Section 10.5

1.) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ so

$P_1(x) = 1 + x$, $P_2(x) = 1 + x + \frac{x^2}{2}$,
$P_3(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$, $P_4(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

2.) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ so

$P_1(x) = x$, $P_2(x) = x - \frac{x^2}{2}$,
$P_3(x) = x - \frac{x^2}{2} + \frac{x^3}{3}$, $P_4(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$

3.) $\sqrt{x+1} = 1 + \frac{1}{2}x + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)}{2!}x^2 + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{3!}x^3 + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right)}{4!}x^4$

$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \cdots$ so

$P_1(x) = 1 + \frac{1}{2}x$, $P_2(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2$,
$P_3(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$,
$P_4(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4$

4.) $\frac{1}{(x+1)^2} = -D\left(\frac{1}{x+1}\right) = -D\left(\frac{1}{1-(-x)}\right)$

$= -D\left(1 - x + x^2 - x^3 + x^4 - x^5 + \cdots\right)$
$= -\left(-1 + 2x - 3x^2 + 4x^3 - 5x^4 + \cdots\right)$
$= 1 - 2x + 3x^2 - 4x^3 + 5x^4 - \cdots$ so

$P_1(x) = 1 - 2x$, $P_2(x) = 1 - 2x + 3x^2$,
$P_3(x) = 1 - 2x + 3x^2 - 4x^3$,
$P_4(x) = 1 - 2x + 3x^2 - 4x^3 + 5x^4$

5.) $\frac{x}{x+1} = x \cdot \frac{1}{x+1} = x(1 - x + x^2 - x^3 + \cdots)$

$= x - x^2 + x^3 - x^4 + \cdots$ so

$P_1(x) = x$, $P_2(x) = x - x^2$,

$P_3(x) = x - x^2 + x^3$, $P_4(x) = x - x^2 + x^3 - x^4$

9.) $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 - \cdots$ so

$P_2(x) = 1 - x^2$, $P_4(x) = 1 - x^2 + x^4$,

$P_6(x) = 1 - x^2 + x^4 - x^6$,

$P_8(x) = 1 - x^2 + x^4 - x^6 + x^8$

10.) $e^{-x^2} = 1 + (-x^2) + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} + \frac{(-x^2)^4}{4!} + \cdots$

$= 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \frac{x^8}{24} - \cdots$ so

$P_2(x) = 1 - x^2$, $P_4(x) = 1 - x^2 + \frac{x^4}{2}$,

$P_6(x) = 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6}$,

$P_8(x) = 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \frac{x^8}{24}$

17.) $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \frac{x^6}{6!} - \cdots$ so

$P_6(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{120} + \frac{x^6}{720}$; then

$e^{-1/2} \approx P_6(1/2) = 1 - (\frac{1}{2}) + \frac{(1/2)^2}{2} - \frac{(1/2)^3}{6} + \frac{(\frac{1}{2})^4}{24} - \frac{(\frac{1}{2})^5}{120} + \frac{(\frac{1}{2})^6}{720}$

≈ 0.606532118056 ;

calculator : $e^{-1/2} \approx 0.606530659713$

18.) $x^2 e^{-x} = x^2\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots\right)$

$= x^2 - x^3 + \frac{x^4}{2!} - \frac{x^5}{3!} + \frac{x^6}{4!} - \cdots$ so

$P_6(x) = x^2 - x^3 + \frac{x^4}{2} - \frac{x^5}{6} + \frac{x^6}{24}$; then

$\left(\frac{1}{4}\right)^2 e^{-\frac{1}{4}} \approx P_6\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^3 + \frac{1}{2}\left(\frac{1}{4}\right)^4 - \frac{1}{6}\left(\frac{1}{4}\right)^5 + \frac{1}{24}\left(\frac{1}{4}\right)^6$

≈ 0.0486755371 ;

calculator : $\frac{1}{16} e^{-1/4} \approx 0.0486750489$