

Section 10.2 (cont'd.)

$$6.) \quad \lim_{n \rightarrow \infty} \frac{n}{2n+3} = \lim_{n \rightarrow \infty} \frac{1}{2 + \frac{3}{n}} = \frac{1}{2+0} = \frac{1}{2} \neq 0$$

so $\sum_{n=1}^{\infty} \frac{n}{2n+3}$ diverges.

$$7.) \quad \lim_{n \rightarrow \infty} \frac{n^2}{n^2+1} = \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{n^2}} = \frac{1}{1+0} = 1 \neq 0$$

so $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$ diverges.

$$8.) \quad \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n \rightarrow \infty} \sqrt{\frac{n^2}{n^2+1}} = \lim_{n \rightarrow \infty} \sqrt{\frac{1}{1 + \frac{1}{n^2}}} = \sqrt{1} = 1 \neq 0$$

so $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$ diverges.

$$9.) \quad \lim_{n \rightarrow \infty} 3 \left(\frac{3}{2}\right)^n = \infty \neq 0 \quad \text{(since } \frac{3}{2} > 1 \text{) so}$$

$\sum_{n=0}^{\infty} 3 \left(\frac{3}{2}\right)^n$ diverges

$$12.) \quad \lim_{n \rightarrow \infty} 2(1.03)^n = +\infty \text{ (since}$$

$1.03 > 1$) so $\lim_{n \rightarrow \infty} 2(-1.03)^n$ DNE

(oscillation) so $\lim_{n \rightarrow \infty} 2(-1.03)^n \neq 0$

and $\sum_{n=0}^{\infty} 2(-1.03)^n$ diverges.

$$31.) \quad \lim_{n \rightarrow \infty} \frac{n+10}{10n+1} = \lim_{n \rightarrow \infty} \frac{1 + \frac{10}{n}}{10 + \frac{1}{n}} = \frac{1+0}{10+0} = \frac{1}{10} \neq 0$$

so series $\sum_{n=1}^{\infty} \frac{n+10}{10n+1}$ diverges by nth-term test

$$32.) \quad \sum_{n=0}^{\infty} \frac{4}{2^n} = 4 \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 4 \left(1 + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \dots\right)$$

$$= 4 \cdot \frac{1}{1 - \frac{1}{2}} = 4 \cdot 2 = 8$$

converges by geom. series test

$$33.) \quad \lim_{n \rightarrow \infty} \frac{n+1}{n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) = 1+0 = 1 \neq 0$$

so series $\sum_{n=1}^{\infty} \frac{n+1}{n}$ diverges by nth-term test

$$34.) \quad \lim_{n \rightarrow \infty} \frac{n+1}{2n-1} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{2 - \frac{1}{n}} = \frac{1}{2} \neq 0$$

so series $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$ diverges by nth-term test

$$35.) \quad \lim_{n \rightarrow \infty} \frac{3n-1}{2n+1} = \lim_{n \rightarrow \infty} \frac{3 - \frac{1}{n}}{2 + \frac{1}{n}} = \frac{3}{2} \neq 0$$

so series $\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$ diverges by nth-term test

$$36.) \quad \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = 1 + \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + \dots$$

$$= \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

converges by geom. series test

37.) $\sum_{n=0}^{\infty} (1.075)^n$ diverges since $1.075 \geq 1$
by geom. series test

38.) $\sum_{n=1}^{\infty} \frac{2^n}{100} = \frac{1}{100} \sum_{n=1}^{\infty} 2^n$ diverges

since $2 \geq 1$ by geom. series test

39.) $\sum_{n=0}^{\infty} \frac{3}{4^n} = 3 \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$ converges

by geometric series test since $-1 < \frac{1}{4} < 1$

40.) $\sum_{n=0}^{\infty} n!$ diverges since

$\lim_{n \rightarrow \infty} n! \neq 0$ by n th-term test

45.)

a.)	<u>after # of years</u>	<u>units in use</u>
	1	$(0.9) 8000$
	2	$(0.9) 8000 + (0.9)^2 8000$
	3	$(0.9) 8000 + (0.9)^2 8000 + (0.9)^3 8000$
	$\vdots$	$\vdots$
	n	$(0.9) 8000 + (0.9)^2 8000 + (0.9)^3 8000 + \dots + (0.9)^n 8000$

↙

$$= (0.9) 8000 [1 + (0.9) + (0.9)^2 + (0.9)^3 + \dots + (0.9)^{n-1}]$$

$$= (0.9) 8000 \cdot \frac{1 - (0.9)^n}{1 - (0.9)} = 72,000 (1 - (0.9)^n) \text{ units}$$

b.) $\lim_{n \rightarrow \infty} 72,000 (1 - (0.9)^n) = \underline{72,000} \text{ units}$

$$\begin{aligned}
 49.) \quad A &= 100\left(1 + \frac{0.10}{12}\right) + 100\left(1 + \frac{0.10}{12}\right)^2 + \dots + 100\left(1 + \frac{0.10}{12}\right)^{60} \\
 &= 100\left(1 + \frac{0.10}{12}\right) \cdot \left[1 + \left(1 + \frac{0.10}{12}\right) + \left(1 + \frac{0.10}{12}\right)^2 + \dots + \left(1 + \frac{0.10}{12}\right)^{59}\right] \\
 &= 100\left(1 + \frac{0.10}{12}\right) \cdot \frac{1 - \left(1 + \frac{0.10}{12}\right)^{59+1}}{1 - \left(1 + \frac{0.10}{12}\right)} \approx \$7808.24
 \end{aligned}$$

$$\begin{aligned}
 50.) \quad A &= P\left(1 + \frac{r}{12}\right) + P\left(1 + \frac{r}{12}\right)^2 + \dots + P\left(1 + \frac{r}{12}\right)^N \\
 &= P\left(1 + \frac{r}{12}\right) \left[1 + \left(1 + \frac{r}{12}\right) + \left(1 + \frac{r}{12}\right)^2 + \dots + \left(1 + \frac{r}{12}\right)^{N-1}\right] \\
 &= P\left(1 + \frac{r}{12}\right) \cdot \frac{1 - \left(1 + \frac{r}{12}\right)^N}{1 - \left(1 + \frac{r}{12}\right)} = P\left(1 + \frac{r}{12}\right) \cdot \frac{-12}{r} \left[1 - \left(1 + \frac{r}{12}\right)^N\right] \\
 &= P\left(\frac{12}{r} + 1\right) \left[\left(1 + \frac{r}{12}\right)^N - 1\right].
 \end{aligned}$$

53.)

<u>day</u>	<u>\$ (in cents)</u>
1	1
2	$2 = 2^1$
3	$4 = 2^2$
4	$8 = 2^3$
$\vdots$	$\vdots$
n	2^{n-1}

so total \$ for 20 days

is $T = 1 + 2 + 2^2 + 2^3 + 2^4 + \dots + 2^{19}$

$$= \frac{1 - 2^{20}}{1 - 2} = 1,048,575 \text{¢} = \$10,485.75$$

Section 10.3

7.) $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges since $p=3 > 1$.

8.) $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ diverges since $p=\frac{1}{3} \leq 1$.

9.) $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ diverges since $p=\frac{1}{3} \leq 1$.

10.) $\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$ converges since $p=\frac{4}{3} > 1$.

11.) $\sum_{n=1}^{\infty} \frac{1}{n^{1.03}}$ converges since $p=1.03 > 1$.

12.) $\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$ converges since $p=\pi > 1$.

13.) $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \dots = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$
diverges since $p=\frac{1}{2} \leq 1$.

14.) $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges
since $p=2 > 1$.

15.) $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \dots$
 $= 1 + \frac{1}{2^{3/2}} + \frac{1}{3^{3/2}} + \frac{1}{4^{3/2}} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

converges since $p=\frac{3}{2} > 1$.

16.) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots = \sum_{n=1}^{\infty} \frac{1}{n} = \sum_{n=1}^{\infty} \frac{1}{n^1}$

diverges since $p=1 \leq 1$.