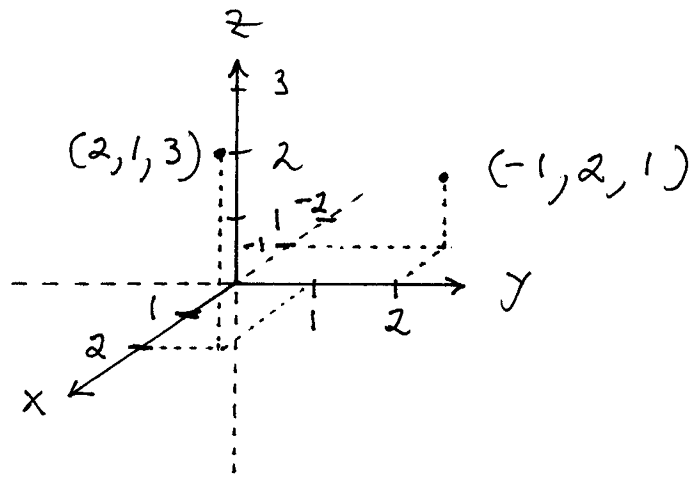
Section 7.1

1.)



6.) Distance $L = \sqrt{(-4-2)^2 + (-1-{}^-1)^2 + (1-5)^2}$
$= \sqrt{36+0+16} = \sqrt{52}$.

8.) Distance $L = \sqrt{(8-8)^2 + (-2-{}^-2)^2 + (2-4)^2}$
$= \sqrt{0+0+4} = 2$

9.) midpoint $= \left(\frac{6-2}{2}, \frac{{}^-9-1}{2}, \frac{1+5}{2}\right) = (2, {}^-5, 3)$

12.) midpoint $= \left(\frac{0+4}{2}, \frac{{}^-2+2}{2}, \frac{5+7}{2}\right) = (2, 0, 6)$

13.) $\left(\frac{x-2}{2}, \frac{y+1}{2}, \frac{z+1}{2}\right) = (2, {}^-1, 3) \rightarrow$

$\frac{x-2}{2} = 2 \rightarrow x-2 = 4 \rightarrow x = 6$;

$\frac{y+1}{2} = -1 \rightarrow y+1 = -2 \rightarrow y = {}^-3$;

$\frac{z+1}{2} = 3 \rightarrow z+1 = 6 \rightarrow z = 5$.

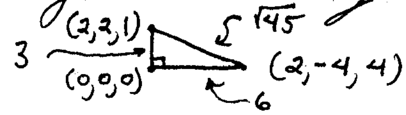
15.) $\left(\frac{x+2}{2}, \frac{y+0}{2}, \frac{z+3}{2}\right) = \left(\frac{3}{2}, 1, 2\right) \rightarrow$

$\frac{x+2}{2} = \frac{3}{2} \rightarrow x+2 = 3 \rightarrow x=1$;

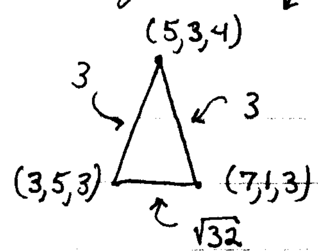
$\frac{y}{2} = 1 \rightarrow y=2$; $\frac{z+3}{2} = 2 \rightarrow z+3=4 \rightarrow$

$z=1$.

17.) $d_1 = \sqrt{2^2+2^2+1^2} = 3$, $d_2 = \sqrt{0^2+6^2+3^2} = \sqrt{45}$,
$d_3 = \sqrt{2^2+4^2+4^2} = \sqrt{36} = 6$ so right triangle
since $3^2 + 6^2 = (\sqrt{45})^2$:



18.) $d_1 = \sqrt{2^2+2^2+1^2} = 3$, $d_2 = \sqrt{4^2+4^2+0^2} = \sqrt{32}$,
$d_3 = \sqrt{2^2+2^2+1} = 3$ so isosceles triangle:



21.) $(x-0)^2+(y-2)^2+(z-2)^2 = 2^2$

$\rightarrow x^2 + (y-2)^2 + (z-2)^2 = 4$

24.) Distance $= \sqrt{(0-{}^-1)^2 + (3-{}^-2)^2 + (3-1)^2}$
$= \sqrt{1+25+4} = \sqrt{30}$ so radius is
$\frac{\sqrt{30}}{2}$; midpoint $= \left(\frac{0-1}{2}, \frac{3-2}{2}, \frac{3+1}{2}\right) = \left(\frac{-1}{2}, \frac{1}{2}, 2\right)$
is center so sphere is

$\left(x-{}^-\frac{1}{2}\right)^2 + \left(y-\frac{1}{2}\right)^2 + (z-2)^2 = \left(\frac{\sqrt{30}}{2}\right)^2 \rightarrow$

$\left(x+\frac{1}{2}\right)^2 + \left(y-\frac{1}{2}\right)^2 + (z-2)^2 = \frac{30}{4} \rightarrow$

$\left(x+\frac{1}{2}\right)^2 + \left(y-\frac{1}{2}\right)^2 + (z-2)^2 = \frac{15}{2}$.

26.) $(x-4)^2+(y--1)^2+(z-1)^2=5^2 \rightarrow$

$(x-4)^2+(y+1)^2+(z-1)^2=25$

27.) Distance $=\sqrt{(2-0)^2+(0-6)^2+(0-0)^2}$

$=\sqrt{4+36}=\sqrt{40}$ so radius is

$\frac{\sqrt{40}}{2}$; midpoint $=\left(\frac{2+0}{2},\frac{0+6}{2},\frac{0+0}{2}\right)=(1,3,0)$

is center so sphere is

$(x-1)^2+(y-3)^2+(z-0)^2=\left(\frac{\sqrt{40}}{2}\right)^2 \rightarrow$

$(x-1)^2+(y-3)^2+z^2=\frac{40}{4}=10$.

29.) radius $=1$ (since $z=1$) so sphere is

$(x--2)^2+(y-1)^2+(z-1)^2=1^2 \rightarrow$

$(x+2)^2+(y-1)^2+(z-1)^2=1$.

32.) $x^2+y^2+z^2-8y=0 \rightarrow$

$x^2+(y^2-8y+16)+z^2=16 \rightarrow$

$(x-0)^2+(y-4)^2+(z-2)^2=4^2 \rightarrow$

center is $(0,4,2)$ and radius $=4$.

34.) $x^2+y^2+z^2-4y+6z+4=0 \rightarrow$

$x^2+(y^2-4y+4)+(z^2+6z+9)=-4+4+9 \rightarrow$

$x^2+(y-2)^2+(z+3)^2=3^2 \rightarrow$

$(x-0)^2+(y-2)^2+(z--3)^2=3^2$ so

center is $(0,2,-3)$ and radius $=3$.

36.) $4x^2+4y^2+4z^2-8x+16y+11=0 \rightarrow$

$4(x^2-2x+1)+4(y^2+4y+4)+4z^2=-11+4+16 \rightarrow$

$4(x-1)^2+4(y+2)^2+4z^2=9 \rightarrow$

$(x-1)^2+(y-{}^-2)^2+(z-0)^2=\frac{9}{4}=\left(\frac{3}{2}\right)^2 \rightarrow$

center is $(1,{}^-2,0)$ and radius $= 3/2$.

39.) $x^2+y^2+z^2-6x-10y+6z+30=0$

so xy-trace means $z=0 \rightarrow$

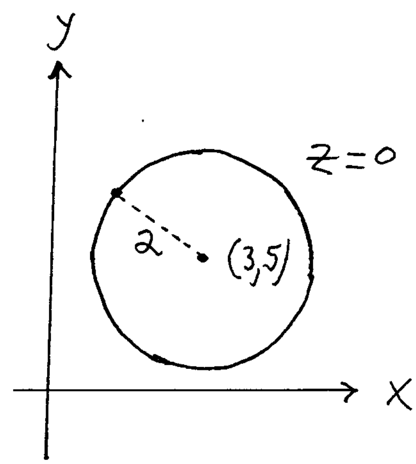
$x^2+y^2-6x-10y+30=0 \rightarrow$

$(x^2-6x+9)+(y^2-10y+25)={}^-30+9+25 \rightarrow$

$\underline{(x-3)^2+(y-5)^2=64=2^2} \rightarrow$

circle, center $(3,5)$ and

radius $= 2$.



41.) $x^2+y^2+z^2-4x-4y-6z-12=0$

so YZ-trace means $x=0 \rightarrow$

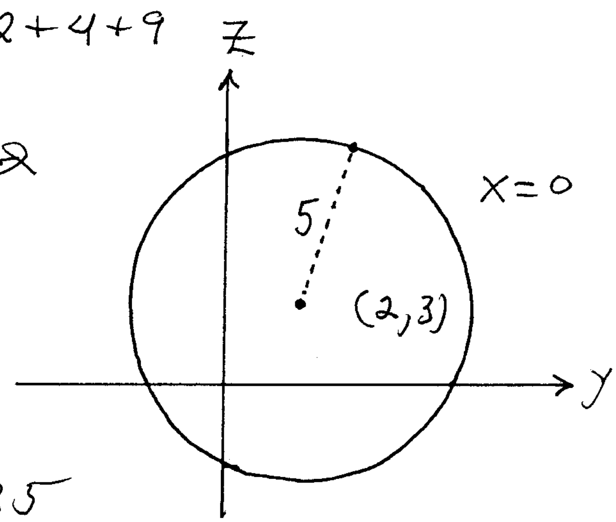
$y^2+z^2-4y-6z-12=0 \rightarrow$

$(y^2-4y+4)+(z^2-6z+9)=12+4+9$

$\rightarrow \underline{(y-2)^2+(z-3)^2=5^2} \rightarrow$

circle, center $(2,3)$ and

radius $= 5$.



43.) $x^2+y^2+z^2=25$

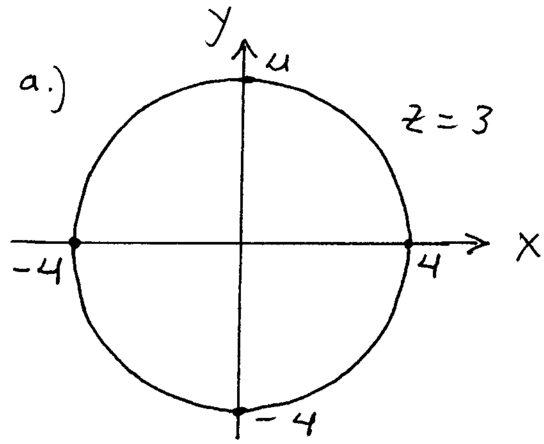
a.) $z=3 \rightarrow x^2+y^2+9=25$

$\rightarrow x^2+y^2=16=4^2 \rightarrow$

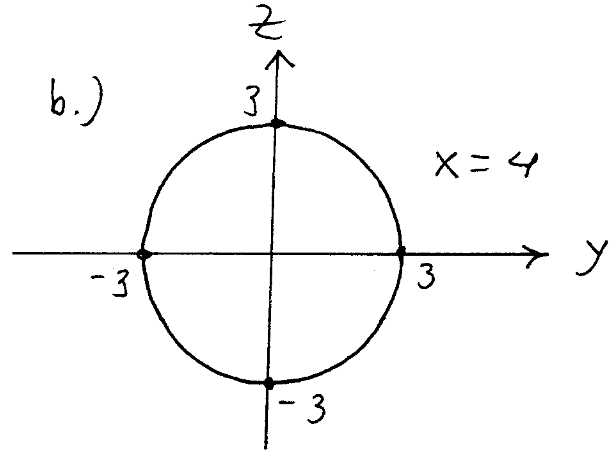
circle, center $(0,0)$ and radius $= 4$.

b.) $x=4 \rightarrow 16+y^2+z^2=25 \rightarrow$
$y^2+z^2=9=3^2 \rightarrow$ circle, center
$(0,0)$ and radius $=3$.

a.)



b.)

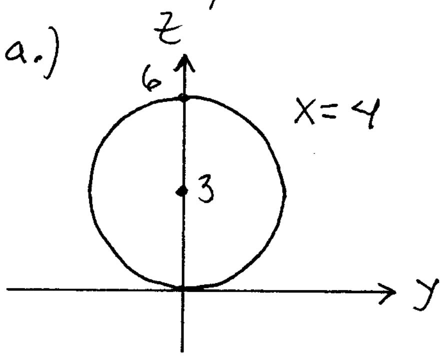


46.) $x^2+y^2+z^2-8x-6z+16=0$

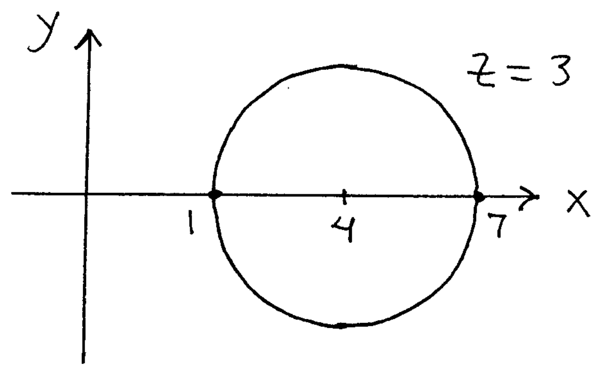
a.) $x=4 \rightarrow 16+y^2+z^2-32-6z+16=0 \rightarrow$
$y^2+(z^2-6z+9)=0+9 \rightarrow y^2+(z-3)^2=3^2 \rightarrow$
circle, center $(0,3)$ and radius $=3$.

b.) $z=3 \rightarrow x^2+y^2+9-8x-18+16=0 \rightarrow$
$(x^2-8x+16)+y^2=-7+16 \rightarrow (x-4)^2+y^2=3^2 \rightarrow$
circle, center $(4,0)$ and radius $=3$.

a.)



b.)



47.) $(x,y,z)=(3,3,3)$

Worksheet 2

1.) $Y' + Y^3 = Y \rightarrow \quad Y' = Y - Y^3 = Y(1-Y)(1+Y) \rightarrow$

$\int \frac{1}{Y(1-Y)(1+Y)}\,dy = \int 1\,dx \quad ;$

$$\frac{1}{Y(1-Y)(1+Y)} = \frac{A}{Y} + \frac{B}{1-Y} + \frac{C}{1+Y} \rightarrow$$

$$A(1-Y)(1+Y) + BY(1+Y) + CY(1-Y) = 1$$

$Y=0:\ A=1,\ Y=1:\ 2B=1 \rightarrow B=\frac{1}{2},\ Y=-1:\ -2C=1 \rightarrow C=\frac{-1}{2}$

then

$$\int \left[\frac{1}{Y} + \frac{\frac{1}{2}}{1-Y} + \frac{\frac{-1}{2}}{1+Y}\right] dy = x + c \rightarrow$$

$\ln|Y| - \frac{1}{2}\ln|1-Y| - \frac{1}{2}\ln|1+Y| = x + c$ then $x=0,\ Y=2 \rightarrow$

$\ln 2 - \frac{1}{2}\cancel{\ln 1}^{\,0} - \frac{1}{2}\ln 3 = c \rightarrow \quad c = \ln 2 - \frac{1}{2}\ln 3$ so

$$\boxed{\ln|Y| - \frac{1}{2}\ln|1-Y| - \frac{1}{2}\ln|1+Y| = x + \left(\ln 2 - \frac{1}{2}\ln 3\right)}$$

2.) $Y' + 2Y = e^{-2x}\cot^2(7x)$, let $\mu = e^{\int 2\,dx} = e^{2x} \rightarrow$

$e^{2x}Y' + 2e^{2x}Y = e^{2x}e^{-2x}\cot^2(7x) = e^0 \cdot \cot^2(7x) = \cot^2(7x) \rightarrow$

$D(e^{2x}Y) = \cot^2(7x) \rightarrow \quad e^{2x}Y = \int \cot^2(7x)\,dx \rightarrow$

$e^{2x}Y = \int [\csc^2(7x) - 1]\,dx \rightarrow$

$$\boxed{e^{2x}Y = \frac{-1}{7}\cot(7x) - x + c}$$

3.) $Y'\cos^2 x + Y = 1 \rightarrow \quad Y'\cos^2 x = 1 - Y \rightarrow$

$\int \frac{1}{1-Y}\,dy = \int \frac{1}{\cos^2 x}\,dx = \int \sec^2 x\,dx \rightarrow$

$$\boxed{-\ln|1-Y| = \tan x + c}.$$

4.) $xY' + 2Y = x\cos x \rightarrow \underbrace{Y' + \left(\frac{2}{x}\right)Y = \cos x}_{} \rightarrow$

$\mu = e^{\int \frac{2}{x}dx} = e^{2\ln x} = e^{\ln x^2} = x^2 \rightarrow$

$x^2Y' + 2xY = x^2\cos x \rightarrow D(x^2Y) = x^2\cos x \rightarrow$

$x^2Y = \int x^2\cos x\, dx$ (Let $u = x^2$, $dv = \cos x\, dx$
$du = 2x\, dx$, $v = \sin x$)

$x^2Y = x^2\sin x - 2\int x\sin x\, dx$ (Let $u = x$, $dv = \sin x\, dx$
$du = dx$, $v = -\cos x$)

$x^2Y = x^2\sin x - 2\left[-x\cos x + \int \cos x\, dx\right]$

$$\boxed{x^2Y = x^2\sin x + 2x\cos x - 2\sin x + C}.$$

5.) $\tan x \cdot Y' = Y^2(Y+1)\cot x \rightarrow$

$\int \frac{1}{Y^2(Y+1)}dy = \int \frac{\cot x}{\tan x}dx = \int \frac{\cos x}{\sin x}\cdot\frac{\cos x}{\sin x}dx = \int \frac{\cos^2 x}{\sin^2 x}dx;$

$\frac{1}{Y^2(Y+1)} = \frac{A}{Y} + \frac{B}{Y^2} + \frac{C}{Y+1} \rightarrow AY(Y+1) + B(Y+1) + C(Y^2) = 1 \rightarrow$

$Y=0: \; B=1$, $Y=-1: \; C=1$, $Y=1: \; 2A+2+1=1 \rightarrow A=-1$ so

$\int \left[\frac{-1}{Y} + \frac{1}{Y^2} + \frac{1}{Y+1}\right]dy = \int \frac{1-\sin^2 x}{\sin^2 x}dx = \int \left[\csc^2 x - 1\right]dx \rightarrow$

$$\boxed{-\ln|Y| - \frac{1}{Y} + \ln|Y+1| = -\cot x - x + C}.$$

6.) $\cos(5x^2)\cdot Y' = x\sec^2(3Y) \rightarrow$

$\int \frac{1}{\sec^2(3Y)}dy = \int \frac{x}{\cos(5x^2)}dx \longrightarrow$

$$\int \cos^2(3y)\,dy = \int x \sec(5x^2)\,dx \rightarrow$$

Let $u = 5x^2 \rightarrow \ldots$

$$\int \frac{1}{2}(1+\cos(6y))\,dy = \int x \sec(5x^2)\,dx \rightarrow$$

$$\boxed{\frac{1}{2}\left(y + \frac{1}{6}\sin(6y)\right) = \frac{1}{10}\cdot \ln|\sec(5x^2)+\tan(5x^2)| + c}$$

7.) $(e^{2x} - e^x)\,e^{2y}\sin(e^y)\,y' = (1+e^x)\cdot e^x \rightarrow$

$$\int e^y e^y \sin(e^y)\,dy = \int \frac{(1+e^x)e^x}{e^x e^x - e^x}\,dx \rightarrow$$

$$\int e^y \sin(e^y)\cdot e^y\,dy = \int \frac{(1+e^x)\cancel{e^x}}{(e^x-1)\cancel{e^x}}\,dx = \int \frac{1+e^x}{e^x-1}\,dx \rightarrow$$

(Let $u = e^y \rightarrow du = e^y\,dy$)

$$\int u \sin u\,du = \int \frac{(e^x-1)+1+1}{e^x-1}\,dx = \int \left[1 + \frac{2}{e^x-1}\right]dx \rightarrow$$

(Let $w = u$, $dv = \sin u\,du$

$dw = du$, $v = -\cos u$)

$$-u\cos u + \int \cos u\,du = x + \int \frac{2e^{-x}}{(e^x-1)\,e^{-x}}\,dx \rightarrow$$

$$-u\cos u + \sin u = x + 2\int \frac{e^{-x}}{1-e^{-x}}\,dx \rightarrow$$

$$\boxed{-e^y\cos(e^y) + \sin(e^y) = x + 2\cdot \ln|1-e^{-x}| + c}$$

8.) $\int \cos^3 y \sin y\,dy = \int \tan^3(10x)\,dx \rightarrow$

Let $u = \cos y \rightarrow \ldots$

$$-\frac{1}{4}\cos^4 y = \int \tan(10x)\cdot \tan^2(10x)\,dx \rightarrow$$

$$-\frac{1}{4}\cos^4 y = \int \tan(10x)\left[\sec^2(10x) - 1\right]dx \rightarrow$$

$$-\frac{1}{4}\cos^4 y = \int \left(\sec(10x)\cdot\sec(10x)\tan(10x) - \tan(10x)\right) dx \rightarrow$$

Let $u = \sec(10x)$

$$\boxed{-\frac{1}{4}\cos^4 y = \frac{1}{10}\cdot\frac{1}{2}\sec^2(10x) - \frac{1}{10}\ln|\sec(10x)| + c}$$