

Math 21A  
Vogler  
Discussion Sheet 3

1.) Evaluate the following limits.

a.)  $\lim_{x \rightarrow 4} \frac{\sqrt{5+x} - 3}{\sqrt{x} - 2}$

b.)  $\lim_{x \rightarrow \infty} (x - x \cos(4/x))$

c.)  $\lim_{x \rightarrow 0} \frac{2x}{\sin x - x}$  ( HINT :  $\frac{\sin x}{x} \leq 1$  . )

2.) Use one-sided limits and limits to infinity to find all vertical and horizontal asymptotes for the following functions. Use the asymptotes together with x- and y-intercepts to sketch graphs of each function.

a.)  $y = \frac{x-2}{x^2-9}$

b.)  $y = \frac{x+2}{x^2(x+1)}$

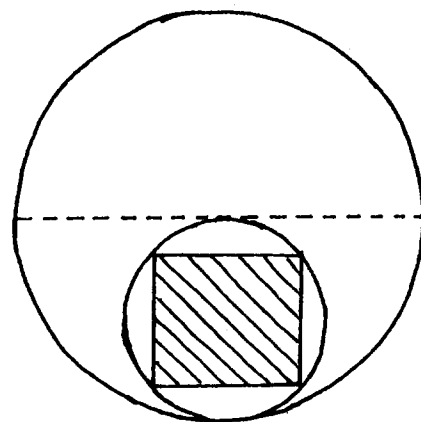
3.) Give an  $\epsilon, \delta$  -proof for  $\lim_{x \rightarrow 1} \frac{x+3}{1+\sqrt{x}} = 2$  .

4.) Use the IMVT to determine if the following equation is solvable. This is a writing exercise :  $x^3 + x - \sqrt{x+4} = 0$  .

5.) In the given diagram the smaller circle is the largest one that can be inscribed in the given semi-circle. If the larger circle has circumference  $4\pi$  in., what is the area of the inscribed shaded square ?

6.) Determine the x-values for which the following function is continuous. It is not necessary to graph the function :

$$f(x) = \begin{cases} \frac{\sin 3x}{x}, & \text{if } x < 0 \\ 3.01, & \text{if } x = 0 \\ \frac{x-1}{\sqrt{x}-1}, & \text{if } 0 < x < 1 \\ 2, & \text{if } x \geq 1. \end{cases}$$



7.) Use the limit definition of derivative,

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ , to differentiate each of the following functions.

a.)  $f(x) = \cos x$

b.)  $f(x) = \frac{7+x}{3x-5}$

c.)  $f(x) = \sqrt{x^2+x}$

8.) Let  $f(x) = \begin{cases} \sin 2x, & \text{if } x \geq 0 \\ 2x, & \text{if } x < 0 \end{cases}$ . Use the limit definition of derivative to determine  $f'(0)$

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The following problem is for recreational purposes only.

9.) Without lifting your pencil, join all sixteen dots with six straight lines.

