x\,dx$)

$$= \int_y^3 y \cdot \tfrac{1}{2} u^{-1/2}\, du = y \cdot \cancel{\tfrac{1}{2}} \frac{u^{1/2}}{\cancel{1/2}} \Big|_{x=y}^{x=3}$$

$$y(x^2+1)^{1/2} \Big|_{x=y}^{x=3} = y \cdot \sqrt{10} - y(y^2+1)^{1/2}$$

11.) $$\int_0^2 \int_0^1 (x-y)\, dy\, dx = \int_0^2 \left(xy - \tfrac{1}{2}y^2\right)\Big|_{y=0}^{y=1} dx$$

$$= \int_0^2 \left(\left(x - \tfrac{1}{2}\right) - (0-0)\right) dx = \int_0^2 \left(x - \tfrac{1}{2}\right) dx$$

$$= \left(\tfrac{1}{2}x^2 - \tfrac{1}{2}x\right)\Big|_0^2 = (2-1) - (0-0) = 1$$

14.) $$\int_0^1 \int_0^x \sqrt{1-x^2}\, dy\, dx = \int_0^1 \left(y\sqrt{1-x^2}\Big|_{y=0}^{y=x}\right) dx$$

$$= \int_0^1 x(1-x^2)^{1/2}\, dx = \tfrac{2}{3}\left(\tfrac{-1}{2}\right)(1-x^2)^{3/2}\Big|_0^1$$

$$= \tfrac{-1}{3}(0)^{3/2} - \tfrac{-1}{3}(1)^{3/2} = \tfrac{1}{3}$$

15.) $$\int_0^1 \int_0^{\sqrt{1-y^2}} (x+y)\, dx\, dy = \int_0^1 \left(\tfrac{1}{2}x^2 + xy\right)\Big|_{x=0}^{x=\sqrt{1-y^2}} dy$$

$$= \int_0^1 \left[\tfrac{1}{2}(1-y^2) + y(1-y^2)^{1/2}\right] dy = \left[\tfrac{1}{2}\left(y - \tfrac{1}{3}y^3\right) - \tfrac{1}{3}(1-y^2)^{3/2}\right]\Big|_0^1$$

$$= \tfrac{1}{2}\left(\tfrac{2}{3}\right) - \tfrac{-1}{3}(1)^{3/2} = \tfrac{2}{3}$$

16.) $$\int_0^2 \int_{3y^2-6y}^{2y-y^2} 3y\, dx\, dy = \int_0^2 \left(3yx\Big|_{x=3y^2-6y}^{x=2y-y^2}\right) dy$$

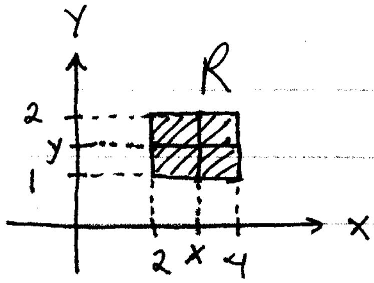
$$= \int_0^2 \left[3y(2y-y^2) - 3y(3y^2-6y)\right] dy = \int_0^2 (24y^2 - 12y^3)\, dy$$

$$= (8y^3 - 3y^4)\Big|_0^2 = 16$$

20.) $$\int_0^4 \int_0^x \frac{2}{(x+1)(y+1)}\, dy\, dx = \int_0^4 \left(\frac{2}{x+1} \ln(y+1)\Big|_{y=0}^{y=x}\right) dx$$

$$= \int_0^4 \frac{2}{x+1} \ln(x+1)\, dx = \left[\ln(x+1)\right]^2\Big|_0^4 = (\ln 5)^2$$

26.)

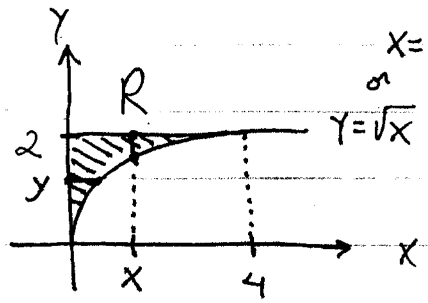



$$\int_1^2\int_2^4 1\,dx\,dy = \int_1^2 \left(x\Big|_2^4\right) dy$$

$$= \int_1^2 2\,dy = 2Y\Big|_1^2 = 2 \quad \underline{\text{AND}}$$

$$\int_2^4\int_1^2 1\,dy\,dx = \int_2^4 \left(Y\Big|_1^2\right) dx = \int_2^4 1\,dx = x\Big|_2^4 = 2.$$

30.)




$$\int_0^4\int_{\sqrt{x}}^2 1\,dy\,dx = \int_0^4 \left(Y\Big|_{\sqrt{x}}^2\right) dx$$

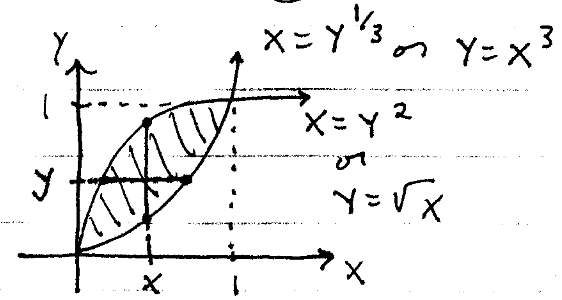
$$= \int_0^4 \left(2 - x^{1/2}\right) dx = \left(2x - \frac{2}{3}x^{3/2}\right)\Big|_0^4$$

$$= \left(8 - \frac{2}{3}4^{3/2}\right) - (0-0) = 8 - \frac{2}{3}\cdot 8 = \frac{8}{3} \quad \underline{\text{AND}}$$

$$\int_0^2\int_0^{Y^2} 1\,dx\,dy = \int_0^2 \left(x\Big|_{X=0}^{X=Y^2}\right) dy = \int_0^2 Y^2\,dy$$

$$= \frac{1}{3}Y^3\Big|_0^2 = \frac{8}{3}.$$

31.)




$$\int_0^1\int_{Y^2}^{Y^{1/3}} 1\,dx\,dy$$

$$= \int_0^1 \left(x\Big|_{X=Y^2}^{X=Y^{1/3}}\right) dy$$

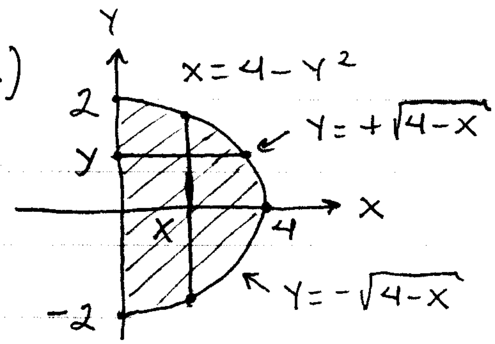
$$= \int_0^1 \left(Y^{1/3} - Y^2\right) dy$$

$$= \left(\frac{3}{4}Y^{4/3} - \frac{1}{3}Y^3\right)\Big|_0^1 = \frac{3}{4} - \frac{1}{3} = \frac{5}{12} \quad \underline{\text{AND}}$$

$$\int_0^1\int_{x^3}^{\sqrt{x}} 1\,dy\,dx = \int_0^1 \left(Y\Big|_{Y=x^3}^{Y=\sqrt{x}}\right) dx$$

$$= \int_0^1 \left(\sqrt{x} - x^3\right) dx = \left(\frac{2}{3}x^{3/2} - \frac{1}{4}x^4\right)\Big|_0^1 = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$

32.)




$$\int_{-2}^{2}\int_{0}^{4-y^2} 1 \, dx \, dy$$

$$= \int_{-2}^{2} x \Big|_{x=0}^{x=4-y^2} dy$$

$$= \int_{-2}^{2} (4-y^2) \, dy$$

$$= \left(4y - \frac{y^3}{3}\right)\Big|_{-2}^{2} = \left(8-\frac{8}{3}\right) - \left(-8+\frac{8}{3}\right) = 16 - \frac{16}{3} = \frac{32}{3}$$ AND

$$\int_{0}^{4}\int_{-\sqrt{4-x}}^{+\sqrt{4-x}} 1 \, dy \, dx = \int_{0}^{4} y \Big|_{y=-\sqrt{4-x}}^{y=+\sqrt{4-x}} dx$$

$$= \int_{0}^{4} \left(\sqrt{4-x} - -\sqrt{4-x}\right) dx = \int_{0}^{4} 2\sqrt{4-x} \, dx$$

$$= -2\left(\frac{2}{3}\right)(4-x)^{3/2}\Big|_{0}^{4} = \frac{-4}{3}\left(0^{3/2} - 4^{3/2}\right)$$

$$= \frac{-4}{3}(0-8) = \frac{32}{3}$$

35.) Area $= \int_0^8 \int_0^3 1 \, dy \, dx = \int_0^8 \left( y \Big|_0^3 \right) dx$

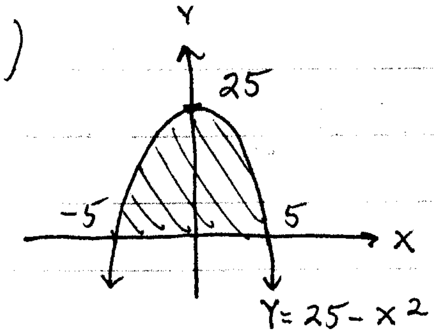
$$= \int_0^8 (3-0) \, dx = \int_0^8 3 \, dx = 3x \Big|_0^8 = 24 - 0 = 24$$

40.) Area $= \int_2^5 \int_0^{\frac{1}{\sqrt{x-1}}} 1 \, dy \, dx = \int_2^5 \left( y \Big|_{y=0}^{y=\frac{1}{\sqrt{x-1}}} \right) dx$

$$= \int_2^5 \left(\frac{1}{\sqrt{x-1}} - 0\right) dx = \int_2^5 (x-1)^{-1/2} \, dx = \frac{(x-1)^{1/2}}{1/2} \Big|_2^5$$

$$= 2\sqrt{4} - 2\sqrt{1} = 4 - 2 = 2$$

41.)



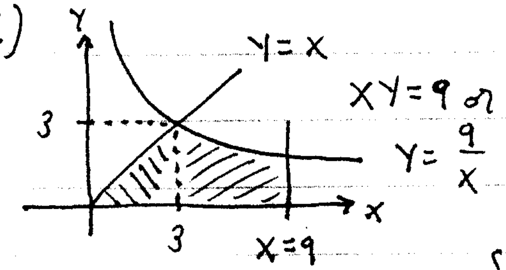

$$\text{Area} = \int_{-5}^{5}\int_{0}^{25-x^2} 1 \, dy \, dx$$

$$= \int_{-5}^{5} \left( Y \Big|_{Y=0}^{Y=25-x^2} \right) dx$$

$$= \int_{-5}^{5} (25 - x^2) \, dx$$

$$= \left(25x - \tfrac{1}{3}x^3\right)\Big|_{-5}^{5} = \left(125 - \tfrac{125}{3}\right) - \left(-125 + \tfrac{125}{3}\right) = \frac{500}{3}$$

44.)



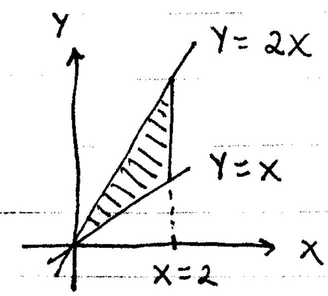

$$\text{Area} = \int_0^3\int_0^x 1 \, dy \, dx + \int_3^9\int_0^{\frac{9}{x}} 1 \, dy \, dx$$

$$= \int_0^3 \left( Y \Big|_0^x \right) dx + \int_3^9 \left( Y \Big|_0^{9/x} \right) dx$$

$$= \int_0^3 x \, dx + \int_3^9 \frac{9}{x} \, dx$$

$$= \tfrac{1}{2}x^2 \Big|_0^3 + 9 \ln x \Big|_3^9 = \frac{9}{2} + 9(\ln 9 - \ln 3) = \frac{9}{2} + 9 \ln 3$$

45.)




$$\text{Area} = \int_0^2 \int_x^{2x} 1 \, dy \, dx$$

$$= \int_0^2 \left( Y \Big|_{Y=x}^{Y=2x} \right) dx$$

$$= \int_0^2 (2x - x) \, dx$$

$$= \int_0^2 x \, dx = \tfrac{1}{2}x^2 \Big|_0^2 = 2$$