# Section 7.3

2.) $f(x,y) = 4 - x^2 - 4y^2$

a.) $f(0,0) = 4 - 0 - 0 = 4$

b.) $f(0,1) = 4 - 0 - 4 = 0$

c.) $f(2,3) = 4 - 4 - 36 = -36$

d.) $f(1,y) = 4 - 1 - 4y^2 = 3 - 4y^2$

e.) $f(x,0) = 4 - x^2 - 0 = 4 - x^2$

f.) $f(t,1) = 4 - t^2 - 4 = -t^2$

3.) $f(x,y) = xe^y$

a.) $f(5,0) = 5e^0 = 5$

b.) $f(3,2) = 3e^2$

c.) $f(2,-1) = 2e^{-1}$

d.) $f(5,y) = 5e^y$

e.) $f(x,2) = xe^2$

f.) $f(t,t) = te^t$

6.) $f(x,y,z) = \sqrt{x+y+z}$

a.) $f(0,5,4) = \sqrt{9} = 3$

b.) $f(6,8,-3) = \sqrt{11}$

11.) $f(x,y) = \int_x^y (2t-3)\,dt = (t^2 - 3t)\Big|_x^y$

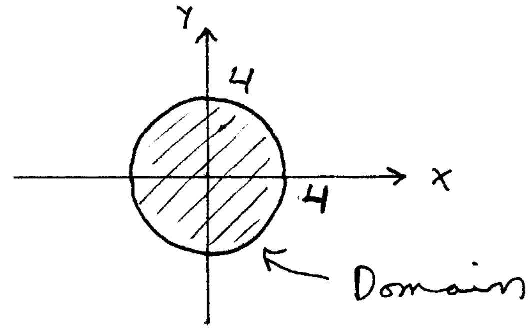
$= (y^2 - 3y) - (x^2 - 3x)$

$= y^2 - 3y - x^2 + 3x$

a.) $f(1,2) = 4 - 6 - 1 + 3 = 0$

b.) $f(1,4) = 16 - 12 - 1 + 3 = 6$

15.) $f(x,y) = \sqrt{16 - x^2 - y^2}$ , $16 - x^2 - y^2 \geq 0 \rightarrow$




$x^2 + y^2 \leq 16$ so

<u>Domain</u> : all $(x, y)$ satisfying $x^2 + y^2 \leq 4^2$ (circle and interior)

<u>Range</u>: all values $z \geq 0$ satisfying $z = \sqrt{4 - x^2 - y^2} \rightarrow z^2 = 16 - x^2 - y^2 \rightarrow$ $x^2 + y^2 + z^2 = 4^2$ ( top $\frac{1}{2}$ of sphere), so range is all $z$-values satisfying $0 \leq z \leq 4$ .

16.) $f(x,y) = x^2 + y^2 - 1$ ,

<u>Domain</u>: all points $(x, y)$ ;

<u>Range</u> : $z = x^2 + y^2 - 1 = (x^2 + y^2) - 1$ ; since $x^2 + y^2 \geq 0$ , the range is all values $z \geq -1$ .

17.) $f(x,y) = e^{\frac{x}{y}}$ , $y \neq 0$ so

<u>Domain</u>: all $(x, y)$ with $y \neq 0$ (everything

in 2-space except the x-axis);

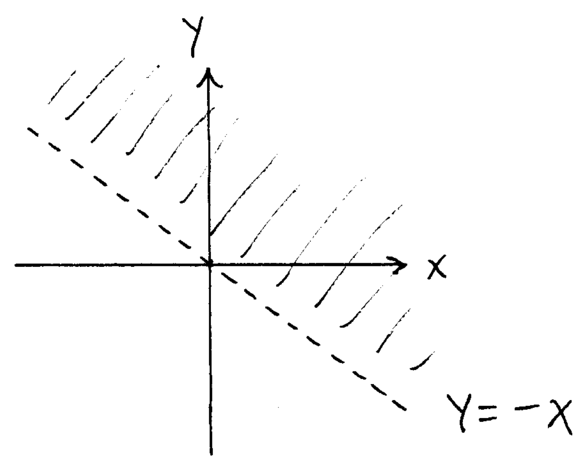
Range: $z = e^{x/y} > 0$ and for fixed $y\ (>0)$

$\lim_{x \to +\infty} e^{\frac{x}{y}} = +\infty$ so range is all

values $z > 0$. and $\lim_{x \to -\infty} e^{\frac{x}{y}} = 0$.

18.) $f(x,y) = \ln(x+y)$, $x + y > 0 \to y > -x$ so

Domain: all $(x,y)$ with $y > -x$ (above line $y = -x$)

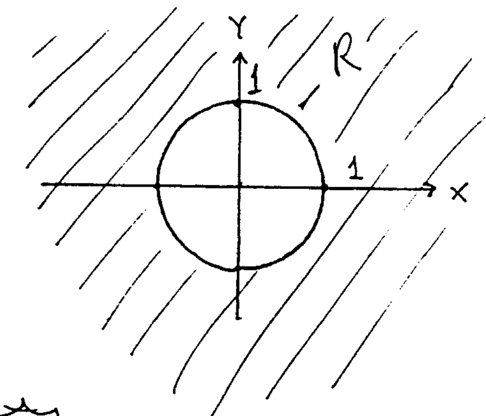



Range: If we let $y = 0$,

then $z = \ln(x+y) = \ln(x+0) = \ln x$ assumes all real values, so range is all real numbers $z$.

20.) $f(x,y) = \sqrt{x^2+y^2-1}$ then

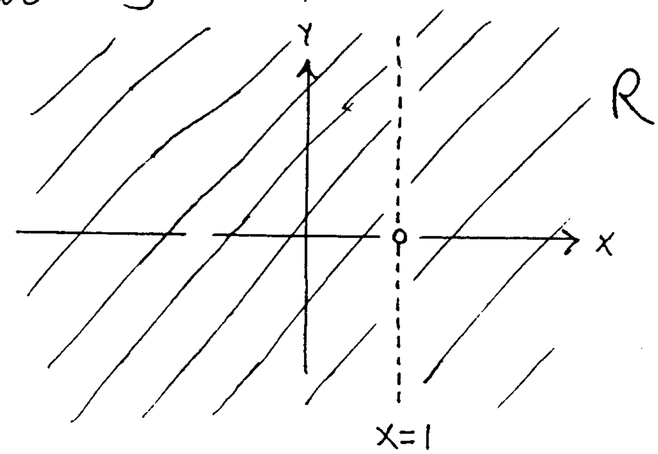
$x^2+y^2-1 \geq 0 \rightarrow x^2+y^2 \geq 1$ (a circle and its exterior) so domain is all points $(x,y)$ satisfying $x^2+y^2 \geq 1$ :




Range : all $z \geq 0$ .

22.) $f(x,y) = \dfrac{4y}{x-1}$ then

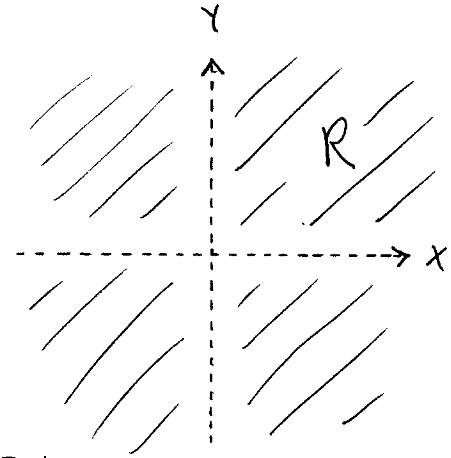
$x \neq 1$ so domain is all points $(x,y)$ satisfying $x \neq 1$ :



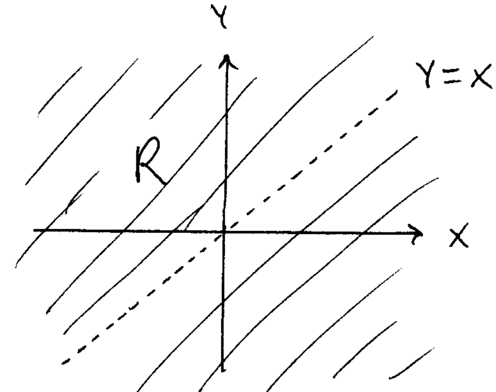

(domain is all points $(x,y)$ not lying on the line $x=1$ );

Range : all values of $z$ .

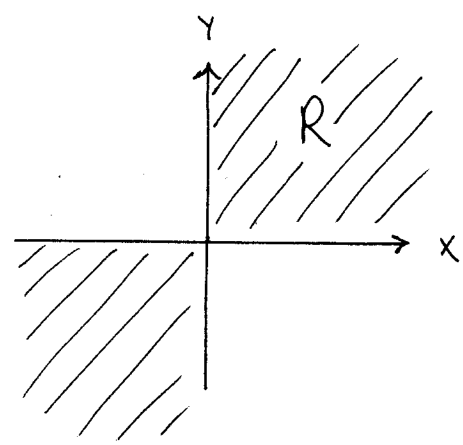
23.) $f(x,y) = \frac{1}{xy}$ then $x \neq 0$ or $y \neq 0$ so domain is all points $(x,y)$ with $x \neq 0$ or $y \neq 0$ : (domain is all points $(x,y)$ not lying on x-axis or y-axis); Range: all $z \neq 0$.




24.) $g(x,y) = \frac{1}{x-y}$ then $x - y \neq 0$ or $y \neq x$ so domain is all points $(x,y)$ with $y \neq x$ : (domain is all points $(x,y)$ not lying on line $y = x$); Range: all $z \neq 0$.



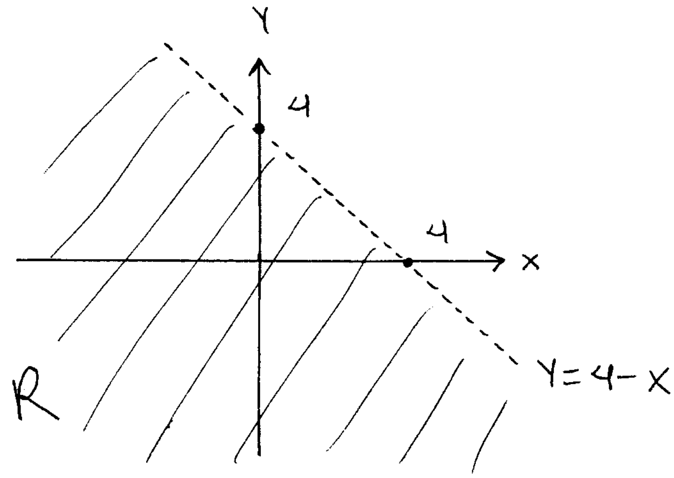

26.) $f(x,y) = \sqrt{xy}$ then $xy \geq 0$ so $x \geq 0$ and $y \geq 0$ OR $x \leq 0$ and $y \leq 0$ so domain is all points $(x,y)$ on x-axis or y-axis or in 1st or 3rd quadrants; Range: all $z \geq 0$.



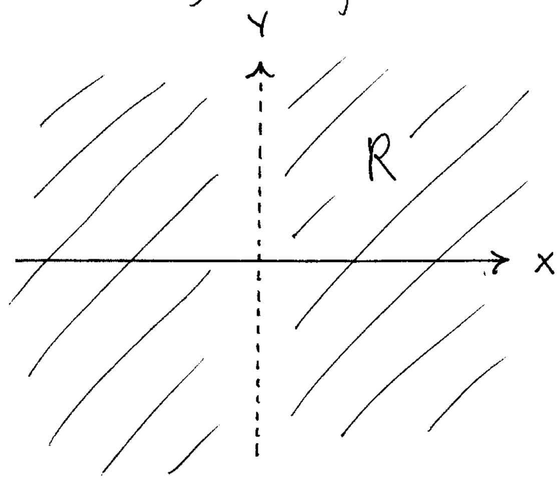

27.) $g(x,y) = \ln(4-x-y)$ then $4 - x - y > 0 \longrightarrow 4 - x > y$ so domain is all points $(x,y)$ satisfying $y < 4 - x$ : (domain is all points $(x,y)$ lying below

the line
$Y = 4 - X$ )

<u>Range</u>: all
values of $Z$.




28.) $f(x,y) = y e^{\frac{1}{x}}$ then $x \neq 0$ so
domain is all points $(x,y)$ with $x \neq 0$ :
(domain is all points $(x,y)$ not lying
on the $Y$-axis ) ;




<u>Range</u>: all values of $Z$.

48.) $$W(x,y) = \frac{1}{x-y}$$

a.) $W(15,10) = \frac{1}{5}$ hr. $=$ 12 min.

b.) $W(12,9) = \frac{1}{3}$ hr. $=$ 20 min.

c.) $W(12,6) = \frac{1}{6}$ hr. $=$ 10 min.

d.) $W(4,2) = \frac{1}{2}$ hr. $=$ 30 min.