Section 7.2

2.) $3X+6Y+2Z=6$;

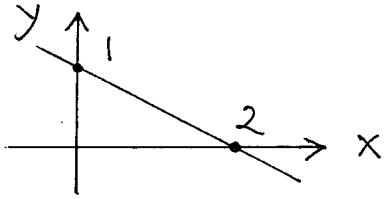
$X=0, Y=0 \rightarrow Z=3$

$X=0, Z=0 \rightarrow Y=1$

$Y=0, Z=0 \rightarrow X=2$

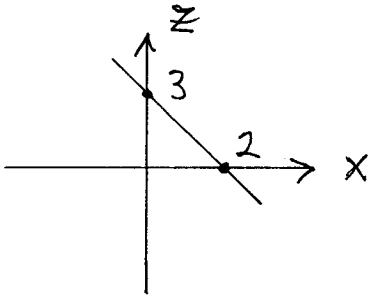
<u>XY-trace</u> ($Z=0$):

$3X+6Y=6 \rightarrow X+2Y=2$ (line)



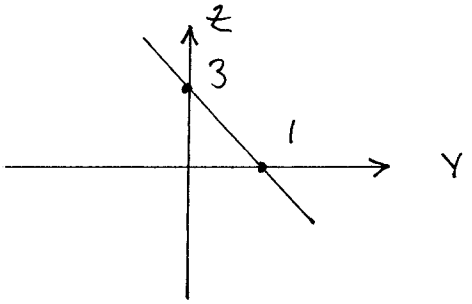
<u>XZ-trace</u> ($Y=0$)

$3X+2Z=6$ (line)



<u>YZ-trace</u> ($X=0$):

$6Y+2Z=6 \rightarrow 3Y+Z=3$ (line)



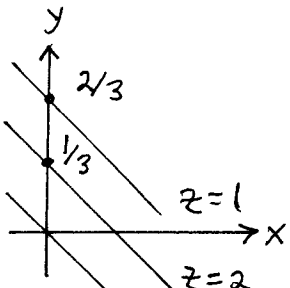
<u>Level Curves</u> :

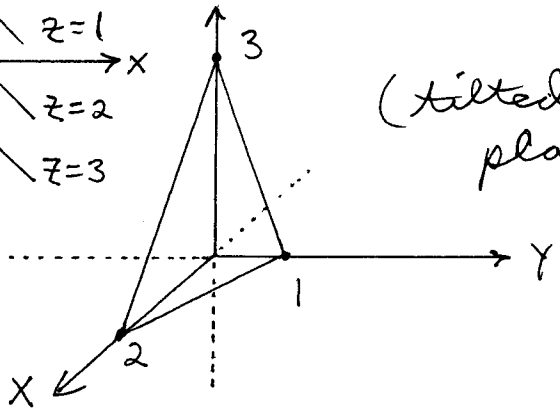
$Z=1$: $3X+6Y=4$

$Z=2$: $3X+6Y=2$

$Z=3$: $3X+6Y=0$

(all are lines.)





(tilted plane)

4.) $X+Y+Z=3$;

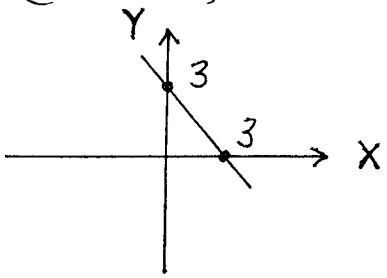
$X=0, Y=0 \rightarrow Z=3$

$X=0, Z=0 \rightarrow Y=3$

$Y=0, Z=0 \rightarrow X=3$

<u>XY-trace</u> ($Z=0$):

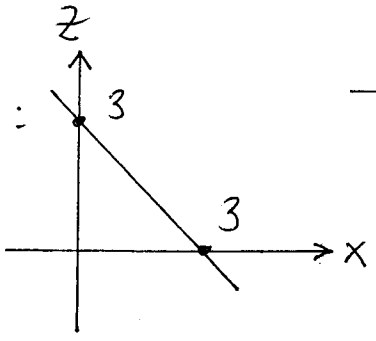
$X+Y=3$ (line)

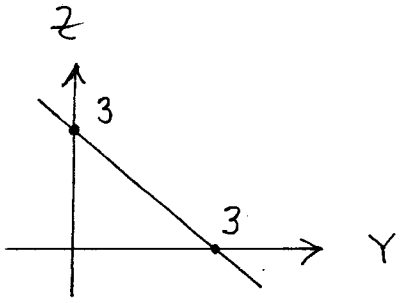


<u>XZ-trace</u> ($Y=0$):

$X+Z=3$

(line)





YZ-trace ($x=0$):

$y+z=3$

(line)

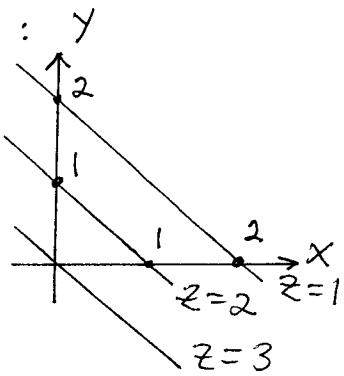
Level Curves:

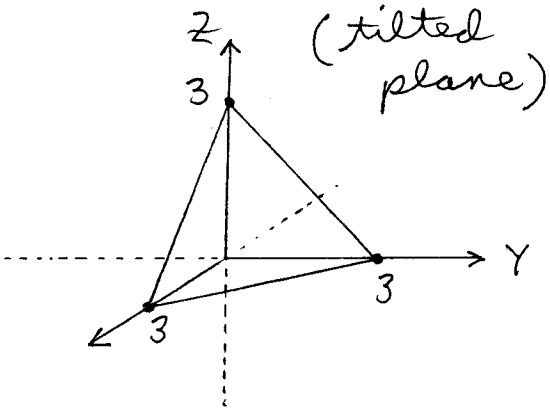
$z=1$: $x+y=2$

$z=2$: $x+y=1$

$z=3$: $x+y=0$

(all are lines.)

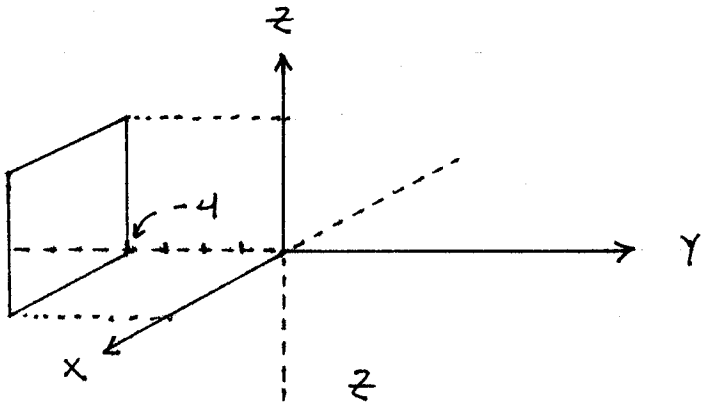




(tilted plane)

8.) $y=-4$

"vertical" plane



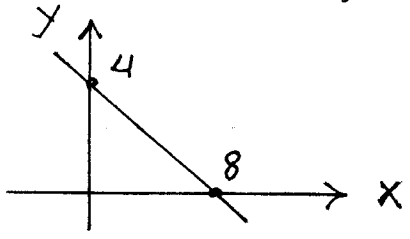
10.) $x+2y=8$

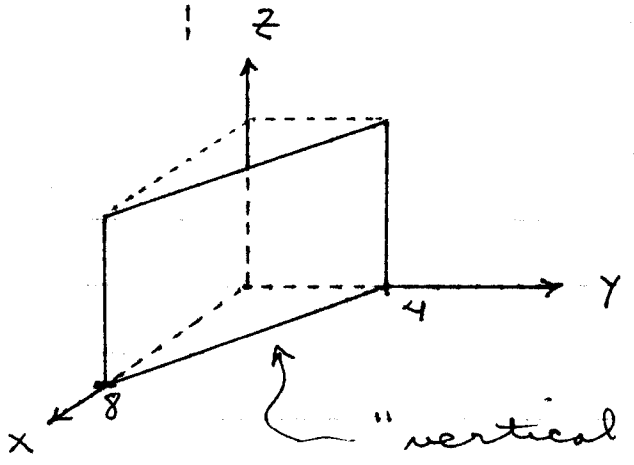
$x=0 \rightarrow y=4$

$y=0 \rightarrow x=8$

XY-trace ($z=0$):

$x+2y=8$ (line)





"vertical" plane

31.) $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{9} = 1$;

$x=0, y=0 \rightarrow z = \pm 3$

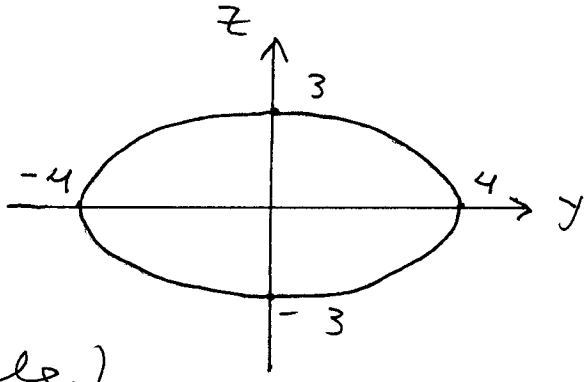
$x=0, z=0 \rightarrow y = \pm 4$

$y=0, z=0 \rightarrow x = \pm 3$

yz-trace

$(x=0)$: $\frac{y^2}{16} + \frac{z^2}{9} = 1$

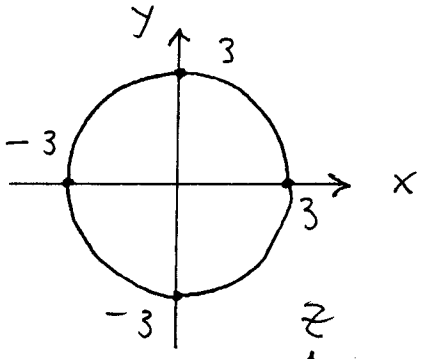
(ellipse)



xz-trace

$(y=0)$: $\frac{x^2}{9} + \frac{z^2}{9} = 1$

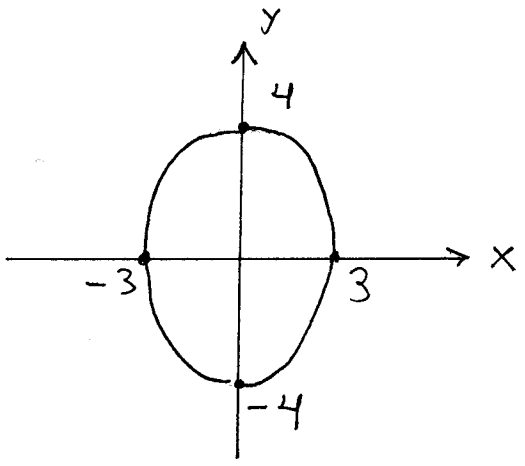
$\rightarrow x^2 + y^2 = 3^2$ (circle)

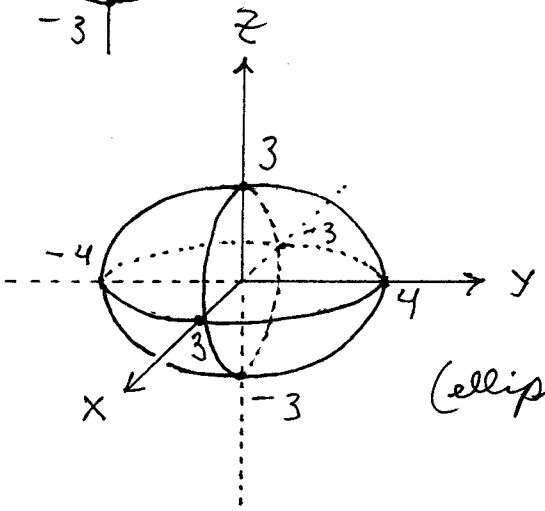


xy-trace

$(z=0)$: $\frac{x^2}{9} + \frac{y^2}{16} = 1$

(ellipse)





(ellipsoid)

33.) $4x^2 + 4y^2 - z^2 = 4$;

$x=0, y=0 \rightarrow -z^2 = 4$ (NO!? ★)

$x=0, z=0 \rightarrow 4y^2 = 4 \rightarrow y = \pm 1$

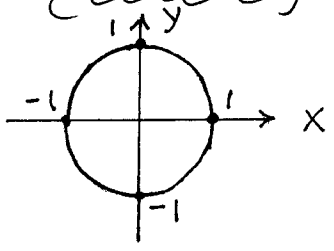
$y=0, z=0 \rightarrow 4x^2 = 4 \rightarrow x = \pm 1$

xy-trace $(z=0)$:

$4x^2 + 4y^2 = 4 \rightarrow$

$x^2 + y^2 = 1$

(circle)

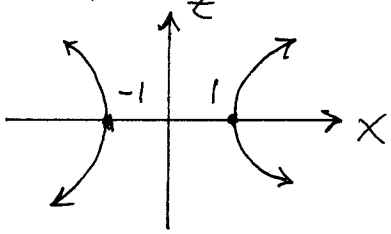


xz-trace ($y=0$):

$4x^2 - z^2 = 4 \rightarrow$

$x^2 - \frac{z^2}{4} = 1$

(hyperbola)

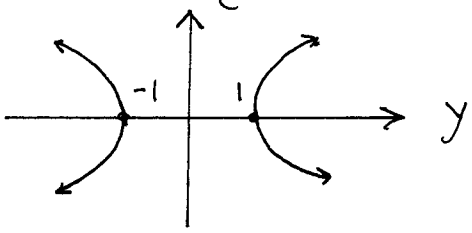


yz-trace ($x=0$):

$4y^2 - z^2 = 4 \rightarrow$

$y^2 - \frac{z^2}{4} = 1$

(hyperbola)



Level Curves:

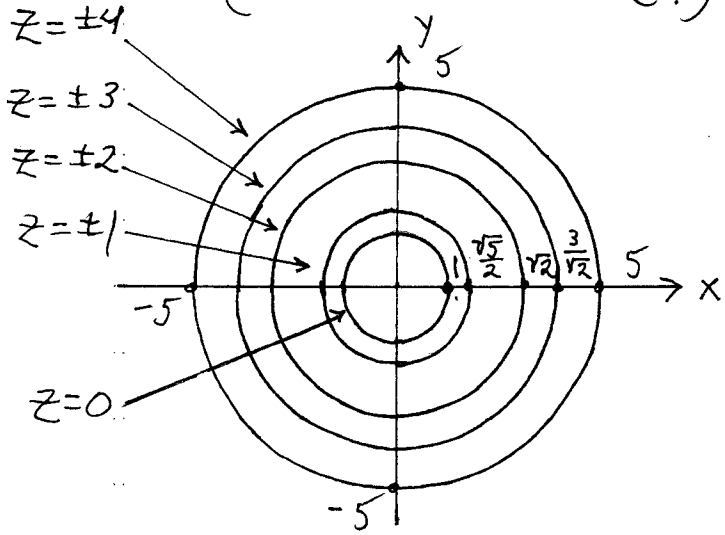
$z = \pm 1$: $4x^2 + 4y^2 - 1 = 4 \rightarrow x^2 + y^2 = \frac{5}{4} = \left(\frac{\sqrt{5}}{2}\right)^2$

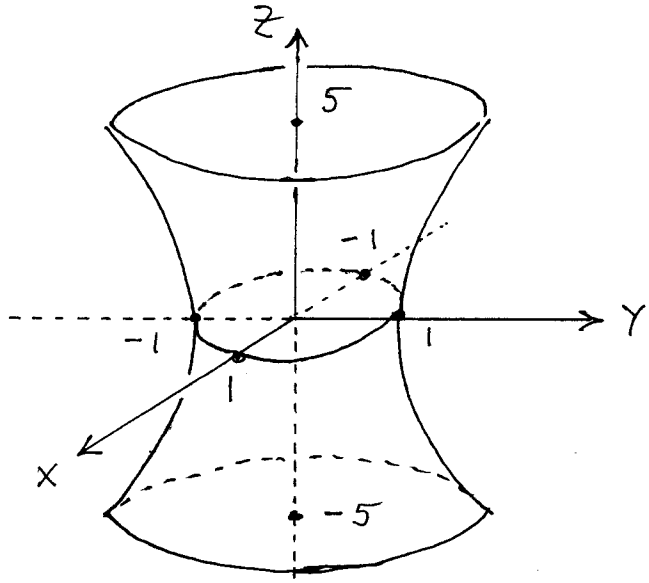
$z = \pm 2$: $4x^2 + 4y^2 - 4 = 4 \rightarrow x^2 + y^2 = 2 = (\sqrt{2})^2$

$z = \pm 3$: $4x^2 + 4y^2 - 9 = 9 \rightarrow x^2 + y^2 = \frac{9}{2} = \left(\frac{3}{\sqrt{2}}\right)^2$

$z = \pm 4$: $4x^2 + 4y^2 - 16 = 9 \rightarrow x^2 + y^2 = 25 = 5^2$

(all are circles.)



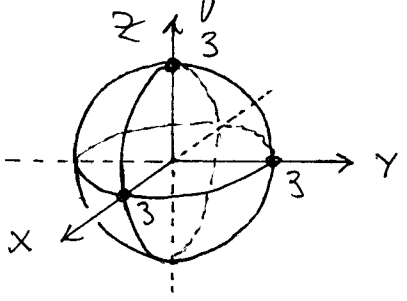


(hyperboloid of one sheet)

38.) $x^2 + y^2 + z^2 = 9 = 3^2$

sphere : center $(0,0,0)$

radius $= 3$



48.) $z = 4x^2 + y^2$;

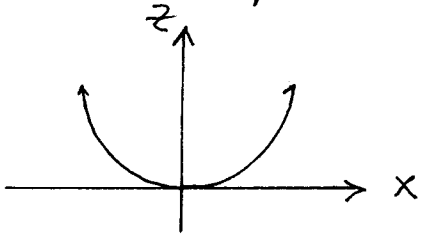
$x=0, y=0 \rightarrow z=0$

$x=0, z=0 \rightarrow y=0$

$y=0, z=0 \rightarrow x=0$

<u>xz-trace</u> $(y=0)$

$z = 4x^2$ (parabola)

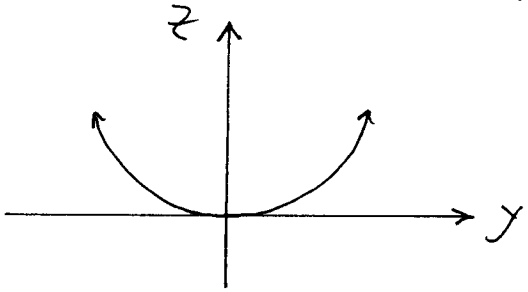


<u>xy-trace</u> $(z=0)$:

$4x^2 + y^2 = 0 \rightarrow x=0, y=0$

<u>yz-trace</u> $(x=0)$:

$z = y^2$ (parabola)



<u>Level Curves</u> :

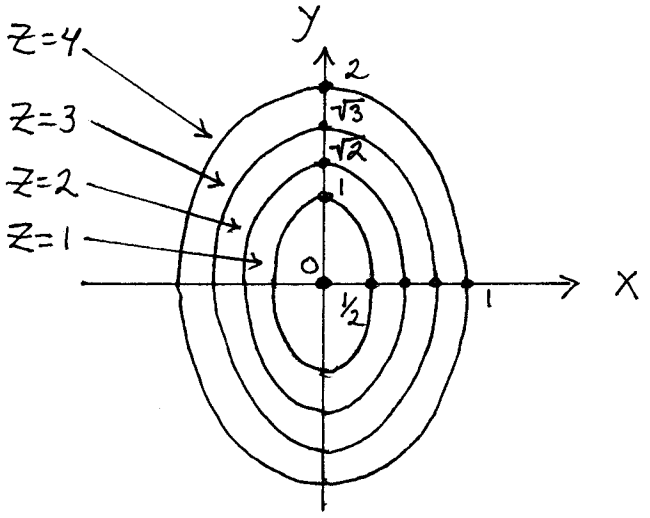
$z=1: 1 = 4x^2 + y^2$

$z=2: 2 = 4x^2 + y^2$

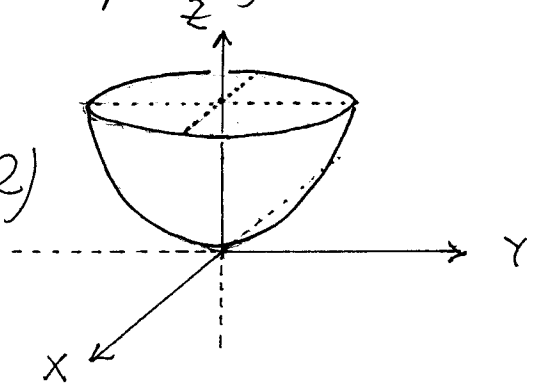
$z=3: 3 = 4x^2 + y^2$

$z=4: 4 = 4x^2 + y^2$

(all are ellipses)



(ellipsoid)



51.) $4x^2 - y^2 + 4z^2 = -16 \rightarrow 4x^2 + 4z^2 = y^2 - 16$;

$x=0, y=0 \rightarrow 4z^2 = -16$ (NO!? ✱)

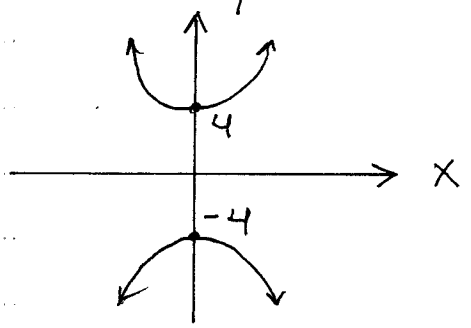
$x=0, z=0 \rightarrow y^2 - 16 = 0 \rightarrow y = \pm 4$

$y=0, z=0 \rightarrow 4x^2 = -16$ (NO!? ✱)

XY-trace ($z=0$):

$4x^2 - y^2 = -16 \rightarrow$

$\frac{y^2}{16} - \frac{x^2}{4} = 1$ (hyperbola)



XZ-trace ($y=0$):

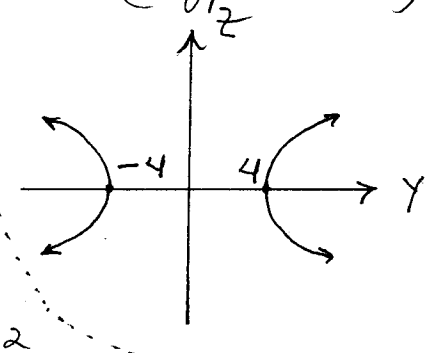
$4x^2 + 4z^2 = -16 \rightarrow$

$x^2 + z^2 = -4$ (NO!? ✱)

YZ-trace ($x=0$):

$-y^2 + 4z^2 = -16 \rightarrow$

$\frac{y^2}{16} - \frac{z^2}{4} = 1$ (hyperbola)



Level Curves (USING Y-values!):

$y = \pm 4$: $4x^2 + 4z^2 = 0 \rightarrow x=0, y=0$

$y = \pm 5$: $4x^2 + 4z^2 = 9 \rightarrow x^2 + z^2 = (\frac{3}{2})^2$

$y = \pm 6$: $4x^2 + 4z^2 = 20 \rightarrow x^2 + z^2 = (\sqrt{5})^2$

$y = \pm 7$: $4x^2 + 4z^2 = 33 \rightarrow x^2 + z^2 = (\frac{\sqrt{33}}{2})^2$

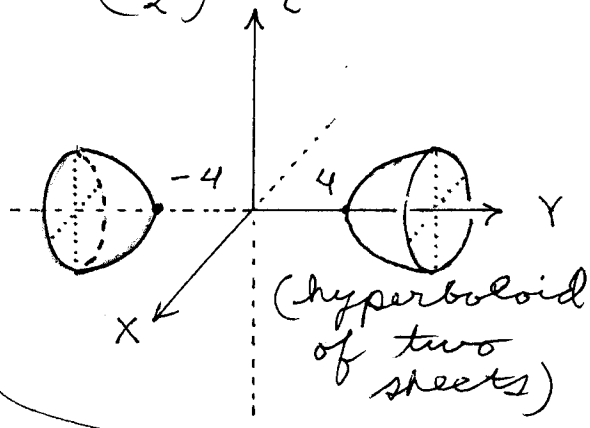
(all are circles)

54.) $4y = x^2 + z^2$;

$x=0, y=0 \rightarrow z=0$

$x=0, z=0 \rightarrow y=0$

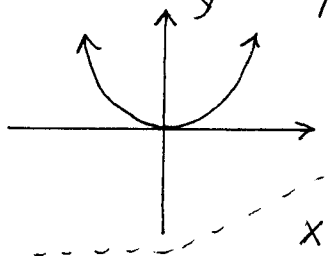
$y=0, z=0 \rightarrow x=0$;



(hyperboloid of two sheets)

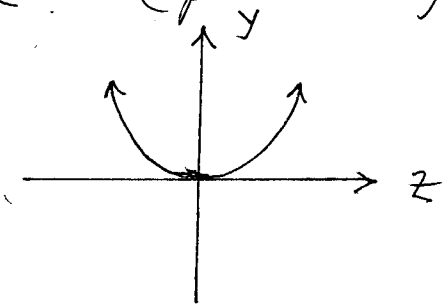
XY-trace ($z=0$):

$4y = x^2$ (parabola)



YZ-trace ($x=0$):

$4y = z^2$ (parabola)



XZ-trace ($y=0$):

$x^2 + z^2 = 0 \rightarrow x=0, z=0$

Level Curves (USING Y-values!)

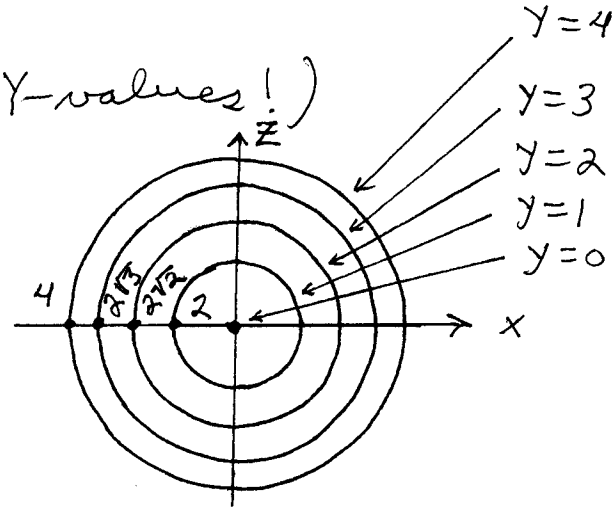
$y=1$: $x^2+z^2=2^2$

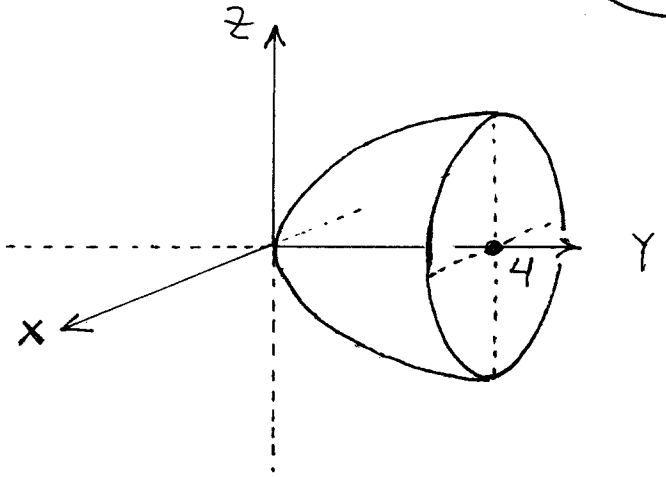
$y=2$: $x^2+z^2=(2\sqrt{2})^2$

$y=3$: $x^2+z^2=(2\sqrt{3})^2$

$y=4$: $x^2+z^2=4^2$

(all are circles)





(paraboloid)

56.) $z^2=2x^2+2y^2$;

$x=0, y=0 \rightarrow z=0$

$x=0, z=0 \rightarrow y=0$

$y=0, z=0 \rightarrow x=0$

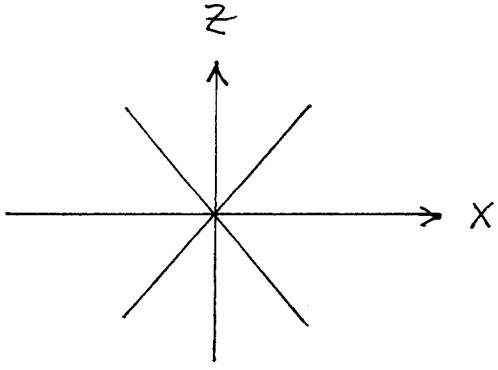
xy-trace ($z=0$) :

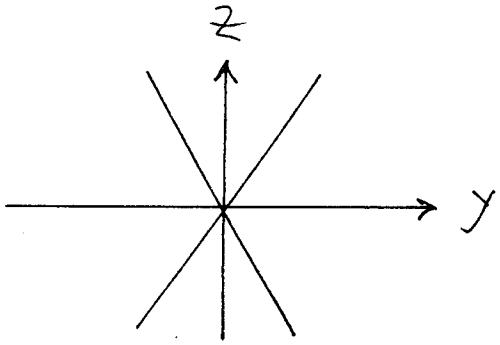
$2x^2+2y^2=0 \rightarrow x=0, y=0$

xz-trace ($y=0$) :

$z^2=2x^2 \rightarrow$

$z=\pm\sqrt{2}x$ (lines)





yz-trace ($x=0$):

$z^2 = 2y^2 \rightarrow$

$z = \pm\sqrt{2}\,y$ (lines)

Level Curves

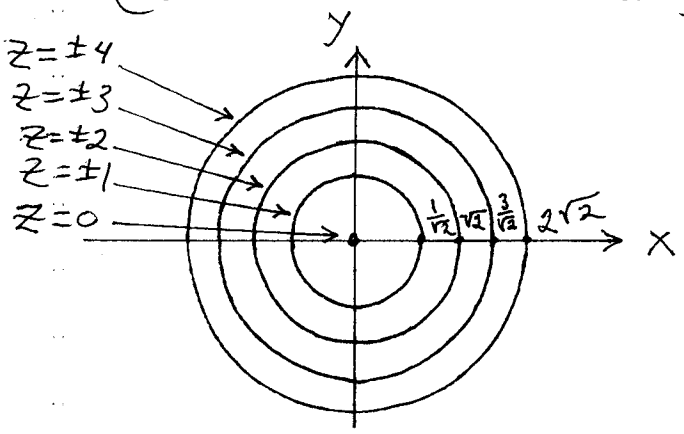
$z = \pm 1: \; 1 = 2x^2 + 2y^2 \rightarrow x^2 + y^2 = \left(\frac{1}{\sqrt{2}}\right)^2$

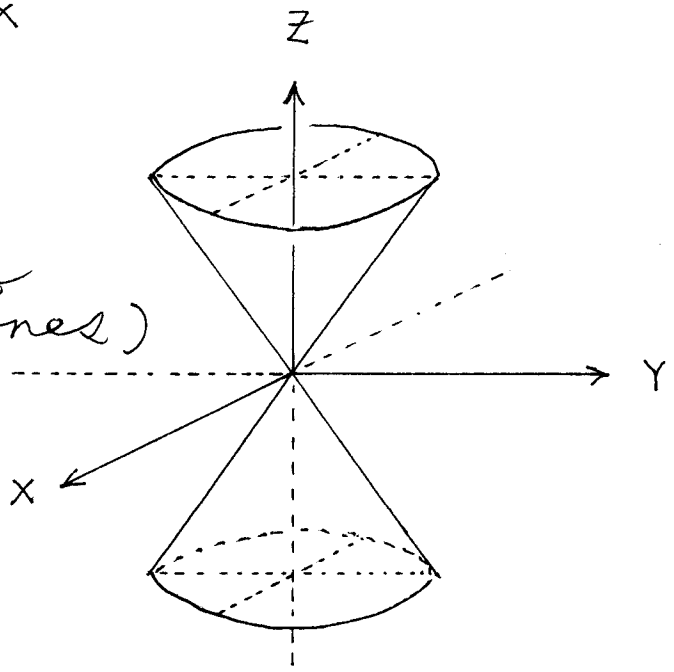
$z = \pm 2: \; 4 = 2x^2 + 2y^2 \rightarrow x^2 + y^2 = (\sqrt{2})^2$

$z = \pm 3: \; 9 = 2x^2 + 2y^2 \rightarrow x^2 + y^2 = \left(\frac{3}{\sqrt{2}}\right)^2$

$z = \pm 4: \; 16 = 2x^2 + 2y^2 \rightarrow x^2 + y^2 = (2\sqrt{2})^2$

(all are circles.)





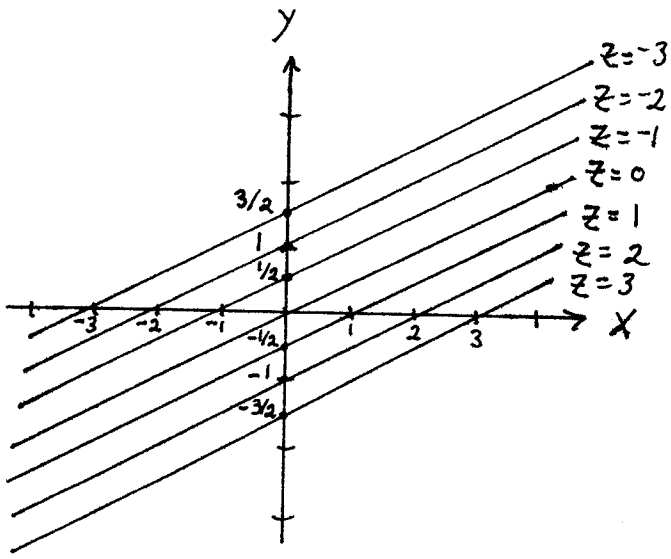
(two cones)

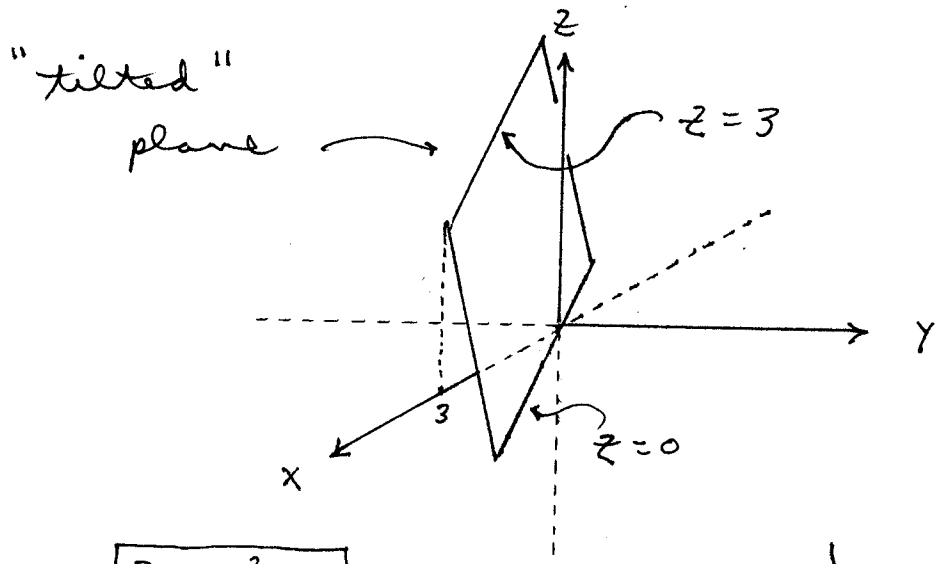
Worksheet 3

$\boxed{z = x - 2y}$

1.)

z-value	level curve
3	$3 = x - 2y$
2	$2 = x - 2y$
1	$1 = x - 2y$
0	$0 = x - 2y$
-1	$-1 = x - 2y$
-2	$-2 = x - 2y$
-3	$-3 = x - 2y$

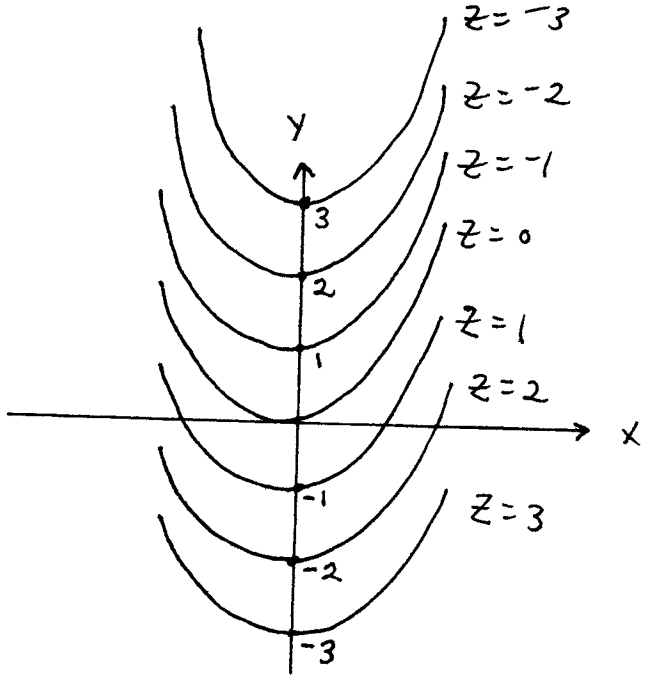


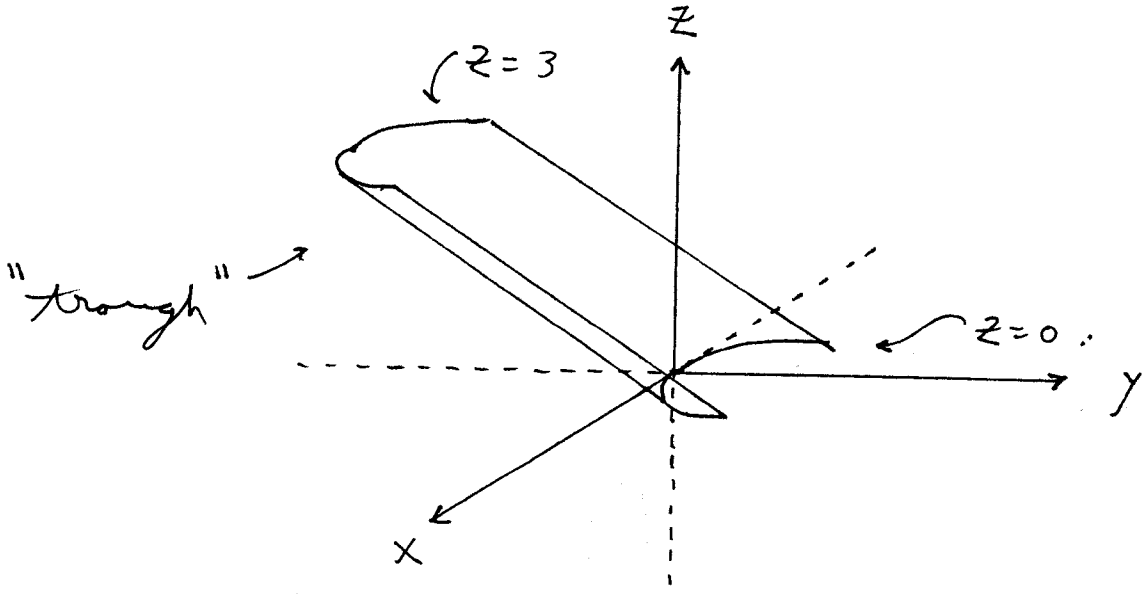


$\boxed{z = x^2 - y}$

2.)

z-value	level curve	
3	$3 = x^2 - y \rightarrow$	$y = x^2 - 3$
2	$2 = x^2 - y \rightarrow$	$y = x^2 - 2$
1	$1 = x^2 - y \rightarrow$	$y = x^2 - 1$
0	$0 = x^2 - y \rightarrow$	$y = x^2$
-1	$-1 = x^2 - y \rightarrow$	$y = x^2 + 1$
-2	$-2 = x^2 - y \rightarrow$	$y = x^2 + 2$
-3	$-3 = x^2 - y \rightarrow$	$y = x^2 + 3$

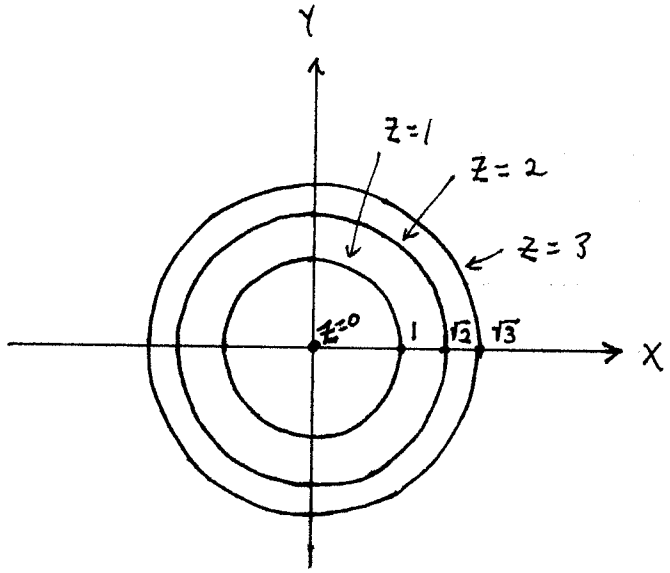


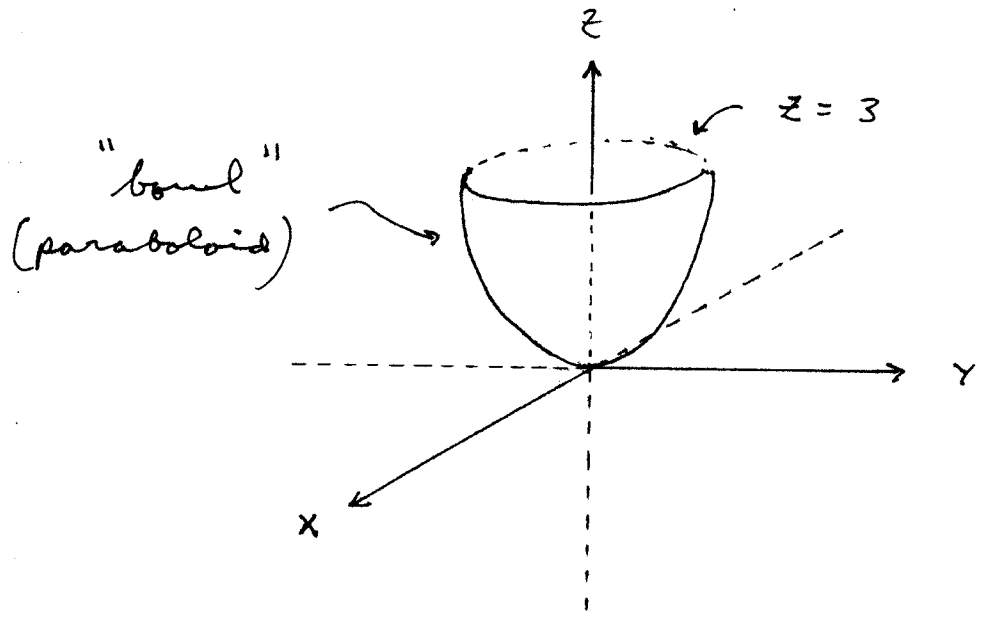


$\boxed{z = x^2 + y^2}$

3.)

z-value	level curve
3	$3 = x^2 + y^2$
2	$2 = x^2 + y^2$
1	$1 = x^2 + y^2$
0	$0 = x^2 + y^2$
-1	impossible
-2	impossible
-3	impossible

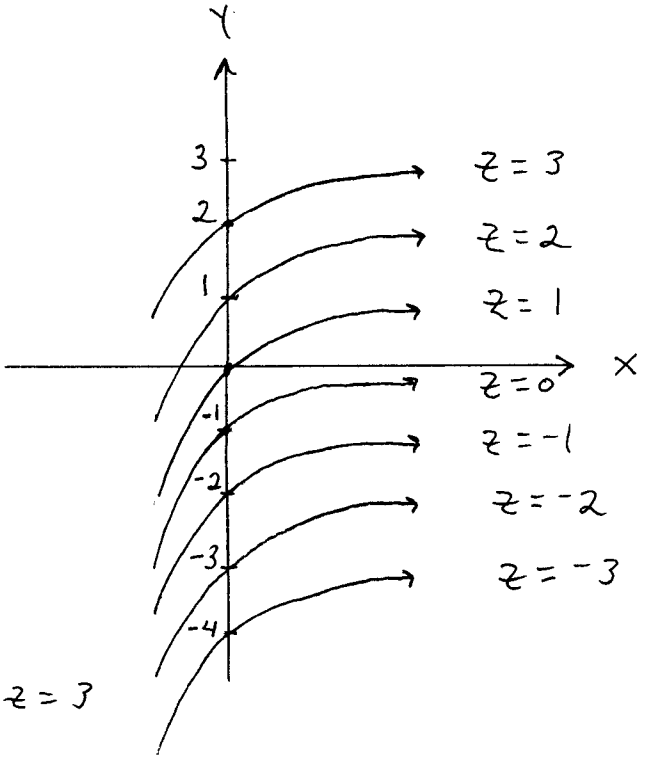


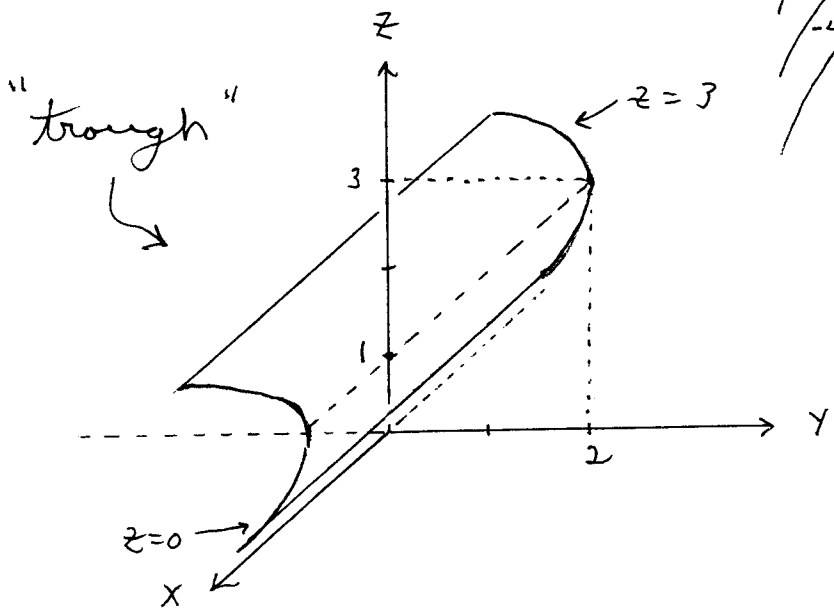


$$z = e^{-x} + y$$

4.)

z-value	level curve
3	$y = 3 - e^{-x}$
2	$y = 2 - e^{-x}$
1	$y = 1 - e^{-x}$
0	$y = -e^{-x}$
-1	$y = -1 - e^{-x}$
-2	$y = -2 - e^{-x}$
-3	$y = -3 - e^{-x}$

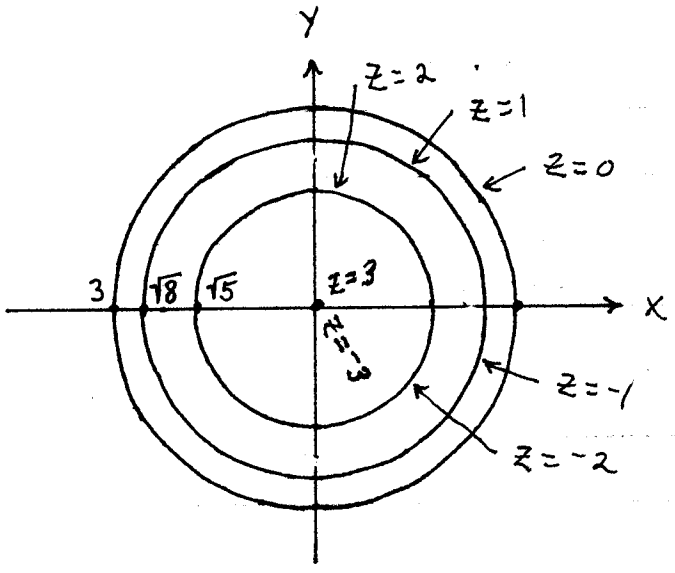


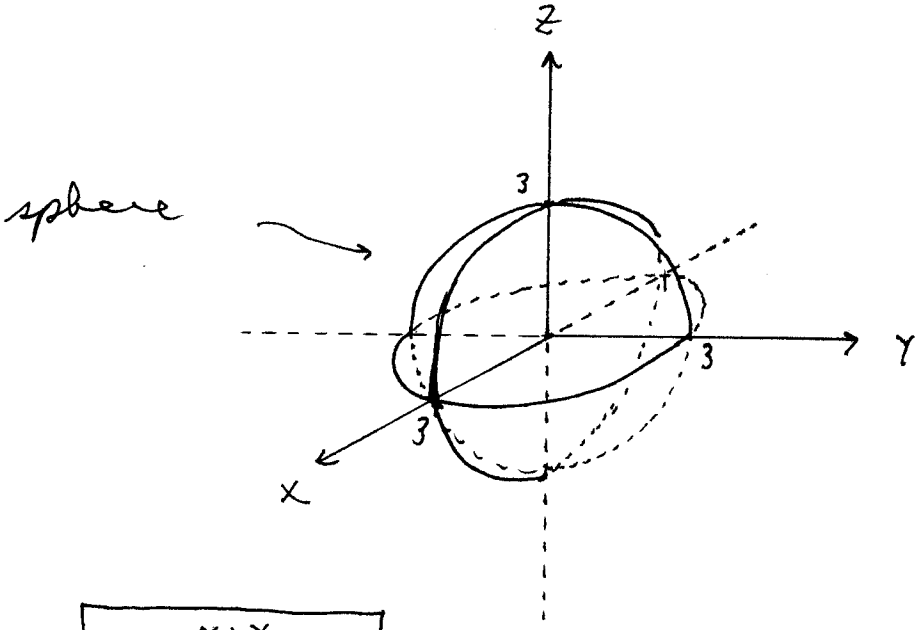


$$x^2 + y^2 + z^2 = 9$$

5.)

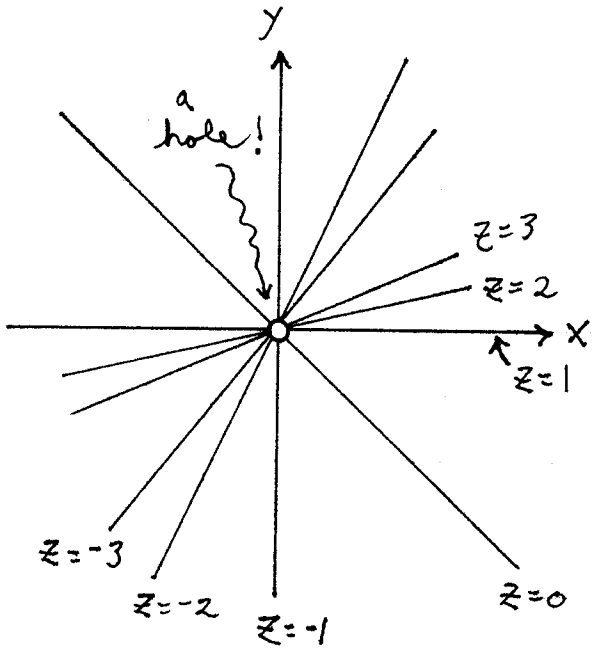
z-value	level curve
3	$x^2 + y^2 = 0$
2	$x^2 + y^2 = 5$
1	$x^2 + y^2 = 8$
0	$x^2 + y^2 = 9$
-1	$x^2 + y^2 = 8$
-2	$x^2 + y^2 = 5$
-3	$x^2 + y^2 = 0$





6.) $\boxed{z = \frac{x+y}{x-y}}$

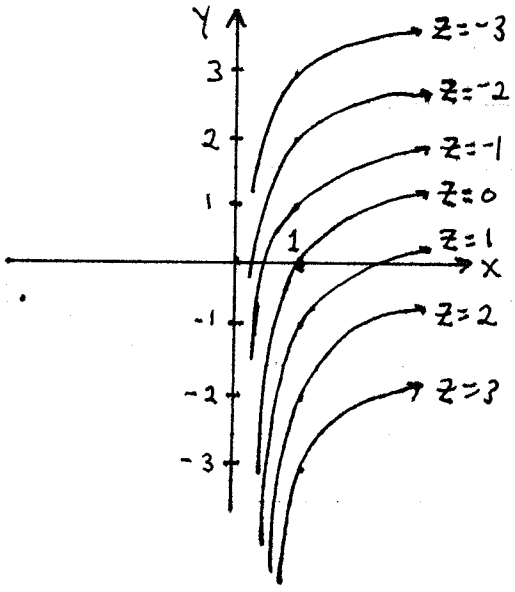
z-value	level curve
3	$3 = \frac{x+y}{x-y} \rightarrow y = \frac{1}{2}x$
2	$2 = \frac{x+y}{x-y} \rightarrow y = \frac{1}{3}x$
1	$1 = \frac{x+y}{x-y} \rightarrow y = 0$
0	$0 = \frac{x+y}{x-y} \rightarrow y = -x$
-1	$-1 = \frac{x+y}{x-y} \rightarrow x = 0$
-2	$-2 = \frac{x+y}{x-y} \rightarrow y = 3x$
-3	$-3 = \frac{x+y}{x-y} \rightarrow y = 2x$

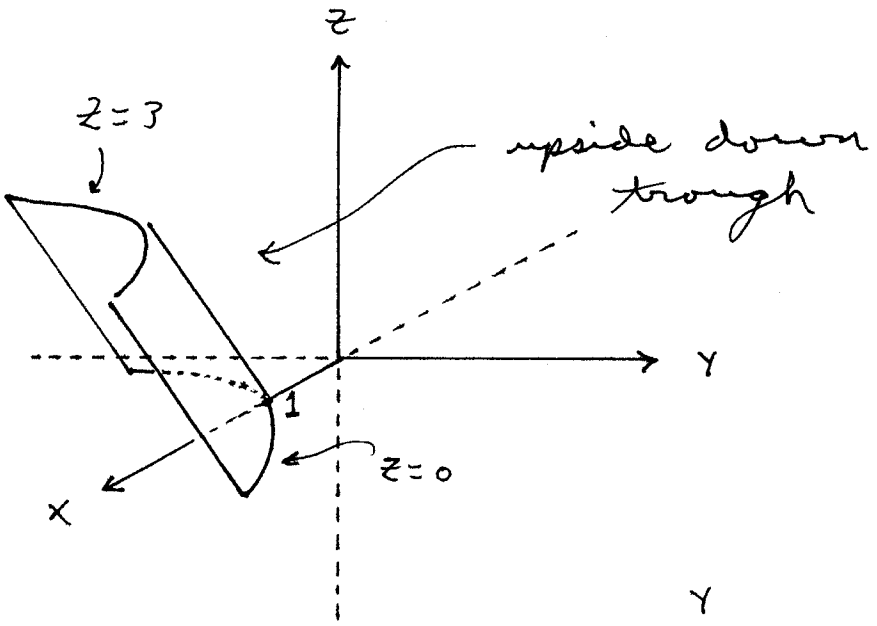


The surface is a "twisted ribbon."

7.) $\boxed{z = \ln x - y}$

z-value	level curve
3	$3 = \ln x - y \rightarrow y = \ln x - 3$
2	$2 = \ln x - y \rightarrow y = \ln x - 2$
1	$1 = \ln x - y \rightarrow y = \ln x - 1$
0	$0 = \ln x - y \rightarrow y = \ln x$
-1	$-1 = \ln x - y \rightarrow y = \ln x + 1$
-2	$-2 = \ln x - y \rightarrow y = \ln x + 2$
-3	$-3 = \ln x - y \rightarrow y = \ln x + 3$

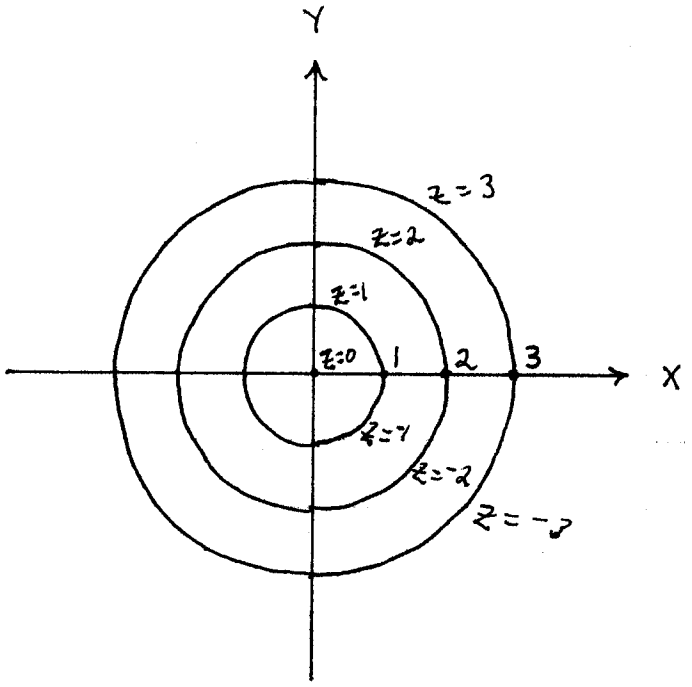




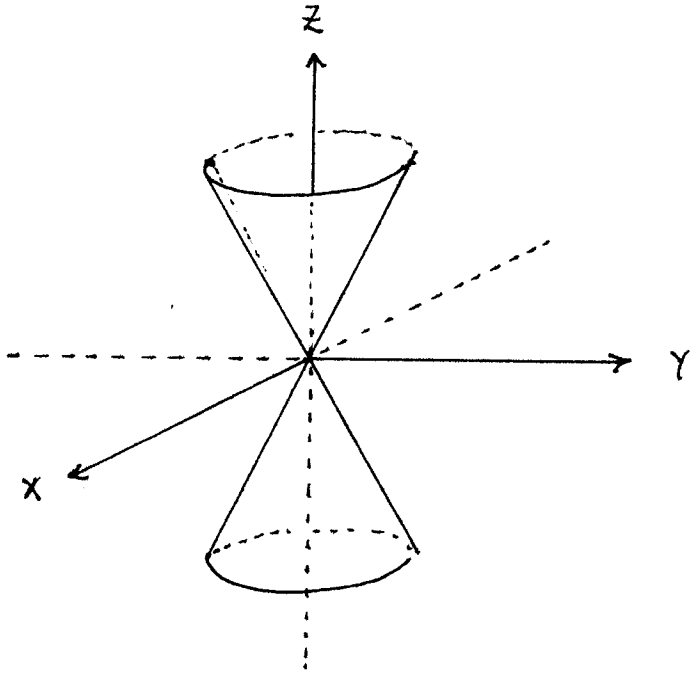
$z^2 = x^2 + y^2$

8.)

z-value	level curve
3	$9 = x^2 + y^2$
2	$4 = x^2 + y^2$
1	$1 = x^2 + y^2$
0	$0 = x^2 + y^2$
-1	$1 = x^2 + y^2$
-2	$4 = x^2 + y^2$
-3	$9 = x^2 + y^2$



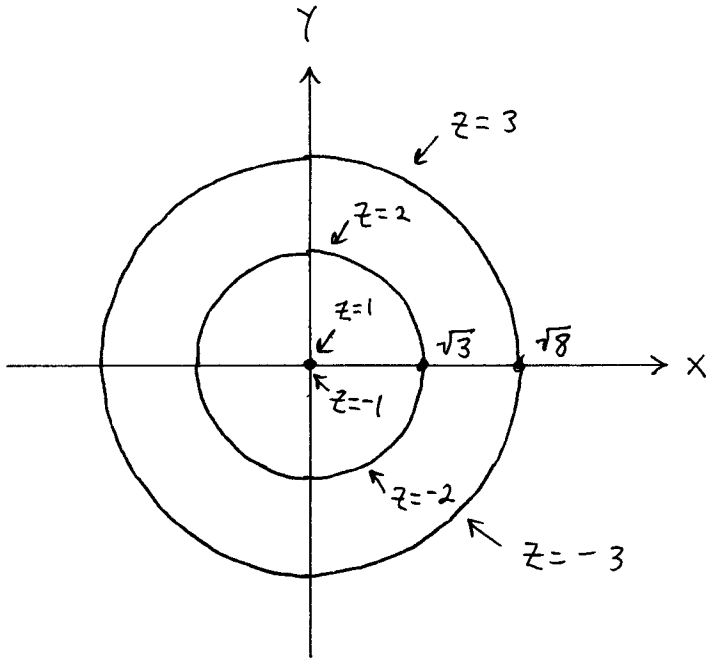
two cones



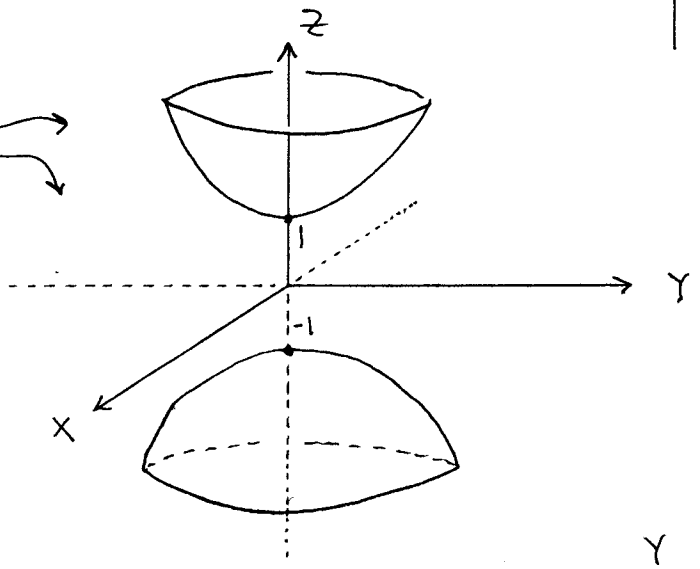
$$z^2 = x^2 + y^2 + 1$$

9.)

z-value	level curve
3	$8 = x^2 + y^2$
2	$3 = x^2 + y^2$
one point → 1	$0 = x^2 + y^2$
can't! → 0	$-1 = x^2 + y^2$
one point → -1	$0 = x^2 + y^2$
-2	$3 = x^2 + y^2$
-3	$8 = x^2 + y^2$

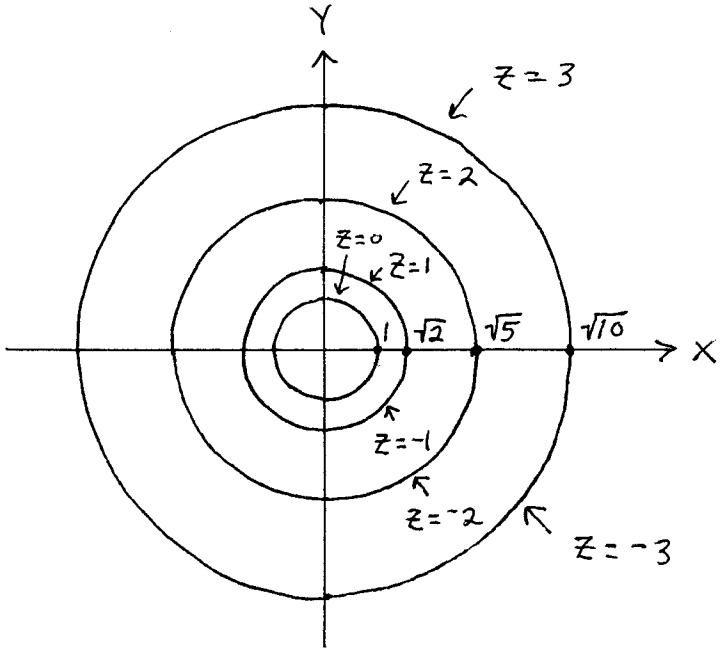


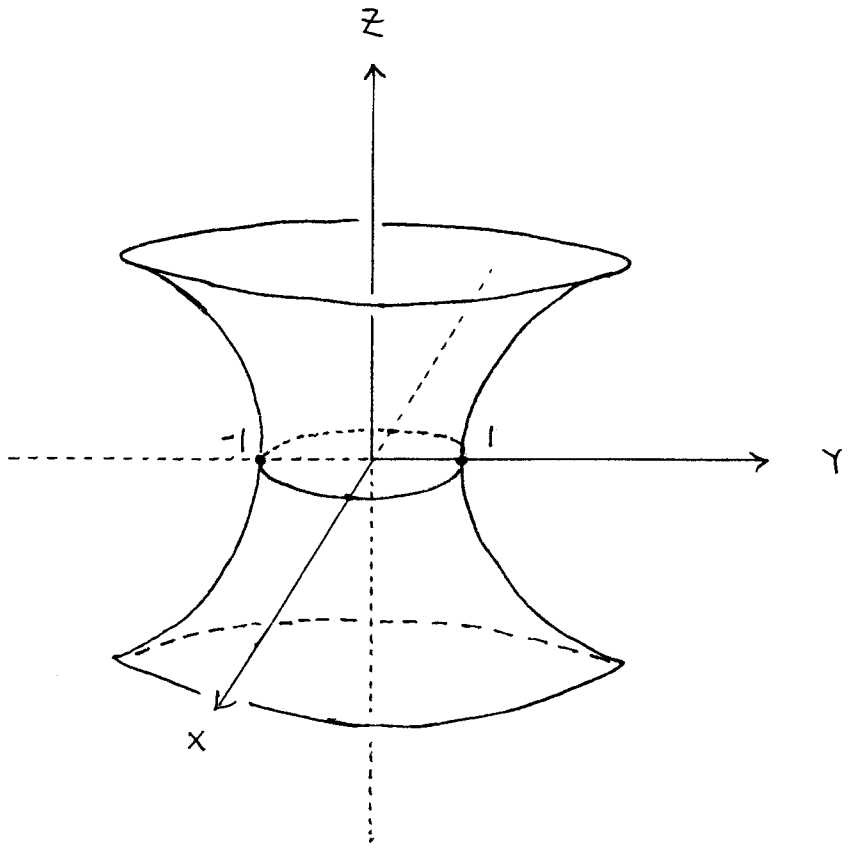
two "bowls" (hyperboloid of two sheets)



10.) $$z^2 = x^2 + y^2 - 1$$

z-value	level curve
3	$10 = x^2 + y^2$
2	$5 = x^2 + y^2$
1	$2 = x^2 + y^2$
0	$1 = x^2 + y^2$
-1	$2 = x^2 + y^2$
-2	$5 = x^2 + y^2$
-3	$10 = x^2 + y^2$





"tube"

(hyperboloid of one sheet)