

Section 7.4

1.) $z = 2x - 3y + 5 \rightarrow$

$$z_x = 2, \quad z_y = -3$$

2.) $z = x^2 - 3y^2 + 7 \rightarrow$

$$z_x = 2x, \quad z_y = -6y$$

3.) $z = 5\sqrt{x} - 6y^2 \rightarrow$

$$z_x = 5 \cdot \frac{1}{2} x^{-1/2}, \quad z_y = -12y$$

4.) $z = x^{-1/2} + 4y^{3/2} \rightarrow$

$$z_x = -\frac{1}{2} x^{-3/2} + 4 \cdot \frac{3}{2} y^{1/2}$$

5.) $z = \frac{x}{y} \rightarrow$

$$z_x = \frac{1}{y}, \quad z_y = -\frac{x}{y^2}$$

6.) $z = x\sqrt{y} \rightarrow$

$$z_x = \sqrt{y}, \quad z_y = x \cdot \frac{1}{2} y^{-1/2}$$

7.) $z = \sqrt{x^2 + y^2} \rightarrow$

$$z_x = \frac{1}{2} (x^2 + y^2)^{-1/2} \cdot (2x), \quad z_y = \frac{1}{2} (x^2 + y^2)^{-1/2} \cdot (2y)$$

8.) $z = \frac{xy}{x^2 + y^2} \rightarrow$

$$z_x = \frac{(x^2 + y^2) \cdot y - xy(2x)}{(x^2 + y^2)^2}, \quad z_y = \frac{(x^2 + y^2) \cdot x - xy(2y)}{(x^2 + y^2)^2}$$

9.) $z = x^2 e^{2y} \rightarrow$

$$z_x = 2x e^{2y}, \quad z_y = x^2 \cdot 2e^{2y}$$

10.) $z = x e^{x+y} \rightarrow$

$$z_x = x \cdot e^{x+y} + 1 \cdot e^{x+y}, \quad z_y = x \cdot e^{x+y}$$

11.) $z = e^{-x^2 - y^2} \rightarrow$

$$z_x = e^{-x^2 - y^2} \cdot (-2x), \quad z_y = e^{-x^2 - y^2} \cdot (-2y)$$

12.) $z = e^{x/y} \rightarrow$

$$z_x = e^{x/y} \cdot \frac{1}{y}, \quad z_y = e^{x/y} \cdot \frac{-x}{y^2}$$

$$13.) z = \ln \frac{x-y}{(x+y)^2} \rightarrow$$

$$z_x = \frac{1}{\frac{(x-y)}{(x+y)^2}} \cdot \frac{(x+y)^2 \cdot (1) - (x-y) \cdot 2(x+y)}{(x+y)^4},$$

$$z_y = \frac{1}{\frac{(x-y)}{(x+y)^2}} \cdot \frac{(x+y)^2 \cdot (-1) - (x-y) \cdot 2(x+y)}{(x+y)^4}$$

$$14.) z = \ln(x^2 + y^2)^{1/2} = \frac{1}{2} \ln(x^2 + y^2) \rightarrow$$

$$z_x = \frac{1}{2} \cdot \frac{1}{x^2 + y^2} \cdot (2x), \quad z_y = \frac{1}{2} \cdot \frac{1}{x^2 + y^2} \cdot (2y)$$

$$f(x, y) = 3x^2 y \cdot e^{x-y}, \quad g(x, y) = 3xy^2 \cdot e^{y-x} \rightarrow$$

$$15.) f_x(x, y) = (3x^2 y) \cdot e^{x-y} + (6xy) \cdot e^{x-y}$$

$$16.) f_y(x, y) = (3x^2 y) \cdot e^{x-y} (-1) + (3x^2) \cdot e^{x-y}$$

$$20.) g_x(x, y) = (3xy^2) \cdot e^{y-x} (-1) + (3y^2) \cdot e^{y-x} \rightarrow$$

$$g_x(-2, -2) = (-24) \cdot e^{\overset{1}{-2} \cdot (-1)} + (12) \cdot e^{\overset{1}{-2} \cdot (-1)} = 36$$

$$21.) z = 3x^2 + xy - y^2 \rightarrow$$

$$z_x = 6x + y \rightarrow z_x(2, 1) = 12 + 1 = 13,$$

$$z_y = x - 2y \rightarrow z_y(2, 1) = 2 - 2 = 0.$$

$$24.) z = e^x \cdot y^2 \rightarrow$$

$$z_x = e^x \cdot y^2 \rightarrow z_x(0, 2) = e^{\overset{1}{0}} \cdot (4) = 4,$$

$$z_y = e^x \cdot 2y \rightarrow z_y(0, 2) = e^{\overset{1}{0}} \cdot 4 = 4$$

$$26.) \quad z = \frac{4xy}{(x^2+y^2)^{1/2}} \rightarrow$$

$$z_x = \frac{(x^2+y^2)^{1/2} \cdot 4y - 4xy \cdot \frac{1}{2}(x^2+y^2)^{-1/2} \cdot (2x)}{((x^2+y^2)^{1/2})^2} \rightarrow$$

$$z_x(1,0) = \frac{(1)(0) - 0}{1} = 0,$$

$$z_y = \frac{(x^2+y^2)^{1/2} \cdot (4x) - 4xy \cdot \frac{1}{2}(x^2+y^2)^{-1/2} \cdot (2y)}{((x^2+y^2)^{1/2})^2} \rightarrow$$

$$z_y(1,0) = \frac{(1)(4) - (0)}{1} = 4.$$

$$27.) \quad z = \ln(x^2+y^2) \rightarrow$$

$$z_x = \frac{2x}{x^2+y^2} \rightarrow z_x(1,0) = \frac{2}{1} = 2,$$

$$z_y = \frac{2y}{x^2+y^2} \rightarrow z_y(1,0) = \frac{0}{1} = 0$$

$$29.) \quad w = 3x^2y - 5xyz + 10yz^2 \rightarrow$$

$$w_x = 6xy - 5yz$$

$$w_y = 3x^2 - 5xz + 10z^2,$$

$$w_z = -5xy + 20yz$$

$$31.) \quad w = \frac{xy}{x+y+z} \rightarrow$$

$$w_x = \frac{(x+y+z)(y) - (xy) \cdot (1)}{(x+y+z)^2},$$

$$w_y = \frac{(x+y+z)(x) - (xy) \cdot (1)}{(x+y+z)^2},$$

$$w_z = \frac{(x+y+z)(0) - xy(1)}{(x+y+z)^2}$$

$$38.) w = xy e^{z^2} \rightarrow$$

$$w_x = y e^{z^2} \rightarrow w_x(2,1,0) = 1 \cdot e^0 = 1,$$

$$w_y = x e^{z^2} \rightarrow w_y(2,1,0) = 2 \cdot e^0 = 2,$$

$$w_z = xy \cdot e^{z^2} \cdot 2z \rightarrow w_z(2,1,0) = (2)(1)e^0(0) = 0$$

$$39.) z = x^2 + 4xy + y^2 - 4x + 16y + 3 \rightarrow$$

$$z_x = 2x + 4y - 4 = 2(x + 2y - 2) = 0 \rightarrow$$

$$\boxed{x + 2y - 2 = 0}$$

$$z_y = 4x + 2y + 16 = 2(2x + y + 8) = 0 \rightarrow$$

$$\boxed{2x + y + 8 = 0}$$

; then

$$x = 2 - 2y \xrightarrow{\text{sub.}} 2(2 - 2y) + y + 8 = 0 \rightarrow$$

$$4 - 4y + y + 8 = 0 \rightarrow -3y = -12 \rightarrow \boxed{y = 4, x = -6}$$

$$41.) z = \frac{1}{x} + \frac{1}{y} + xy \rightarrow$$

$$z_x = -\frac{1}{x^2} + y = 0 \rightarrow \boxed{y = \frac{1}{x^2}};$$

$$z_y = -\frac{1}{y^2} + x = 0 \rightarrow \boxed{x = \frac{1}{y^2}}; \text{ then (sub.)}$$

$$x = \frac{1}{\left(\frac{1}{x^2}\right)^2} = \frac{1}{\frac{1}{x^4}} = x^4 \rightarrow x = x^4 \rightarrow$$

$$0 = x^4 - x = x(x^3 - 1) \rightarrow \cancel{x=0}^{\text{No}} \text{ or } \boxed{x=1} \rightarrow$$

$$\boxed{y=1}$$

$$42.) z = \ln(x^2 + y^2 + 1) \rightarrow$$

$$z_x = \frac{2x}{x^2+y^2+1} = 0 \rightarrow 2x = 0 \rightarrow \boxed{x=0} ;$$

$$z_y = \frac{2y}{x^2+y^2+1} = 0 \rightarrow 2y = 0 \rightarrow \boxed{y=0} .$$

$$43.) z = 2x - 3y + 5 \rightarrow$$

$$z_x = 2, \quad z_y = -3$$

$$a.) z_x(2,1) = 2$$

$$b.) z_y(2,1) = -3$$

$$49.) z = 4 - x^2 - y^2 \rightarrow$$

$$z_x = -2x, \quad z_y = -2y$$

$$a.) z_x(1,1) = -2$$

$$b.) z_y(1,1) = -2$$

$$52.) z = x^4 - 3x^2y^2 + y^4 \rightarrow$$

$$\frac{\partial z}{\partial y} = -6x^2y + 4y^3 \rightarrow \frac{\partial^2 z}{\partial x \partial y} = -12xy ,$$

$$\frac{\partial z}{\partial x} = 4x^3 - 6xy^2 \rightarrow \frac{\partial^2 z}{\partial y \partial x} = -12xy$$

$$55.) z = x^3 - 4y^2 \rightarrow$$

$$\frac{\partial z}{\partial x} = 3x^2 \rightarrow \frac{\partial^2 z}{\partial x^2} = 6x \text{ and } \frac{\partial^2 z}{\partial y \partial x} = 0 ,$$

$$\frac{\partial z}{\partial y} = -8y \rightarrow \frac{\partial^2 z}{\partial y^2} = -8 \text{ and } \frac{\partial^2 z}{\partial x \partial y} = 0$$

$$59.) z = \frac{xy}{x-y} \rightarrow$$

$$\frac{\partial z}{\partial x} = \frac{(x-y)(y) - (xy)(1)}{(x-y)^2} = \frac{xy - y^2 - xy}{(x-y)^2} = \frac{-y^2}{(x-y)^2} ;$$

$$\frac{\partial z}{\partial y} = \frac{(x-y)(x) - (xy)(-1)}{(x-y)^2} = \frac{x^2 - xy + xy}{(x-y)^2} = \frac{x^2}{(x-y)^2} ;$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{(x-y)^2(0) - (-y^2) \cdot 2(x-y)}{(x-y)^4} = \frac{2y^2}{(x-y)^3} ;$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{(x-y)^2(0) - (x^2) \cdot 2(x-y)(-1)}{(x-y)^4} = \frac{2x^2}{(x-y)^3} ;$$

$$\begin{aligned} \frac{\partial^2 z}{\partial y \partial x} &= \frac{(x-y)^2(-2y) - (-y^2) \cdot 2(x-y)(-1)}{(x-y)^4} \\ &= \frac{2(x-y) \cdot [-y(x-y) - y^2]}{(x-y)^4} = \frac{-2xy}{(x-y)^3} ; \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 z}{\partial x \partial y} &= \frac{(x-y)^2(2x) - x^2 \cdot 2(x-y)}{(x-y)^4} \\ &= \frac{2(x-y) \cdot [(x-y)x - x^2]}{(x-y)^4} = \frac{-2xy}{(x-y)^3} . \end{aligned}$$

$$62.) z = xe^y + ye^x \rightarrow$$

$$\frac{\partial z}{\partial x} = e^y + ye^x, \quad \frac{\partial z}{\partial y} = xe^y + e^x \rightarrow$$

$$\frac{\partial^2 z}{\partial x^2} = ye^x, \quad \frac{\partial^2 z}{\partial y^2} = xe^y,$$

$$\frac{\partial^2 z}{\partial y \partial x} = e^y + e^x, \quad \frac{\partial^2 z}{\partial x \partial y} = e^y + e^x$$

$$72.) IQ(M, c) = \frac{100M}{c} \rightarrow$$

$$IQ_c = -\frac{100M}{c^2}, \quad IQ_M = \frac{100}{c} \rightarrow$$

$$IQ_c(12, 10) = -\frac{1200}{100} = -12 \quad (IQ \text{ pts. per chron. years}),$$

$$IQ_M(12, 10) = \frac{100}{10} = +10 \quad (IQ \text{ pts. per mental years})$$

For a fixed M an increase in chronological age decreases IQ; for a fixed C an increase in mental age M increases IQ.

73.) $N = f(p, t)$; $\frac{\partial N}{\partial p} < 0$ means that for a fixed tuition t an increase in food price p decreases the number of applicants ; $\frac{\partial N}{\partial t} < 0$ means that for a fixed food price p an increase in tuition t decreases the number of applicants.

74.) $T(x, y) = 500 - 0.6x^2 - 1.5y^2$ at $(2, 3) = (x, y)$:

$$T_x = -1.2x \quad \text{and} \quad T_y = -3y \quad \text{so}$$

a.) Rate of change in x -direction at $(2, 3)$ is

$$T_x = (-1.2)(2) = \textcircled{-2.4} \text{ degrees per meter.}$$

b.) Rate of change in y -direction at $(2, 3)$ is

$$T_y = (-3)(3) = \textcircled{-9} \text{ degrees per meter.}$$