Section C.1

3.) $y = e^{-2x} \xrightarrow{D} y' = -2e^{-2x}$ then
$y' + 2y = (-2e^{-2x}) + 2(e^{-2x}) = 0$.

6.) $y = 4x^2 \xrightarrow{D} y' = 8x$ then
$y' - \frac{2}{x}y = 8x - \frac{2}{x}(4x^2) = 8x - 8x = 0$.

8.) $y = \frac{1}{x} \xrightarrow{D} y' = \frac{-1}{x^2}$ and $y'' = \frac{2}{x^3}$ then
$xy'' + 2y' = x\left(\frac{2}{x^3}\right) + 2\left(\frac{-1}{x^2}\right) = \frac{2}{x^2} - \frac{2}{x^2} = 0$.

10.) $y = e^{x^3} \xrightarrow{D} y' = 3x^2 e^{x^3}$ and
$y'' = 3x^2 \cdot 3x^2 e^{x^3} + 6x e^{x^3} = (9x^4 + 6x)e^{x^3}$ then
$y'' - 3x^2 y' - 6xy = (9x^4 + 6x)e^{x^3}$
$- 3x^2(3x^2 e^{x^3}) - 6x\, e^{x^3}$
$= (9x^4 + 6x - 9x^4 - 6x)e^{x^3} = 0 \cdot e^{x^3} = 0$.

11.) $y = \frac{1}{x} + c \xrightarrow{D} y' = \frac{-1}{x^2}$.

12.) $y = (4 - x^2)^{1/2} + c \rightarrow y' = \frac{1}{2}(4 - x^2)^{-\frac{1}{2}} \cdot (-2x)$ then
$y' = \dfrac{-x}{\sqrt{4 - x^2}}$.

16.) $y = ce^{-t} + 10 \xrightarrow{D} y' = -ce^{-t}$ then
$y' + y = (-ce^{-t}) + (ce^{-t} + 10) = 10$.

17.) $y = cx^2 - 3x \xrightarrow{D} y' = 2cx - 3$ then

$$xy' - 3x - 2y = 0 \rightarrow x(2cx-3) - 3x - 2(cx^2 - 3x)$$
$$= 2cx^2 - 3x - 3x - 2cx^2 + 6x = 0 .$$

17.) $y = cx^2 - 3x \xrightarrow{D} y' = 2cx - 3$ then
$$xy' - 3x - 2y = x(2cx-3) - 3x - 2(cx^2 - 3x)$$
$$= 2cx^2 - 3x - 3x - 2cx^2 + 6x = 0 .$$

20.) $y = c_1 + c_2 e^x \xrightarrow{D} y' = c_2 e^x$ and $y'' = c_2 e^x$
then $y'' - y' = c_2 e^x - c_2 e^x = 0 .$

26.) $y = ce^{x-x^2} \xrightarrow{D} y' = c(1-2x)e^{x-x^2}$ then
$$y' + (2x-1)y = c(1-2x)e^{x-x^2} + (2x-1)\cdot ce^{x-x^2}$$
$$= c(1-2x+2x-1)e^{x-x^2} = c(0)e^{x-x^2} = 0 .$$

27.) $y = x\ln x + cx + 4 \xrightarrow{D} y' = x\cdot\frac{1}{x} + \ln x + c$
$= 1 + \ln x + c$ then
$$x(y'-1) - (y-4) = x(\cancel{1} + \ln x + c - \cancel{1}) - (x\ln x + cx + \cancel{4} - \cancel{4})$$
$$= x\ln x + cx - x\ln x - cx = 0$$

28.) $y = x(\ln x + c) \xrightarrow{D} y' = x\cdot\frac{1}{x} + \ln x + c$
$= 1 + \ln x + c$ then
$$x + y - xy' = x + x(\ln x + c) - x(1 + \ln x + c)$$
$$= x + x\ln x + cx - x - x\ln x - cx = 0 .$$

29.) $x^2 + y^2 = cy \xrightarrow{D} 2x + 2yy' = cy' \rightarrow$
$$2x + 2yy' = \frac{x^2+y^2}{y}\cdot y' \rightarrow$$

$2xy + 2y^2y' = (x^2+y^2)\,y' \rightarrow$
$2y^2y' - (x^2-y^2)\,y' = -2xy \rightarrow$
$(2y^2 - x^2 - y^2)\,y' = -2xy \rightarrow$
$(y^2-x^2)\,y' = -2xy \rightarrow y' = \dfrac{-2xy}{y^2-x^2}\cdot\dfrac{-1}{-1}$
$\rightarrow\ y' = \dfrac{2xy}{x^2-y^2}$.

32.) $x^2-y^2 = c \xrightarrow{D} 2x - 2yy' = 0 \rightarrow$
$2x = 2yy' \rightarrow y' = \dfrac{x}{y} \xrightarrow{D}$
$y'' = \dfrac{y\cdot 1 - xy'}{y^2} = \dfrac{y - x\cdot\frac{x}{y}}{y^2} = \dfrac{\frac{y}{1} - \frac{x^2}{y}}{\frac{y^2}{1}}$
$= \dfrac{y^2-x^2}{y}\cdot\dfrac{1}{y^2} = \dfrac{y^2-x^2}{y^3}$, i.e.,
$y'' = \dfrac{y^2-x^2}{y^3} \rightarrow y^3y'' = y^2-x^2 \rightarrow$
$y^3y'' + x^2 - y^2 = 0$.

33.) $y = e^{-2x} \rightarrow y' = -2e^{-2x} \rightarrow y'' = 4e^{-2x} \rightarrow$
$y''' = -8e^{-2x} \rightarrow y^{(4)} = 16e^{-2x} \rightarrow$
$y^{(4)} - 16y = 16e^{-2x} - 16(e^{-2x}) = 0$. (TRUE)

34.) $Y = 5\ln x \rightarrow Y' = \dfrac{5}{x} = 5x^{-1} \rightarrow$
$Y'' = -5x^{-2} \rightarrow Y''' = 10\cdot x^{-3} \rightarrow Y^{(4)} = -30\cdot x^{-4} \rightarrow$
$Y^{(4)} - 16Y = -30x^{-4} - 16(5\ln x) \neq 0$. (FALSE)

39.) $y = xe^x \xrightarrow{D} y' = xe^x + e^x \xrightarrow{D}$

$y'' = xe^x + e^x + e^x = xe^x + 2e^x \xrightarrow{D}$

$y''' = xe^x + e^x + 2e^x = xe^x + 3e^x$ then

$y''' - 3y' + 2y = (xe^x + 3e^x) - 3(xe^x + e^x) + 2(xe^x)$

$xe^x + \cancel{3e^x} - 3xe^x - \cancel{3e^x} + 2xe^x = 0$. (TRUE)

40.) $y = x\ln x \xrightarrow{D} y' = x \cdot \frac{1}{x} + \ln x = 1 + \ln x \xrightarrow{D}$

$y'' = \frac{1}{x} \xrightarrow{D} y''' = \frac{-1}{x^2}$ then

$y''' - 3y' + 2y = \frac{-1}{x^2} - 3(1 + \ln x) + 2(x\ln x)$

$= \frac{-1}{x^2} - 3 - 3\ln x + 2x\ln x \neq 0$. (FALSE)

42.) $2x^2 + 3y^2 = C \rightarrow 4x + 6yy' = 0 \rightarrow$

$2x + 3yy' = 0$ (TRUE); if $x = 1$, $y = 2$ then

$2(1)^2 + 3(2)^2 = C \rightarrow C = 14 \rightarrow 2x^2 + 3y^2 = 14$.

43.) $y = c_1 + c_2 \ln|x| \rightarrow y' = c_2 \cdot \left(\frac{1}{x}\right) \rightarrow y'' = c_2\left(\frac{-1}{x^2}\right)$

then $xy'' + y' = x \cdot \left(\frac{-c_2}{x^2}\right) + \left(\frac{c_2}{x}\right) = \frac{-c_2}{x} + \frac{c_2}{x} = 0$ (TRUE);

$x = 1$, $y' = \frac{1}{2}$, $y' = \frac{c_2}{x} \rightarrow \frac{1}{2} = \frac{c_2}{1} \rightarrow c_2 = \frac{1}{2}$, and

$x = 1$, $y = 5$, $y = c_1 + c_2 \ln|x| \rightarrow 5 = c_1 + c_2 \cancelto{0}{\ln 1} \rightarrow c_1 = 5$

so $y = 5 + \frac{1}{2}\ln|x|$.

44.) $y = c_1 x + c_2 x^3 \xrightarrow{D} y' = c_1 + 3c_2 x^2 \xrightarrow{D} y'' = 6c_2 x$

then $x^2 y'' - 3xy' + 3y = x^2(6c_2 x) - 3x(c_1 + 3c_2 x^2) + 3(c_1 x + c_2 x^3)$

$= \cancel{6c_2 x^3} - \cancel{3c_1 x} - \cancel{9c_2 x^3} + \cancel{3c_1 x} + \cancel{3c_2 x^3} = 0$ (TRUE);

$x = 2$, $y = 0$, $y = c_1 x + c_2 x^3 \rightarrow 0 = 2c_1 + 8c_2 \rightarrow \underline{0 = c_1 + 4c_2}$ and

$x = 2$, $y' = 4$, $y' = c_1 + 3c_2 x^2 \rightarrow \underline{4 = c_1 + 12c_2}$ then

$4 = 8c_2 \rightarrow c_2 = \frac{1}{2}$ so $0 = c_1 + 4\left(\frac{1}{2}\right) \rightarrow c_1 = -2$ and

$$Y = -2x + \frac{1}{2}x^3 .$$

47.) $Y = (c_1 + c_2 x) e^{\frac{2}{3}x} \rightarrow \xrightarrow{D}$

$Y' = (c_1 + c_2 x) e^{\frac{2}{3}x} \cdot \frac{2}{3} + c_2 e^{\frac{2}{3}x} = \left(\frac{2}{3}c_1 + c_2 + \frac{2}{3}c_2 x\right) e^{\frac{2}{3}x} \xrightarrow{D}$

$Y'' = \left(\frac{2}{3}c_1 + c_2 + \frac{2}{3}c_2 x\right) e^{\frac{2}{3}x} \cdot \frac{2}{3} + \frac{2}{3}c_2 e^{\frac{2}{3}x}$

$= \left(\frac{4}{9}c_1 + \frac{4}{3}c_2 + \frac{4}{9}c_2 x\right) e^{\frac{2}{3}x}$ then

$9Y'' - 12Y' + 4Y = 9\left(\frac{4}{9}c_1 + \frac{4}{3}c_2 + \frac{4}{9}c_2 x\right) e^{\frac{2}{3}x}$

$- 12\left(\frac{2}{3}c_1 + c_2 + \frac{2}{3}c_2 x\right) e^{\frac{2}{3}x} + 4(c_1 + c_2 x) e^{\frac{2}{3}x}$

53.) $y' = 3x^2 \rightarrow y = x^3 + c$.

54.) $y' = \frac{1}{1+x} \rightarrow y = \ln|1+x| + c$.

56.) $Y' = \frac{x-2}{x} = 1 - \frac{2}{x} \rightarrow Y = x - 2\ln|x| + c$

57.) $Y' = \frac{1}{x^2-1} \rightarrow Y = \int \frac{1}{x^2-1}\,dx$

$= \int \frac{1}{(x-1)(x+1)}\,dx = \int \left[\frac{A}{x-1} + \frac{B}{x+1}\right] dx = \cdots$

$= \int \left[\frac{\frac{1}{2}}{x-1} + \frac{\frac{-1}{2}}{x+1}\right] dx = \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + c$ -

59.) $Y' = x\sqrt{x-3} \rightarrow Y = \int x(x-3)^{\frac{1}{2}}\,dx$

(Let $u = x-3$, $x = u+3$, and $du = 1 \cdot dx$)

$= \int (u+3) \cdot u^{\frac{1}{2}}\,du = \int \left[u^{3/2} + 3u^{\frac{1}{2}}\right] du$

$$= \frac{2}{5} u^{5/2} + 3 \cdot \frac{2}{3} u^{3/2} + c$$

$$= \frac{2}{5} (x-3)^{5/2} + 2 (x-3)^{3/2} + c$$

60.) $y' = xe^x \rightarrow y = \int xe^x \, dx$

(Let $u = x$, $dv = e^x \, dx$
$du = dx$, $v = e^x$)

$$= xe^x - \int e^x \, dx = xe^x - e^x + c$$

61.) $y^2 = cx^3$ and $x = 4, y = 4 \longrightarrow$
$(4)^2 = c(4)^3 \longrightarrow c = 1/4$ so part. sol. is
$y^2 = \frac{1}{4} x^3$.

63.) $y = ce^x$ and $x = 0, y = 3 \longrightarrow$
$3 = ce^0 = c \cdot 1 = c$ so part. sol. is
$y = 3e^x$.

66.) $A = Ce^{kt}$ and $t = 0$ yrs., $A = \$1000 \rightarrow$
$1000 = Ce^{k(0)} = Ce^0 = C \cdot 1 = C \rightarrow$
$A = 1000 e^{kt}$ and $t = 10$ yrs, $A = \$3320.12 \rightarrow$
$3320.12 = 1000 e^{10k} \rightarrow 3.32012 = e^{10k} \rightarrow$
$\ln 3.32012 = \ln e^{10k} \longrightarrow$
$\ln 3.32012 = 10k \rightarrow k = \frac{1}{10} \ln 3.32012$
so part. sol. is

$$A = 1000 \, e^{\left(\frac{1}{10} \ln 3.32012\right) t}.$$