Section C.3

1.) $x^3 - 2x^2y' + 3y = 0 \rightarrow -2x^2y' + 3y = -x^3 \rightarrow$
$y' + \frac{3}{-2x^2}\, y = \frac{-x^3}{-2x^2} \rightarrow y' + \frac{-3}{2x^2} \cdot y = \frac{x}{2}$

2.) $y' - 5(2x - y) = 0 \rightarrow y' - 10x + 5y = 0 \rightarrow$
$y' + 5 \cdot y = 10x$

3.) $xy' + y = xe^x \rightarrow y' + \frac{1}{x} \cdot y = \frac{xe^x}{x} \rightarrow$
$y' + \frac{1}{x} \cdot y = e^x$

4.) $xy' + y = x^3y \rightarrow xy' + y - x^3y = 0 \rightarrow$
$xy' + (1 - x^3)y = 0 \rightarrow y' + \frac{1-x^3}{x} \cdot y = 0$

5.) $y + 1 = (x-1)y' \rightarrow 0 = (x-1)y' - y - 1 \rightarrow$
$(x-1)y' - y = 1 \rightarrow y' + \frac{-1}{x-1} \cdot y = \frac{1}{x-1}$

6.) $x = x^2(y' + y) \rightarrow y' + y = \frac{x}{x^2} \rightarrow$
$y' + (1) \cdot y = \frac{1}{x}$

7.) $y' + 3 \cdot y = 6 \rightarrow \mu = e^{\int 3\, dx} = e^{3x} \rightarrow$
$e^{3x}y' + 3e^{3x} \cdot y = 6e^{3x} \rightarrow D(e^{3x}y) = 6e^{3x} \rightarrow$
$e^{3x}y = \int 6e^{3x}\, dx = 6 \cdot \frac{1}{3}e^{3x} + c \rightarrow$
$\boxed{y = 2 + ce^{-3x}}$

8.) $y' + 5 \cdot y = 15 \rightarrow \mu = e^{\int 5\,dx} = e^{5x} \rightarrow$

$e^{5x}y' + 5e^{5x}y = 15e^{5x} \rightarrow D(e^{5x}y) = 15e^{5x} \rightarrow$

$e^{5x}y = \int 15e^{5x}\,dx = 15 \cdot \frac{1}{5}e^{5x} + c \rightarrow$

$\boxed{y = 3 + ce^{-5x}}$

9.) $y' + (1) \cdot y = e^{-x} \rightarrow \mu = e^{\int 1\,dx} = e^{x} \rightarrow$

$e^{x}y' + e^{x}y = e^{x}e^{-x} = e^{0} = 1 \rightarrow D(e^{x}y) = 1 \rightarrow$

$e^{x}y = \int 1\,dx = x + c \rightarrow y = xe^{-x} + ce^{-x}$

10.) $y' + 3 \cdot y = e^{-3x} \rightarrow \mu = e^{\int 3\,dx} = e^{3x} \rightarrow$

$e^{3x}y' + 3e^{3x}y = e^{3x}e^{-3x} = e^{0} = 1 \rightarrow D(e^{3x}y) = 1 \rightarrow$

$e^{3x}y = \int 1\,dx = x + c \rightarrow \boxed{y = xe^{-3x} + ce^{-3x}}$

11.) $y' + \frac{1}{x} \cdot y = 3x + 4 \rightarrow \mu = e^{\int \frac{1}{x}\,dx} = e^{\ln x} = x \rightarrow$

$xy' + y = 3x^2 + 4x \rightarrow D(xy) = 3x^2 + 4x \rightarrow$

$xy = \int (3x^2 + 4x)\,dx = x^3 + 2x^2 + c \rightarrow$

$\boxed{y = x^2 + 2x + c/x}$

12.) $y' + \frac{2}{x} \cdot y = 3x + 1 \rightarrow \mu = e^{\int \frac{2}{x}\,dx} = e^{2\ln x} = e^{\ln x^2} = x^2$

$\rightarrow x^2y' + 2xy = 3x^3 + x^2 \rightarrow D(x^2y) = 3x^3 + x^2 \rightarrow$

$x^2y = \int (3x^3 + x^2)\,dx = \frac{3}{4}x^4 + \frac{1}{3}x^3 + c \rightarrow$

$\boxed{y = \frac{3}{4}x^2 + \frac{1}{3}x + \frac{c}{x^2}}$

13.) $y' + 5x \cdot y = x \rightarrow \mu = e^{\int 5x\,dx} = e^{\frac{5}{2}x^2} \rightarrow$

$e^{\frac{5}{2}x^2} y' + 5x e^{\frac{5}{2}x^2} y = x e^{\frac{5}{2}x^2} \rightarrow$

$D\left(e^{\frac{5}{2}x^2} y\right) = x e^{\frac{5}{2}x^2} \rightarrow e^{\frac{5}{2}x^2} y = \int x e^{\frac{5}{2}x^2} dx$

$= \frac{1}{5} e^{\frac{5}{2}x^2} + c \rightarrow y = \frac{1}{5} + c e^{-\frac{5}{2}x^2}$.

14.) $y' + 5 \cdot y = e^{5x} \rightarrow \mu = e^{\int 5\,dx} = e^{5x} \rightarrow$

$e^{5x} y' + 5 e^{5x} y = e^{5x} e^{5x} = e^{10x} \rightarrow D(e^{5x} y) = e^{10x} \rightarrow$

$e^{5x} y = \int e^{10x} dx = \frac{1}{10} e^{10x} + c \rightarrow \boxed{y = \frac{1}{10} e^{5x} + c e^{-5x}}$

15.) $y' + \frac{1}{x-1} \cdot y = \frac{x^2 - 1}{x-1} = x + 1 \rightarrow$

$\mu = e^{\int \frac{1}{x-1} dx} = e^{\ln(x-1)} = x - 1 \rightarrow$

$(x-1) y' + y = x^2 - 1 \rightarrow D\left((x-1) y\right) = x^2 - 1 \rightarrow$

$(x-1) y = \int (x^2 - 1)\,dx = \frac{1}{3} x^3 - x + c \rightarrow$

$y = \frac{1}{3} x (x+1) + \frac{c}{x-1}$

16.) $x y' + y = x^2 + 1 \rightarrow y' + \frac{1}{x} \cdot y = x + \frac{1}{x} \rightarrow$

$\mu = e^{\int \frac{1}{x} dx} = e^{\ln x} = x \rightarrow x y' + y = x^2 + 1 \rightarrow$

$D(xy) = x^2 + 1 \rightarrow xy = \int (x^2 + 1)\,dx = \frac{x^3}{3} + x + c$

$\rightarrow y = \frac{x^2}{3} + 1 + \frac{c}{x}$

17.) $x^3 y' + 2y = e^{1/x^2} \rightarrow$

$$y' + \frac{2}{x^3}\cdot y = \frac{1}{x^3} e^{\frac{1}{x^2}} \rightarrow \mu = e^{\int \frac{2}{x^3}dx} = e^{\frac{-1}{x^2}} \rightarrow$$

$$e^{\frac{-1}{x^2}} y' + \frac{2}{x^3} e^{\frac{-1}{x^2}} y = \frac{1}{x^3} \rightarrow D\left(e^{\frac{-1}{x^2}} y\right) = \frac{1}{x^3} \rightarrow$$

$$e^{\frac{-1}{x^2}} y = \int \frac{1}{x^3}dx = \frac{-1}{2x^2} + c \rightarrow \boxed{y = \left(\frac{-1}{2x^2} + c\right) e^{1/x^2}}$$

18.) $xy' + y = x^2 \ln x \rightarrow y' + \left(\frac{1}{x}\right)\cdot y = x \ln x \rightarrow$

$$\mu = e^{\int \frac{1}{x}dx} = e^{\ln x} = x \rightarrow xy' + y = x^2 \ln x \rightarrow$$

$$D(xy) = x^2 \ln x \rightarrow xy = \int x^2 \ln x \, dx$$

(Let $u = \ln x$, $dv = x^2 dx$

$du = \frac{1}{x}dx$, $v = \frac{1}{3}x^3$)

$$\rightarrow xy = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx = \frac{1}{3}x^3 \ln x - \frac{1}{3}\cdot\frac{1}{3}x^3 + c \rightarrow$$

$$y = \frac{1}{3}x^2 \ln x - \frac{1}{9}x^2 + \frac{c}{x}$$

$$y' + y = 4 \rightarrow \mu = e^{\int 1 dx} = e^x \rightarrow$$

$$e^x y' + e^x y = 4e^x \rightarrow$$

$$D(e^x \cdot y) = 4e^x \rightarrow e^x \cdot y = \int 4e^x dx \rightarrow$$

$$e^x y = 4e^x + c \rightarrow y = 4 + ce^{-x}$$

19.) $y' + y = 4 \rightarrow$ (two ways)

<u>1st Order</u>: $\mu = e^{\int 1 dx} = e^x \rightarrow$

$$e^x y' + e^x y = 4e^x \rightarrow D(e^x y) = 4e^x \rightarrow$$

$e^x y = \int 4e^x dx = 4e^x + c \rightarrow \boxed{y = 4 + ce^{-x}}$

OR

<u>Separable</u>: $y' + y = 4 \rightarrow y' = 4 - y \rightarrow$

$\frac{dy}{dx} = 4 - y \rightarrow \int \frac{1}{4-y}\, dy = \int 1\, dx \rightarrow$

$-\ln|4-y| = x + c \rightarrow \ln|4-y| = -x - c \rightarrow$

$|4-y| = e^{-x-c} = e^{-c}e^{-x} = c_1 e^{-x}$ where

$c_1 > 0 \rightarrow 4 - y = c_2 e^{-x}$ where c_2 is $\pm$

$\rightarrow \boxed{y = 4 - c_2 e^{-x}}$.

22.) $y' + 4x \cdot y = x \rightarrow$ (two ways)

<u>1st Order</u>: $\mu = e^{\int 4x\, dx} = e^{2x^2} \rightarrow$

$e^{2x^2} y' + 4x e^{2x^2} y = x e^{2x^2} \rightarrow$

$D(e^{2x^2} y) = x e^{2x^2} \rightarrow e^{2x^2} y = \int x e^{2x^2} dx$

$= \frac{1}{4} e^{2x^2} + c \rightarrow \boxed{y = \frac{1}{4} + ce^{-2x^2}}$

OR

<u>Separable</u>: $y' + 4xy = x \rightarrow$

$y' = x - 4xy = (1 - 4y)x \rightarrow$

$\int \frac{1}{1-4y}\, dy = \int x\, dx \rightarrow \frac{-1}{4}\ln|1-4y| = \frac{x^2}{2} + c$

$\rightarrow \ln|1-4y| = -2x^2 - 4c \rightarrow$

$|1-4y| = e^{-2x^2 - 4c} = e^{-2x^2} \cdot e^{-4c}$

$= c_1 e^{-2x^2} \rightarrow 1 - 4y = c_2 e^{-2x^2} \rightarrow$

$1 - c_2 e^{-2x^2} = 4y \rightarrow \boxed{y = \frac{1}{4} + c_3 e^{-2x^2}}$.

28.) $Y' + (2)Y = e^{-2x} \rightarrow$

$\mu = e^{\int 2\,dx} = e^{2x} \rightarrow e^{2x}Y' + 2e^{2x}Y = e^{2x}e^{-2x} \rightarrow$

$D\left(e^{2x}Y\right) = e^0 = 1 \rightarrow e^{2x}Y = \int 1\,dx \rightarrow$

$e^{2x}Y = x + C \rightarrow \underline{Y = xe^{-2x} + Ce^{-2x}}$;

31.) $Y' + (3x^2)Y = 3x^2 \rightarrow \mu = e^{\int 3x^2\,dx} = e^{x^3} \rightarrow$

$e^{x^3} \cdot Y' + 3x^2 e^{x^3} \cdot Y = 3x^2 e^{x^3} \rightarrow D\left(e^{x^3}Y\right) = 3x^2 e^{x^3} \rightarrow$

$e^{x^3}Y = \int 3x^2 e^{x^3}\,dx = e^{x^3} + C \rightarrow$

$\underline{Y = 1 + Ce^{-x^3}}$; if $x=0$, $Y=6$ then

$6 = 1 + C \rightarrow C = 5 \rightarrow$ $Y = 1 + 5e^{-x^3}$.

34.) $x^2Y' - 4xY = 10 \rightarrow Y' + \left(\frac{-4}{x}\right)Y = \frac{10}{x^2} \rightarrow$

$\mu = e^{\int \frac{-4}{x}\,dx} = e^{-4\ln x} = e^{\ln x^{-4}} = x^{-4} \rightarrow$

$x^{-4}Y' - 4x^{-5}Y = 10x^{-6} \rightarrow D\left(x^{-4}Y\right) = 10x^{-6} \rightarrow$

$x^{-4}Y = \int 10x^{-6}\,dx = -2x^{-5} + C \rightarrow \underline{Y = -2x^{-1} + Cx^4}$;

if $x=1$, $Y=10$ then $10 = -2 + C \rightarrow C = 12 \rightarrow$

$Y = -2x^{-1} + 12x^4$.

35.) $\frac{dS}{dt} = 20 - 0.2S + 0.2t \rightarrow$

$\frac{dS}{dt} + 0.2S = 20 + 0.2t \rightarrow e^{\int 0.2\,dt} = e^{0.2t} \rightarrow$

$e^{0.2t}\frac{dS}{dt} + 0.2e^{0.2t}S = (20+0.2t)e^{0.2t} \rightarrow$

$D\left(e^{0.2t}S\right) = 20e^{0.2t} + 0.2te^{0.2t} \rightarrow$

$e^{0.2t}S = \int\left(20e^{0.2t} + 0.2te^{0.2t}\right)dt$

$= 20\cdot\frac{1}{0.2}e^{0.2t} + 0.2\int te^{0.2t}\,dt$

(Let $u = t$, $dV = e^{0.2t}\,dt$
$du = 1\,dt$, $V = \frac{1}{0.2}e^{0.2t}$)

$= 100e^{0.2t} + 0.2\left[\frac{1}{0.2}te^{0.2t} - \frac{1}{0.2}\int e^{0.2t}\,dt\right]$

$= 100e^{0.2t} + te^{0.2t} - \frac{1}{0.2}e^{0.2t} + c \rightarrow$

$S = 100 + t - 5 + ce^{-0.2t} \rightarrow$

$S = 95 + t + ce^{-0.2t}$;

$t=0, S=0 \rightarrow 0 = 95 + c \rightarrow c = -95 \rightarrow$

$\boxed{S = 95 + t - 95e^{-.2t}}$

t :	0	1	2	3	4	5	6	7	8	9	10
S :	0	18.2	33.3	45.9	56.3	65.1	72.4	78.6	83.8	88.3	92.1

44.) $\frac{dy}{dt} = \frac{1-y}{4} = \frac{1}{4} - \frac{1}{4}y \rightarrow$

$y' + \frac{1}{4}\cdot y = \frac{1}{4} \rightarrow \mu = e^{\int \frac{1}{4}\,dt} = e^{\frac{1}{4}t} \rightarrow$

$e^{\frac{1}{4}t}y' + \frac{1}{4}e^{\frac{1}{4}t}y = \frac{1}{4}e^{\frac{1}{4}t} \rightarrow D(e^{\frac{1}{4}t}y) = \frac{1}{4}e^{\frac{1}{4}t}$

$\rightarrow e^{\frac{1}{4}t}y = \int \frac{1}{4}e^{\frac{1}{4}t}\,dt = e^{\frac{1}{4}t} + c \rightarrow$

$\underline{y = 1 + ce^{-\frac{1}{4}t}}$;

a.) $t=0,\ y=0 \rightarrow 0 = 1 + ce^{0} = 1 + c \rightarrow$

$c = -1 \rightarrow$

$y = 1 - e^{-\frac{1}{4}t}$

b.) If $y = 50\% = 0.5$, then

$0.5 = 1 - e^{-\frac{1}{4}t} \rightarrow e^{-\frac{1}{4}t} = \frac{1}{2} \rightarrow$

$\ln e^{-\frac{1}{4}t} = \ln(1/2) \rightarrow -\frac{1}{4}t = \ln(1/2) \rightarrow$

$t = -4\ln(1/2) \approx$ 2.77 yrs.

c.) If $t = 4$ yrs, then

$y = 1 - e^{-\frac{1}{4}(4)} \approx 0.6321 =$ 63.21 %