

Section 10.3 (cont'd.)

$$17.) \quad \lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n \rightarrow \infty} \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n}$$
$$= \lim_{n \rightarrow \infty} \frac{3}{n+1} = 0 < 1 \quad \text{so series } \text{converges}$$

$$18.) \quad \lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n \rightarrow \infty} \frac{(n+1) \left(\frac{2}{3}\right)^{n+1}}{n \left(\frac{2}{3}\right)^n}$$
$$= \lim_{n \rightarrow \infty} \frac{2}{3} \left(1 + \frac{1}{n}\right) = \frac{2}{3} < 1 \quad \text{so series } \text{converges}$$

$$19.) \quad \lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n \rightarrow \infty} \frac{(n+1)!}{3^{n+1}} \cdot \frac{3^n}{n!}$$
$$= \lim_{n \rightarrow \infty} \frac{n+1}{3} = \infty > 1 \quad \text{so series } \text{diverges}$$

$$20.) \quad \lim_{n \rightarrow \infty} \frac{(n+1) \left(\frac{3}{2}\right)^{n+1}}{n \left(\frac{3}{2}\right)^n} = \lim_{n \rightarrow \infty} \frac{3}{2} \left(1 + \frac{1}{n}\right) = \frac{3}{2} > 1$$

so series diverges

$$21.) \quad \lim_{n \rightarrow \infty} \frac{n+1}{4^{n+1}} \cdot \frac{4^n}{n} = \lim_{n \rightarrow \infty} \frac{1}{4} \left(1 + \frac{1}{n}\right) = \frac{1}{4} < 1$$

so series converges

$$22.) \quad \lim_{n \rightarrow \infty} \frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2} = \lim_{n \rightarrow \infty} \frac{1}{2} \left(\frac{n+1}{n}\right)^2 = \frac{1}{2} < 1$$

so series converges

$$23.) \quad \lim_{n \rightarrow \infty} \frac{2^{n+1}}{(n+1)^5} \cdot \frac{n^5}{2^n} = \lim_{n \rightarrow \infty} 2 \left(\frac{n}{n+1}\right)^5 = 2 \cdot 1 = 2 > 1$$

so series diverges

$$24.) \lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n \rightarrow \infty} \frac{e^{-(n+1)}}{e^{-n}} = \lim_{n \rightarrow \infty} e^{-1} = \frac{1}{e} < 1$$

so series converges

$$25.) \lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n \rightarrow \infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n}$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n+1} = 0 < 1 \text{ so series } \text{converges}$$

$$26.) \lim_{n \rightarrow \infty} \frac{4^{n+1}}{(n+1)!} \cdot \frac{n!}{4^n} = \lim_{n \rightarrow \infty} \frac{4}{n+1} = 0 < 1$$

so series converges

$$28.) \lim_{n \rightarrow \infty} \frac{3^{n+1}}{(n+1)+1} \cdot \frac{n+1}{3^n} = \lim_{n \rightarrow \infty} 3 \cdot \frac{n+1}{n+2}$$

$$= \lim_{n \rightarrow \infty} 3 \cdot \frac{1 + \frac{1}{n}}{1 + \frac{2}{n}} = 3(1) = 3 > 1 \text{ so}$$

series diverges

$$29.) \lim_{n \rightarrow \infty} \frac{(n+1)5^{n+1}}{(n+1)!} \cdot \frac{n!}{n5^n}$$

$$= \lim_{n \rightarrow \infty} 5 \cdot \frac{n+1}{n} \cdot \frac{1}{n+1} = \lim_{n \rightarrow \infty} \frac{5}{n} = 0 < 1$$

so series converges

$$30.) \lim_{n \rightarrow \infty} \frac{2 \cdot (n+1)!}{(n+1)^5} \cdot \frac{n^5}{2 \cdot n!} = \lim_{n \rightarrow \infty} (n+1) \left(\frac{n}{n+1} \right)^5$$

$$= \lim_{n \rightarrow \infty} (n+1) \cdot \left(\frac{1}{1 + \frac{1}{n}} \right)^5 = \infty \cdot (1)^5 = \infty > 0$$

so series diverges.

$$41.) \lim_{n \rightarrow \infty} \frac{2n}{n+1} = 2 \neq 0 \text{ so series}$$

diverges by nth term test.

$$42.) \sum_{n=1}^{\infty} \frac{10}{3 n^{2/3}} = \frac{10}{3} \sum_{n=1}^{\infty} \frac{1}{n^{2/3}} \text{ diverges}$$

by p-series test with $p = \frac{2}{3} \leq 1$.

$$44.) \sum_{n=0}^{\infty} \left(\frac{5}{6} \right)^n = 1 + \left(\frac{5}{6} \right) + \left(\frac{5}{6} \right)^2 + \dots$$

$$= \frac{1}{1 - (5/6)} = \frac{1}{1/6} = 6 \text{ so series}$$

converges by geom. series test
since $-1 < 5/6 < 1$.

$$46.) \lim_{n \rightarrow \infty} (\ln n) = \infty \neq 0 \text{ so series}$$

diverges by nth-term test

$$47.) \sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{1}{n^3} \right) = \left(\sum_{n=1}^{\infty} \frac{1}{n^2} \right) - \left(\sum_{n=1}^{\infty} \frac{1}{n^3} \right)$$

converges since each series

converges by p-series test with $p = 2 > 1$ and $p = 3 > 1$.

$$50.) \lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n \rightarrow \infty} \frac{(n+1)(0.4)^{n+1}}{n(0.4)^n}$$

$$= \lim_{n \rightarrow \infty} (0.4) \left(1 + \frac{1}{n}\right) = 0.4 < 1 \quad \text{so series}$$

converges by ratio test.

$$51.) \lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n \rightarrow \infty} \frac{(n+1)!}{3^{(n+1)-1}} \cdot \frac{3^{n-1}}{n!}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{3} \cdot \frac{n+1}{1} = \infty > 1 \quad \text{so series}$$

diverges by ratio test.

$$52.) \sum_{n=1}^{\infty} \frac{1}{n^{0.95}} \quad \text{diverges by}$$

p-series test with $p = 0.95 \leq 1$.